Birzeit University
Mathematics Department
Math 234

Second Exam Exam — First Semester 2019/2020

Student Name: .... Number:..................Section: . . . .
circle your section number
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Question 1 (2.5 points each). Mark each of the following statements by True or false

(1) (..T...) If $v_1, v_2, \cdots, v_k$ are vectors in a vector space V, and $\text{Span}(v_1, v_2, \cdots, v_k) = \text{Span}(v_1, v_2, \cdots, v_{k-1})$, then $v_1, v_2, \cdots, v_k$ are linearly dependent.

(2) (..T...) If A is an $m \times n$-matrix, then $\text{rank}(A) = \text{rank}(A^T)$.

(3) (..T...) If A, B are row equivalnt matrices, then $R(A) = R(B)$ (A, B have the same raw space).

(4) (..F...) The set $S = \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2 : x_1 x_2 = 0 \right\}$ is a subspace of $\mathbb{R}^2$

(5) (..T...) If A is a 6×4-matrix, $\text{rank}(A) = 4$, b is in the column space of A, then the system $Ax = b$ has exactly one solution.

(6) (...T...) If A is 3×5-matrix, $\text{rank}(A) = 3$, then the system $Ax = b$ has infinitely many solutions for every $b \in \mathbb{R}^3$.

(7) (...T...) If A is a 5×4-matrix, and $Ax = 0$ has only the zero solution, then $\text{rank}(A) = 4$.

(8) (..T...) $S = \{A \in \mathbb{R}^{3\times3} : A \text{ is upper triangular}\}$ is a subspace of $\mathbb{R}^{3\times3}$

(9) (...T...) If S is a subspace of a vector space V, then $0 \in S$

(10) Let A be a 2×4 matrix, and $\text{rank}(A) = 2$. Mark each of the following by true or false

- (..T...) The columns of A form a spanning set for $\mathbb{R}^2$.
- (..T...) The rows of A are linearly independent.
- (..F...) The system $Ax = 0$ has only the zero solution.
- (..T...) The columns of A are linearly dependent.
- (..T...) The system $Ax = b$ has infinitely many solutions for every $b \in \mathbb{R}^2$
- (...F...) $\text{nullity}(A) = 0$

(11) (..T...) If A is an $m \times n$-matrix, $m \neq n$, then either the rows or the columns of A are linearly dependent

Question 2 (2.5 points each). Circle the most correct answer

(1) If A is a 4×4-matrix, and $Ax = 0$ has only the zero solution, then rank$(A) =$

(a) 1
(b) 2
(c) 3
(d) 4

(2) If $f_1, f_2, \cdots, f_n \in C^{n-1}[a, b]$ and $W[f_1, f_2, \cdots, f_n](x) = 0$ for all $x \in [a, b]$, then $f_1, f_2, \cdots, f_n$ are

(a) linearly independent.
(b) linearly dependent
(c) form a spanning set for $C^{n-1}[a, b]$
(d) none of the above

(3) If $T_{n \times n}$ is a transition matrix between two bases for a vector space V, $\dim(V) = n > 0$, then

(a) T is nonsingular
(b) $\det(T) = 1$
(c) rank$(T) = 1$
(d) nullity$(T) = n$

(4) If the columns of $A_{n \times n}$ are linearly independent and $b \in \mathbb{R}^n$, then the system $Ax = b$ has

(a) no solution
(b) exactly one solution
(c) infinitely many solutions
(d) none of the above

(5) if $\{v_1, v_2, \cdots, v_k\}$ is a spanning set for $\mathbb{R}^{2\times 3}$, then

(a) $k = 6$
(b) $k \geq 6$
(c) $k \leq 6$
(d) $k > 6$

(6) If A is an $m \times n$ matrix, then

(a) rank$(A) \leq m$
(b) rank$(A) \leq n$
(c) rank$(A) \leq \min\{m, n\}$
(d) rank$(A) = m = n$

(7) If A is an $m \times n$-matrix, and columns of A are linearly ~~dependent~~ independent, then

(a) $m \leq n$

(b) $n \leq m$

(c) $m = n$

(d) $m = n + 1$

(8) If A is a 3×5-matrix, rows of A are linearly independent, then

(a) $\text{rank}(A) = \text{nullity}(A)$

(b) $\text{rank}(A) = \text{nullity}(A) + 1$

(c) $\text{rank}(A) = \text{nullity}(A) + 2$

(d) $\text{rank}(A) = \text{nullity}(A) + 3$

(9) Let A be a 4×3 matrix, and $\text{rank}(A) = 3$

(a) The columns of A are linearly independent

(b) $\text{nullity}(A) = 0$

(c) The rows of A are linearly dependent

(d) All of the above

(10) If $\{v_1, v_2, v_3, v_4\}$ forms a spanning set for a vector space V, $\dim(V) = 3$, v_4 can be written as a linear combination of v_1, v_2, v_3, then

(a) v_1 can be written as a linear combination of v_2, v_3, v_4

(b) $\{v_1, v_2, v_3\}$ do not form a spanning set for V

(c) $\{v_1, v_2, v_3\}$ are linearly dependent

(d) $\{v_1, v_2, v_3\}$ is a basis for V

(11) The dimension of the space $S = \text{Span}\left\{ A_1 = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}, A_2 \begin{pmatrix} 1 & 0 \\ 3 & -1 \end{pmatrix}, A_3 = \begin{pmatrix} -1 & -3 \\ 6 & -8 \end{pmatrix} \right\}$ is

(a) 0

(b) 1

(c) 2

(d) 3

(12) The rank of $A = \begin{pmatrix} 1 & 4 & 1 & 2 & 1 \\ 2 & 6 & -1 & 2 & -1 \\ 3 & 10 & 0 & 4 & 0 \end{pmatrix}$ is

(a) 2

(b) 3

(c) 0

(d) 1

(13) The transition matrix from the ordered basis $U = \left[u_1 = \begin{pmatrix}1\\2\end{pmatrix}, u_2 = \begin{pmatrix}3\\7\end{pmatrix}\right]$ to the standard basis $S = \left[e_1 = \begin{pmatrix}1\\0\end{pmatrix}, e_2 = \begin{pmatrix}0\\1\end{pmatrix}\right]$ is

(a) $T = \begin{pmatrix}7 & -3\\-2 & 1\end{pmatrix}$

(b) $T = \begin{pmatrix}1 & 3\\2 & 7\end{pmatrix}$

(c) $T = \begin{pmatrix}-7 & 3\\2 & -1\end{pmatrix}$

(d) $T = \begin{pmatrix}1 & -3\\-2 & 7\end{pmatrix}$

(14) Let $E = [2 + x, 1 - x, x^2 + 1]$ be an ordered basis for P_3. If $p(x) = 3x^2 + 5x + 4$, then the coordinate vector of $p(x)$ with respect to E is

(a) $\begin{pmatrix}3\\5\\4\end{pmatrix}$

(b) $\begin{pmatrix}2\\-3\\3\end{pmatrix}$

(c) $\begin{pmatrix}3\\-3\\2\end{pmatrix}$

(d) $\begin{pmatrix}3\\2\\-3\end{pmatrix}$

(15) If A is a 3×4 matrix, then

(a) The columns of A are linearly independent

(b) $Rank(A) = 3$

(c) nullity$(A) \geq 1$

(d) The rows of A are linearly dependent

(16) Let $S = \{\begin{pmatrix}a+b\\a+b\\a+b\end{pmatrix} : a, b \in \mathbb{R}\}$. Then dimension of S equals

(a) 1

(b) 2

(c) 3

(d) 0

(17) If A is a 4×3 matrix such that $N(A) = \{0\}$, and b can be written as a linear combination of the columns of A, then

(a) The system $Ax = b$ has exactly one solution

(b) The system $Ax = b$ is inconsistent

(c) The system $Ax = b$ has infinitely many solutions

(d) None of the above

(18) If A is an $n \times n$-matrix and for each $b \in \mathbb{R}^n$ the system $Ax = b$ has a unique solution, then

(a) A is nonsingular

(b) $\text{rank}(A) = n$

(c) $\text{nullity}(A) = 0$

(d) all of the above

(19) Which of the following is not a basis for the corresponding space

(a) $\{5 - x, x\}$; P_2

(b) $\{(1, -1)^T, (2, -3)^T\}$; $\mathbb{R}^2$

(c) $\{x, 1 - x, 2x + 3\}$; P_3

(d) $\{(1, -1, -1)^T, (2, -3, 0)^T, (-1, 0, 2)^T\}$; $\mathbb{R}^3$

(20) If $\{v_1, v_2, v_3, v_4\}$ is a basis for a vector space V , then the set $\{v_1, v_2, v_3\}$ is

(a) linearly independent and not a spanning set for V.

(b) linearly dependent and not a spanning set for V.

(c) linearly independent and a spanning set for V.

(d) none of the above

Question 3 (10 points). (a) If $U = \left[u_1 = \begin{pmatrix} a \\ b \end{pmatrix}, u_2 = \begin{pmatrix} c \\ d \end{pmatrix}\right]$, $V = \left[v_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, v_2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}\right]$ are ordered bases for $\mathbb{R}^2$, and the transition matrix from U to V is $T = \begin{pmatrix} 3 & 5 \\ 1 & -2 \end{pmatrix}$. Find u_1, u_2

$$T_{U\to V} = V^{-1}U \Longrightarrow VT = U$$

$$\Longrightarrow U = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 3 & 5 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 5 & 1 \\ 9 & 4 \end{pmatrix}$$

$$\Longrightarrow u_1 = \begin{pmatrix} 5 \\ 9 \end{pmatrix}, \quad u_2 = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

(b) If $S = \{p(x) \in P_3 : p(1) = 0\}$, show that is a subspace of P_3

1) $S \neq \phi$ since $0 \in S$

2) let $p(x), q(x) \in S \Longrightarrow p(1) = 0$ and $q(1) = 0$

Now $(p+q)(1) = p(1) + q(1) = 0 + 0 = 0$

so $p + q \in S$.

3) Let $p(x) \in S$, α scalar $\Longrightarrow p(1) = 0$

$\Longrightarrow (\alpha p)(1) = \alpha p(1) = \alpha(0) = 0$

so $(\alpha p)(x) \in S$.

By (1), (2) & (3) $\Longrightarrow S$ is a subspace of P_3.

Question 4 (6 points). If $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & 0 & 0 \\ -2 & -2 & 0 & 0 \end{pmatrix}$ and the row echelon form of A is

$$U = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

(a) Find a basis for the row space of A.

basis for $R(A)$ is $\{(1\ 1\ 1\ 1), (0\ 0\ 1\ 1)\}$

(b) Find a basis for the column space of A.

basis for $C(A)$ is $\left\{ \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\}$

(c) Find a basis for null space A : solutions of $Ax=0$

$x_2 = \alpha,\ x_4 = \beta$ free variables

$x_3 = -\beta$

$x_1 = -\alpha + \beta - \beta = -\alpha$

$$\therefore N(A) = \left\{ \begin{pmatrix} -\alpha \\ \alpha \\ -\beta \\ \beta \end{pmatrix} : \alpha, \beta \text{ scalars} \right\} = \left\{ \alpha \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix} : \alpha, \beta \text{ scalars} \right\}$$

so a basis for $N(A)$ is $\left\{ \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix} \right\}$

(d) Find Rank(A), Nullity(A).

Rank$(A) = 2$.

nullity$(A) = 2$.