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Birzeit University
Department of Mathematics
Math 243

Second hour exam
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Section#...12:30 ⇒ 2:00
Spring 2014
Number: 1120959

Questio#1(30%) Prove or disprove each of the following statements

a)If $A \neq \emptyset$ and $A \times B = A \times C$ then $B=C$ T

⊆) suppose $A \neq \emptyset$ and $A \times B = A \times C$ and $y \in B$

$\Rightarrow \exists\, x \in A$ [since $A \neq \emptyset$]

$\Rightarrow (x,y) \in A \times B$

$\Rightarrow (x,y) \in A \times C$ [since $A \times B \subseteq A \times C$]

$\Rightarrow y \in C \Rightarrow B \subseteq C$

⊇) suppose $A \neq \emptyset$ and $A \times B = A \times C$ and $z \in C$

$\Rightarrow \exists\, x \in A$

$\Rightarrow (x,z) \in A \times C$

$\Rightarrow (x,z) \in A \times B$ [since $A \times C \subseteq A \times B$] $\Rightarrow z \in B$

$\Rightarrow C \subseteq B$

b)If f and g are functions then $f \cup g$ is a function F

Counter example

let $f = \{(1,2),(3,4)\}$ be a function.

$g = \{(1,3),(5,4)\}$ be a function.

but $f \cup g = \{(1,2),(3,4),(1,3),(5,4)\}$

Not function

because $[1] = \{2,3\}$

c)If R and S are equivalence relations on A then $R \cup S$ is an equivalence relations on A F

let $X = \{1,2,3\}$ and R, S define on $X \times X$

$R = \{(1,1),(2,2),(3,3),(1,2),(2,1)\}$ equivelence

$S = \{(1,1),(2,2),(3,3),(2,3),(3,2)\}$ equivelenc

$R \cup S = \{(1,1),(2,2),(3,3),(1,2),(2,1),(2,3),(3,2)\}$

Not equivelence because Not transitive

$(1,2) \wedge (2,3) \in R \cup S$

but $(1,3) \notin R \cup S$.

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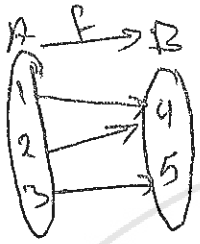
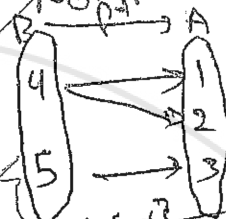
d)If f is a function then f^{-1} is a function F

Counter example let $f: A \to B$ if $A = \{1,2,3\}$, $B = \{4,5\}$ $f = \{(1,4),(2,4),(3,5)\}$ $f^{-1} = \{(4,1),(4,2),(5,3)\}$ not function.



be a function but f^{-1} not function



becaus $f^{-1}(4) = \{1,2\}$

e) If R,S are transitive then $S \circ R$ is transitive F

$X = \{1,2,3,4,5,6\}$

$R = \{(1,5),(2,6)\}$ $S = \{(5,2),(6,3)\}$

$S \circ R = \{(1,2),(2,3)\}$

R is transitive
S is transitive
but $S \circ R$ is not transitive because $(1,3) \notin S \circ R$

~~let $X = \{1,2,3\}$~~
~~R, S define on X×X~~
~~$R = \{(1,1),(2,2),(3,3),(1,2),(2,1)\}$ transitive~~
~~$S = \{(1,1),(2,2),(3,3),(1,3),(3,1)\}$ transitive~~
~~$R \circ S = \{(1,1),(2,2),(3,3),(1,2),(2,1),(1,3),(3,2)\}$~~
~~$S \circ R = \{(1,1),(2,2),(3,3),(1,3),(2,3),(2,1),(2,3)\}$~~

f) If R and S are symmetric relations the $(R \circ S)^{-1} = S \circ R$ T

(⊆) Suppose that R and S symmetric and $(x,y) \in (R \circ S)^{-1}$

$\Rightarrow (y,x) \in R \circ S$

$\Rightarrow \exists b \in B, (y,b) \in S, (b,x) \in R$

$\Rightarrow (b,y) \in S \wedge (x,b) \in R$ [since S,R is symmetric]

$\Rightarrow (x,b) \in R \wedge (b,y) \in S$

$\Rightarrow (x,y) \in S \circ R$

(2) suppose R, S symmetric and $(x,y) \in S \circ R$

$\Rightarrow \exists b \in B, (x,b) \in R \wedge (b,y) \in S$

$\Rightarrow (b,x) \in R \wedge (y,b) \in S$ [since R,S symmetric]

$\Rightarrow (y,b) \in S \wedge (b,x) \in R$

$\Rightarrow (y,x) \in R \circ S$

$\Rightarrow (x,y) \in (R \circ S)^{-1}$

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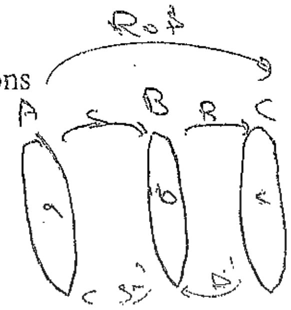
Question #2(20%) a)Prove tha if A,B,C be sets, and let $R \subseteq B\times C, S \subseteq A\times B$ be relations then $(R\circ S)^{-1} = S^{-1}\circ R^{-1}$



① $\subseteq$ suppose that $R \subseteq B\times C$, $S \subseteq A\times B$ are relations and $(c,a) \in (R\circ S)^{-1} \Rightarrow (a,c) \in R\circ S$

$\Rightarrow \exists b \in B ; (a,b) \in S \wedge (b,c) \in R$

$\Rightarrow \exists b \in B , (b,a) \in S^{-1} \wedge (c,b) \in R^{-1}$

$\Rightarrow \exists b \in B , (c,b) \in R^{-1} \wedge (b,a) \in S^{-1}$

$\Rightarrow (c,a) \in S^{-1}\circ R^{-1}$

② suppose that $R \subseteq B\times C$, $S \subseteq A\times B$ are relation. and $(c,a) \in S^{-1}\circ R^{-1}$

$\Rightarrow \exists b \in B , (a,b) \in S \wedge (b,c) \in R$

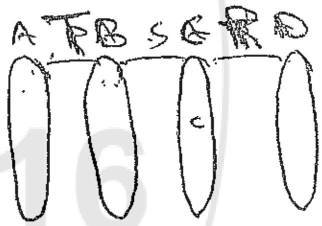
$\Rightarrow \exists b \in B , (b,a) \in S^{-1} \wedge (c,b) \in R^{-1}$

$\Rightarrow \exists b \in B , (a,b) \in R^{-1} \wedge (b,a) \in S^{-1}$

$\Rightarrow (c,a) \in S^{-1}\circ R^{-1}$ (since $(S^{-1}\circ R^{-1})^{-1} = R^{-1}\circ S^{-1^{-1}} = R\circ S$

$\Rightarrow (c,a) \in (R\circ S)^{-1}$

b)Prove tha if R,S,T be relations from A to A then $(R\circ S)\cup(R\circ T) \subseteq R\circ(S\cup T)$



suppose that R,S,T relation on A to A and $(a,d) \in (R\circ S)\cup(R\circ T)$

$\Rightarrow (a,d) \in R\circ S \vee (a,d) \in R\circ T$

$\Rightarrow \exists b \in C , (b,c) \in S \wedge (c,d) \in R \vee \exists b' \in B ; (a,b) \in T \wedge (b,d) \in R$

$\Rightarrow \exists b \in B , \exists c \in C ; ((b,c) \in S \wedge (c,d) \in R) \vee ((a,b) \in T \wedge (b,d) \in R)$

~~$\Rightarrow (a,b) \in$ ...~~

$\Rightarrow (a,c) \in S\cup T \wedge (c,d) \in R$

$\Rightarrow (a,d) \in R\circ(S\cup T)$

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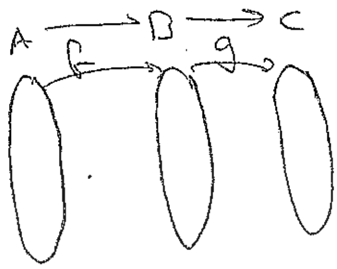
Question #3(20%) Let A, B, C be nonempty set and let $f: A \to B$, $g: B \to C$ be functions.



a) Show that if $g \circ f$ is one to one, then $f: A \to B$ is one to one.

~~suppose that $f: A \to B$, $g: B \to C$~~

suppose that $g \circ f$ is (1-1) and $f(x_1) = f(x_2)$

$\Rightarrow g(f(x_1)) = g(f(x_2))$

$\Rightarrow g \circ f(x_1) = g \circ f(x_2)$

$\Rightarrow x_1 = x_2$ [since $g \circ f$ is (1-1)]

b) Show that if $g \circ f$ is onto, then $g: B \to C$ is onto.

suppose that $g \circ f$ is onto ~~and~~

$\Rightarrow \exists a \in A\ g \circ f(a) = c$

$\Rightarrow \exists\, b \in B \ni (a,b) \in f \wedge (b,c) \in g$

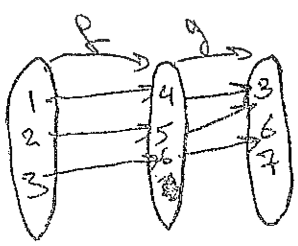
$\Rightarrow (b,c) \in g$

$\Rightarrow g(b) = c$

c) Show that the converse of (a) is not true

Converse if $f: A \to B$ is (1-1) then $g \circ f$ is (1-1)

counter example



f is (1-1) function

but $g \circ f$ is not one to one function

$x_1 = 1$

$x_2 = 2$

$g \circ f(1) = 3$ and $g \circ f(2) = 3$

so $g \circ f$ is not one to one

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Question #4(20%)

Let $X = \mathbb{R}\times\mathbb{R}$, For each real number b let $D_b = \{(x, y) \in X: y = x+b\}$

a) Is $\{D_b : b \in \mathbb{R}\}$ a partion of X? Prove your answer?

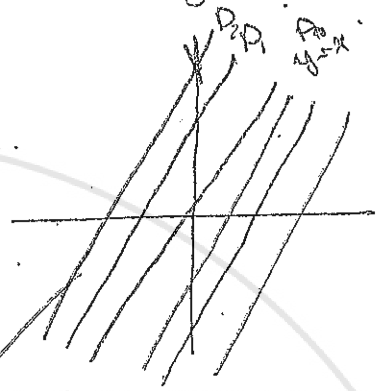
$D_1 = \{(x,y) \in X : y = x+1\}$ $\quad D_0 = \{(x,y) \in X : y = x\}$.

$D_2 = \{(2,y) \in X : y = x+2\}$

$D_3 = \{(3,y) \in X : y = x+3\}$

$\vdots$

$D_n = \{(n,y) \in X : y = x+n\}$



D_b is apartion becase

① $\{\} \notin D_b \; \forall b \in R$

② $\bigcup_{b\in R} D_b = R \times R \; ; \forall b \in R$

③ $D_b \cap D_c = \phi \; \forall b,c \in R$

~~③ $D_b \cap D_c = \phi \; \forall b,$~~

b)Define a relation R on X by (s,t)R(u,v) if and only if there is a real number b such that (s,t)and (u,v) both belongs to D_b, for some $b \in \mathbb{R}$

Is R Reflexive ? Symmetric? Transitive? Explane your answer

$D_b = \{(x,y) \in X : y = x+b\}$

① Reflixive (yes)

~~let~~ $(s,t), (s,t) \in D_b$ since $t = s+b \wedge t = s+b$

so $(s,t) R (s,t)$

② symtric (yes)

let $(s,t), (u,v) \in R$

$\Rightarrow t = s+b \wedge v = u+b$

$\Rightarrow v = u+b \wedge t = s+b$

$\Rightarrow (u,v), (s,t) \in R$

$\Rightarrow$ is symtric.

③ transitive (yes.

~~let (s,t),(u,v)~~

let $(s,t), (m,n) \in R \wedge (m,n),(k,L) \in R$

$\Rightarrow (t = s+b \wedge n = m+b) \wedge (n = m+b \wedge L = k+b)$

$\Rightarrow t = s+b \wedge n = m+b \wedge L = k+b$

$\Rightarrow t = s+b \wedge L = k+b$

$\Rightarrow (s,t), (k,L) \in R$

$\Rightarrow$ so is transitive

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Question #5(10%) a) Let $f(x)\ \dfrac{1}{\sqrt{x-1}}$, $g(x)=\sqrt{4-x}$. Find the domain of f , g , f ∘ g

① $f(x) = \dfrac{1}{\sqrt{x-1}}$

$\sqrt{x-1} = 0$ $\quad\quad$ $x-1 \geqslant 0$

$x-1=0$ $\quad\quad$ $x \geqslant 1$

$\boxed{x \neq 1}$ $\quad\quad$ but 1 is singular point.

Domain of f is ~~x~~ $x > 1 = (1, \infty)$

~~D(x) = { x ... x~~

② $g(x) = \sqrt{4-x}$

$4-x \geqslant 0$

$4 \geqslant x$

Domain of g is $x \leq 4 = (-\infty, 4]$

$f\circ g(x) = f(g(x)) = f(\sqrt{4-x})$ $\quad\quad$ Domain $f\circ g(x) = x < 3 = (-\infty, 3)$

$= \dfrac{1}{\sqrt{\sqrt{4-x}-1}}$

$\sqrt{4-x}-1 \geqslant 0$

$\sqrt{4-x} \geqslant 1$

$4-x \geqslant 1$

$-4+x \leq -1$

$x \leq +3$

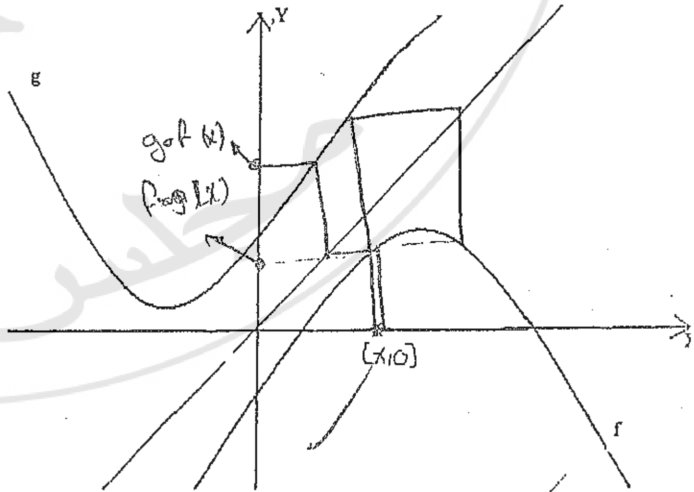
but $x \neq 3$

① $\sqrt{\sqrt{4-x}-1} = 0$

$-4+x = -1$

$x = +3$

b)Use the graph of the functions f , and g
to indicate on the graph
the value of (f ∘ g) (x) and (g ∘ f)(x)



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