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Birzeit University
Mathematics Department
Math 234-1

Second Exam

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Q1: (69 points)

(1) Let V be a vector space. Mark each of the following statements by true or false.

(a) For any $v \in V$, $-v \in V$ T
(b) For any $v, w \in V$, $v \cdot w \in V$ F
(c) For any $v \in V$, $2v \in V$ T
(d) For any $v \in V$, $0v \in R$ F
(e) V could be equal ϕ F

(2) Let V be a vector space, v_1, v_2, v_3, v_4 span V. Mark each of the following statements by true or false.

(a) $dim(V) = 4$ F
(b) $dim(V) \geq 4$ F
(c) $dim(V) \leq 4$ T
(d) any set of more than 5 vector in V are linearly dependent T
(e) Any basis of V has exactly 4 vectors T

(3) Let V be a vector space, $dim(V) = 5$. Mark each of the following statements by true or false.

(a) If v_1, v_2, v_3, v_4, v_5 in V, then v_1, v_2, v_3, v_4, v_5 is a basis for V T
(b) If v_1, v_2 in V, then v_1, v_2 are linearly independent T
(c) If v_1, v_2 in V, then v_1, v_2 can't span V T
(d) If v_1, v_2, v_3, v_4, v_5 in V, and $v \in V$, then $v \in Span(v_1, v_2, v_3, v_4, v_5)$ T
(e) If B is a basis for V, and $v \in V$, then $[v]_B \in R^5$ T

(4) Let A be 3×3 matrix such that $|A| = 0$. Mark each of the following statements by true or false.

(a) The columns of A are linearly dependent T
(b) The rows of A are linearly dependent T
(c) A is invertible F
(d) $N(A) = \{0\}$ F
(e) $A = 0$ F

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$\{1,2\}$ $(2,4)$ $1,0$ $1,3,5$ $2,4,6$

(5) Let A, B be subspaces of a vector space V. Mark each of the following statements by true or false.

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(a) $A \bigcap B$ is a subspace of V T

(b) $A \bigcup B$ is a subspace of V F

(c) $A + B = \{x + y; x \in A, y \in B\}$ is a subspace of V ~~F~~ T

(d) $2A = \{2x : x \in A\}$ is a subspace of V T

(e) $A \bigcap B \neq \phi$ T

(6) Let $V = P_3$. Mark each of the following statements by true or false.

$$\begin{vmatrix} 0 & -1 & -2 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

(a) $x, x-1, x-2$ span V F

(b) $x, x-1, x^2-2$ span V T

(c) $x, x-1, x-2, x^2$ is linearly dependent T

(d) x, x^2-2 does not span V T

(e) $0, x-1, x^2-2$ is linearly dependent T

(7) Let $A_{3\times 4}$ matrix. Mark each of the following statements by true or false.

(a) The columns of A are linearly dependent ~~F~~ T

(b) The rows of A are linearly dependent F

(c) The system $Ax = b$ has infinite solutions for any $b \in R^3$ F

(d) The system $Ax = 0$ has infinite solutions T

(e) $N(A) \neq \{0\}$ T

(f) If $(0,1,2,-1)^t \in N(A)$ and $(1,0,2,1)^t$ a solution of $Ax = b$ then $(1,1,4,0)^t$ is a solution of $Ax = b$ T

(8) Let V be an infinite dimensional vector space. Mark each of the following statements by true or false.

(a) Any subspace of V is infinite dimensional F

(b) Any finite set of vectors of V are linearly independent F

(c) Any finite set of vectors of V can't span V T

(d) A subspace of V could be finite dimensional T

(e) If A, B are subspaces of V, then $A \bigcap B \neq \{0\}$ F

Let A, B be $n \times n$ nonzero matrices such that $AB = 0$. Mark each of the following statements by true or false.

(9) (a) $Ax = 0$ has a nonzero solution F

(b) $span(b_1, b_2, ..., b_n) \subseteq N(A)$ T

(c) $span(a_1, a_2, ..., a_n) \subseteq N(B)$ T

(d) A is singular F

(e) The columns of A are linearly dependent F

Q3: (10 points)

Let $V = P_3$ and let $S = \{f \in V : f(0) = 0, f(1) = 0\}$.

(a) Show that S is a subspace of V $f(0)=0$ $f(1)=0$

$(f+g)(0) = f(0) + g(0) = 0$
$(f+g)(1) = f(1) + g(1) = 0$ the addition codition

$\alpha f(0) = \alpha f(0) = 0 = f(0)$, $\alpha f(1) = 0 = f(1)$
$\alpha g(0) = 0$, $\alpha g(1) = 0$
the multiplication is done

there is a zero vector and the set is not empty so it subspace

(b) Find a basis for S

$P_x = ax^2 + bx + c$

$f(0) = 0$ $c = 0$

$f(1) = 0$ $a + b + c = 0$ $a = -b$

$S = ax^2 - ax + 0$

$a(x^2 - x)$

$S = \{x^2 - x\}$

span and linearly independins

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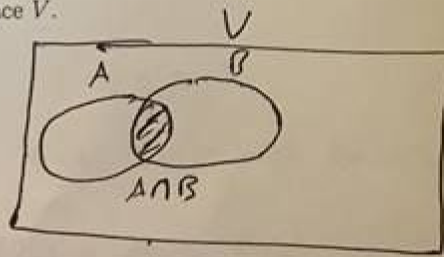
so $S = \{x^2 - x\}$ $\dim(V) = 1$

Q2: **(15 points)** Let A, B be subspaces of a vector space V.

(a) Show that $A \bigcap B$ is a subspace of V



$f(x) = x_1$ $\quad g(x) = x_1$

$(f+g)(x_5) = f(x) + g(x)$

$f(x) + g(x) \in A \cap B$ under addition

$\alpha f(x) = \alpha x_1$ and $\alpha g(x) = \alpha x_1 \in A \cap B$

under multiplication

~~f(0) = 0~~ let $x_1 = 0$ is closed under addition and multiplication

$f(x) = 0 \quad g(x) = 0$

so there is a $\vec{0} \in A \cap B$

(b) Show that $A \bigcup B$ is not always a subspace of V

$f(x) = x_1 \quad g(x) = x_1$

$(f+g)(x) = f(x) + g(x)$ this may be $\in A \cup B$

or $\notin A \cup B$ out the rejion of $A \cup B$

so it not subspace

Give example.

(c) Let A be $m \times n$ matrix. Show that $N(A)$ is a subspace of R^n.

$A\vec{x} = 0 \qquad \vec{x} = (\vec{x}_1, \vec{x}_2, \vec{x}_3, \ldots \vec{x}_n) \in R^n$

$N(A) = (\vec{x}_1, \vec{x}_2, \vec{x}_3, \ldots \vec{x}_n) \in R^n$

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this closed under addition and multiplication

$(N(A), +, \cdot) \in R^n$ so its a subspace

Q4: **(10 points)** Let $V = P_2$, $B = [1-x, 2+x]$, $F = [1+2x, 2-3x]$

(a) Find the transition matrix S from B into F

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 2 \\ 2 & -3 & -1 & 1 \end{array}\right] \simeq \left[\begin{array}{cc|cc} 1 & 2 & 1 & 2 \\ 0 & -7 & -3 & -3 \end{array}\right]$$

$$= \left[\begin{array}{cc|cc} 1 & 2 & 1 & 2 \\ 0 & 1 & \frac{3}{7} & \frac{3}{7} \end{array}\right] \simeq \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{7} & \frac{8}{7} \\ 0 & 1 & \frac{3}{7} & \frac{3}{7} \end{array}\right]$$

$\frac{14}{7} - \frac{6}{7} = \frac{8}{7}$

$$T = \cancel{\begin{bmatrix} -\frac{1}{5} & \frac{4}{5} \\ \frac{3}{5} & \frac{3}{5} \end{bmatrix}} \quad T = \begin{bmatrix} \frac{1}{7} & \frac{8}{7} \\ \frac{3}{7} & \frac{3}{7} \end{bmatrix}$$

(b) Let $v \in V$, where $[v]_B = (2,5)^t$. Use the transition matrix S to find $[v]_F$

$$[v]_F = T \cdot [v_B] = \begin{bmatrix} \frac{1}{7} & \frac{8}{7} \\ \frac{3}{7} & \frac{3}{7} \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$[v]_c = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

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