Key
Form 1

Birzeit University
Mathematics Department
Math 234

First Exam

First Semester 2019/2020

Student Name: .... Number:.................Section: . . . .
circle your section number
Dr. Mohammad Saleh: section (1) & (4)
Dr. Khaled Altakhman: section (2)
Dr. Ala' Talahma: section (3)
Dr. Hasan Yousef: section (5)

Question 1 (2 points each). Mark each of the following statements by True or false

(1) (...T...) If A is a 4×4-matrix, $b = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$, and the system $Ax = b$ has a unique solution, then A is nonsingular

(2) (..F..) Let A, B are $n \times n$-matrices with $AB = 0$, if $B \neq 0$, then A is nonsingular.

(3) (...T..) If A is an $n \times n$-matrix and $A^3 = I$, then $\det(A) = 1$

(4) (...T..) If U is the reduced row echelon form of an $n \times n$ nonsingular matrix, then $U = I_n$.

(5) (...T..) If the matrix B is obtained from A by multiplying a row of A by 4, where A, B are 3×3 matrices, then $\det(B) = 4\det(A)$.

(6) (..T...) Let A be an $n \times n$-matrix such that $A^T = A^{-1}$, then $\det(A) = \pm 1$

(7) (..T...) Let A be a 3×4 matrix, and let B be a 4×4 matrix which has a column of zeros, then AB has a column of zeros.

(8) (..T...) If A is a 3×3-matrix and $A \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, then A is singular.

(9) (.F....) If A is a singular 3×3-matrix, then the reduced row echelon form of A has 2 rows of zeros.

(10) (.F....) If x_1, x_2 are solutions to $Ax = b$, then $x_1 + x_2$ is a solution of the system $Ax = b$.

(11) F(..~~T~~..) If x_1, x_2 are solutions to $Ax = b$, then $\frac{1}{3}x_1 + \frac{3}{4}x_2$ is a solution of the system $Ax = b$.

(12) (...T..) If x_1, x_2 are solutions to $Ax = b$, then $x_1 - x_2$ is a solution of the system $Ax = 0$.

(13) (...T..) Any two $n \times n$-nonsingular matrices are row equivalent.

(14) (...T..) The adjoint of the matrix $\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$ is $\begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}$

(15) (..T...) If A and B are $n \times n$ matrices such that $Ax = Bx$ for some nonzero $x \in \mathbb{R}^n$. Then $A - B$ is singular.

(16) (..F...) If A is row equivalent to B, then $\det(A) = \det(B)$.

(17) (..T...) If A is an $n \times n$-matrix, then AA^T is symmetric.

(18) (..T...) If A is a nonsingular matrix, then A can be written as a product of elementary matrices.

(19) (..F...) If A is an $n \times n$-matrix with positive entries, then $\det(A) \geq 0$.

(20) (..F...) Let A be a square nonsingular $n \times n$ matrix. If $|adj A| = |A|$ then A is a 2×2-matrix.

(21) (..F...) In the linear system $Ax = 0$, if $a_1 = a_2 + 3a_4$ then $x = \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}$ is a solution to $Ax = 0$.

Question 2 (2 points each). Circle the most correct answer

(1) Let U be an $n \times n$-matrix in reduced row echelon form and $U \neq I$, then

(a) $\det(U) = 1$

(b) U is the zero matrix

(c) The system $Ux = 0$ has only the zero solution

(d) U has a row consisting entirely of zeros

(2) If A, B are 2×2-matrices, $|A| = -2$ and $|B| = 4$, then $|-3A^{-1}B^T| =$

(a) 18

(b) -18

(c) 6

(d) -72

(3) If A is a singular matrix and U is the row echelon form of A, then $\det(U) =$.

(a) 0

(b) 1

(c) ± 1

(d) none of the above

(4) If $A = LU$ is the LU-factorization of a matrix A, and A is singular, then .

(a) L and U are both nonsingular

(b) L is singular and U is nonsigular

(c) U is singular and L is nonsigular

(d) L and U are both singular

(5) If A is singular and B is nonsingular $n \times n$-matrices, then AB is

(a) singular

(b) nonsingular

(c) may or may not be singular

(d) none of the above

(6) If A is a 3×3-matrix and the system $Ax = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}$ has a unique solution, then the system

$$Ax = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

(a) is inconsistent

(b) has infinitely many solutions

(c) has only the zero solution.

(d) none of the above

(7) If A is a 3×5 matrix, then the system $Ax = 0$

(a) has no nonzero solution.

(b) has only the zero solution

(c) has infinitely many solutions

(d) is inconsistent

(8) If A is a symmetric $n \times n$-matrix and P any $n \times n$-matrix, then $P^T AP$ is

(a) not defined

(b) not symmetric

(c) symmetric

(d) none of the above

(9) If A is $n \times n$-nonsingular matrix, then

(a) the system $Ax = 0$ has nonzero solutions

(b) the reduced row echelon form of A has a row of zeros

(c) $\det(A) = 0$

(d) A is a product of elementary matrices.

(10) If A is an $n \times n$ matrix and the system $Ax = b$ has infinitely many solutions, then

(a) A is nonsingular

(b) A singular

(c) A is symmetric

(d) A has a row of zeros

(11) Let $A = \begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{pmatrix}$ and $b = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$. The system $Ax = b$

(a) has infinitely many solutions

(b) has a unique solution

(c) has exactly three solutions.

(d) is inconsistent

(12) If E is an elementary matrix of type III, then E^T is

(a) an elementary matrix of type III

(b) an elementary matrix of type II

(c) an elementary matrix of type I

(d) not an elementary matrix

(13) Let A be a 4×3-matrix with $a_2 = a_3$. If $b = a_1 + a_2 + a_3$, where a_j is the jth column of A, then a solution to the system $Ax = b$ is

(a) $x = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

(b) $x = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

(c) $x = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

(d) $x = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

(14) If A is a nonsingular and symmetric matrix, then

(a) A^{-1} is singular and symmetric

(b) A^{-1} is nonsingular and symmetric

(c) A^{-1} is nonsingular and not symmetric

(d) A^{-1} is singular and not symmetric

(15) If $A = \begin{pmatrix} 1 & -2 & 5 \\ 4 & -5 & 8 \\ -3 & 3 & a \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$, then the system $Ax = b$ has infinitely many solutions if and only if

(a) $b_3 = b_1 - b_2$ and $a = -3$

(b) $a \neq -3$

(c) $b_3 = b_1 - b_2$ or $a = -3$

(d) $a = -3$

(16) If B is a 3×3 matrix such that $B^2 = B$. One of the following is always true

(a) $B^5 = B$.

(b) $B = I$.

(c) B is nonsingular.

(d) $\det(B) = 0$.

(17) If $AB = AC$, and $|A| \neq 0$, then

(a) $B = C$.
(b) $B \neq C$
(c) $A = C$
(d) none of the above

(18) If A and B are two nonsingular $n \times n$-matrices, then

(a) The system $(AB)x = 0$ has a nontrivial (nonzero) solution.
(b) $\det(A) = \det(B)$
(c) There is a nonsingular matrix C such that $A = CB$
(d) There is a singular matrix C such that $A = CB$.

(19) In the square linear system $AX = b$, if A is singular and b is a linear combination of the columns of A then the system has

(a) no solution
(b) infinitely many solutions
(c) a unique solution
(d) can not tell

(20) If $AB = 0$, where A and B are $n \times n$ matrices. Then

(a) both A, B are singular.
(b) both A, B are nonsingular.
(c) either A or B is singular
(d) either $A = 0$ or $B = 0$

(21) Let $A = \begin{pmatrix} 2 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & a \end{pmatrix}$. Then the values of a that make A singular are (is)

(a) 2
(b) 0
(c) 1
(d) -1

(22) Let $(1, 2, 0)^T$ and $(2, 1, 1)^T$ be the first two columns of a 3×3 matrix A and $(1, 1, 1)^T$ be a solution of the system $Ax = (4, 2, 5)^T$. Then the third column of the matrix A is

(a) $(1, -1, -4)^T$.
(b) $(4, -1, 1)^T$.
(c) $(1, -1, 4)^T$.
(d) $(1, 1, 4)^T$.

Question 3 (10 points). (a) If $(5A^T)^{-1} = \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$, then $A = \begin{pmatrix} -\frac{2}{5} & 1 \\ -\frac{1}{5} & \frac{3}{5} \end{pmatrix}$

(b) Let A be a nonsingular $n \times n$-matrix. Show that $(A^T)^{-1} = (A^{-1})^T$.

consider $A^T (A^{-1})^T = (A^{-1} A)^T$

$= I^T = I.$

similarly, $(A^{-1})^T A^T = I.$

so $(A^T)^{-1} = (A^{-1})^T$

Question 4 (10 points). Let $A = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 4 & 5 \\ 0 & 3 & 2 \end{pmatrix}$, then

(a) $\operatorname{adj}(A) =$

$$\operatorname{adj}(A) = \begin{pmatrix} -7 & 8 & -13 \\ -2 & 6 & -13 \\ 3 & -9 & 13 \end{pmatrix}$$

$A_{11} = \begin{vmatrix} 4 & 5 \\ 3 & 2 \end{vmatrix} = -7$

$A_{12} = -\begin{vmatrix} 1 & 5 \\ 0 & 2 \end{vmatrix} = -2$

$A_{13} = \begin{vmatrix} 1 & 4 \\ 0 & 3 \end{vmatrix} = 3$

$A_{21} = -\begin{vmatrix} -1 & 2 \\ 3 & 2 \end{vmatrix} = 8$

$A_{22} = \begin{vmatrix} 3 & 2 \\ 0 & 2 \end{vmatrix} = 6$

$A_{23} = -\begin{vmatrix} 3 & -1 \\ 0 & 3 \end{vmatrix} = -9$

$A_{31} = \begin{vmatrix} -1 & 2 \\ 4 & 5 \end{vmatrix} = -13$

$A_{32} = -\begin{vmatrix} 3 & 2 \\ 1 & 5 \end{vmatrix} = -13$

$A_{33} = \begin{vmatrix} 3 & -1 \\ 1 & 4 \end{vmatrix} = 13$

(b) $\det(A) = -13$

(c) $A^{-1} = \frac{-1}{13} \begin{pmatrix} -7 & 8 & -13 \\ -2 & 6 & -13 \\ 3 & -9 & 13 \end{pmatrix}$