

Mathematics Department
Math 234

Second Exam

Summer 2009/2010

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Question 1 (21 points). Mark each of the following by True or False

(1) (T) Let $S = \{v_1, v_2, \dots, v_r\}$ be a set of vectors in \mathbb{R}^n . If $r > n$, then S is linearly dependent.

(2) (F) If A is a nonzero 3×4 matrix such that $Ax = 0$ has only the zero solution, then $\text{rank}(A) = 4$.

(3) (F) A subset of an n -dimensional vector space that contains $n + 1$ elements is a spanning set.

(4) (F) Elementary row operations change the nullspace of a matrix.

(5) (E) If A is an $n \times n$ matrix, $\det(A) \neq 0$, then $\text{rank}(A) \neq n$.

(6) (T) If A is a 4×3 matrix with $\text{rank}(A) = 3$, then the homogeneous system $Ax = 0$ has a nontrivial solution.

(7) (F) If $L: \mathbb{P}_2 \rightarrow \mathbb{P}_2$ is the linear transformation defined by $L(ax^2 + bx + c) = (a - c)x + (b + c)$, then $x^2 - x + 1$ is in $\ker(L)$.

(8) (F) Let $L: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be defined by $L(x, y) = (x - y, 0, 2x + 3)$, then L is a linear transformation.

(9) (T) If U, V are subspaces of a vector space W , then $U \cap V$ is also a subspace of W .

(10) (F) If the columns of $A_{n \times n}$ are linearly dependent and $b \in \mathbb{R}^n$, then the system $Ax = b$ is inconsistent.

(11) (F) Let $L: V \rightarrow W$ be a linear transformation and $\{v_1, v_2, \dots, v_n\}$ be a basis for V , then $\{L(v_1), L(v_2), \dots, L(v_n)\}$ is a basis for W .

(12) (F) It is possible to find a matrix A of size 3×5 such that $\text{nullity}(A) = 1$.

(13) (T) If A, B are two equivalent $n \times n$ -matrices, then $\text{rank}(A) = \text{rank}(B)$.

(14) (T) $L: \mathbb{R}^{3 \times 3} \rightarrow \mathbb{R}^{3 \times 3}$, $L(A) = A + A^T$ is a linear transformation.

Question 2 (42 points). Circle the most correct answer

(1) Let A be an $m \times n$ matrix. If the rows of A span \mathbb{R}^n , then

- (a) $n \leq m$
(b) $m \leq n$
(c) $n = m$
(d) none

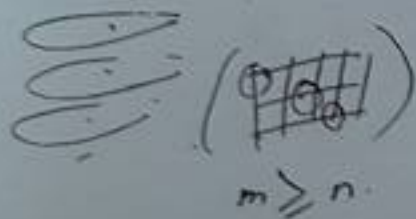
$n \leq m$

$L(x^2 - x + 1) = 0 + 2$

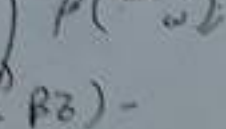
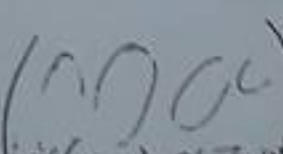
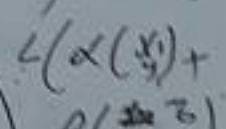
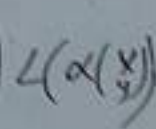
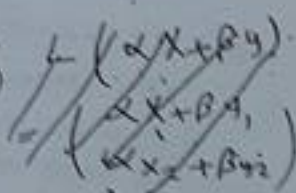
$a = 1$
 $b = -1$
 $c = 1$

$L(\alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}) = \alpha(x - y) + \beta(x + 2y) = \alpha(x - y) + \beta(x + 2y)$

$\alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = (\alpha y + \beta x), 0, 2(\alpha x + \beta y)$

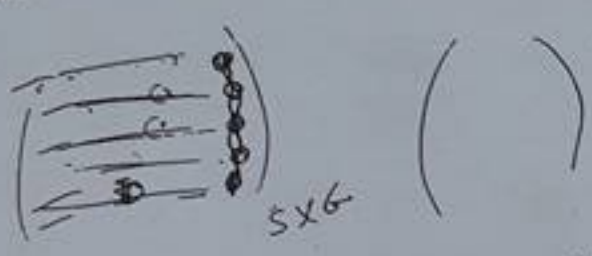


$m > n$



(2) Let A be a 5×6 -matrix such that $Ax = b$ is consistent for every $b \in \mathbb{R}^5$, then

- (a) Nullity(A) = 6
- (b) Nullity(A) = 0
- (c) rank(A) = 1
- (d) rank(A) = 5



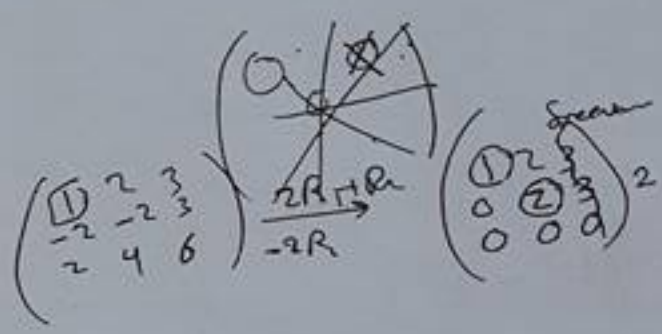
(3) If A is an $n \times n$ singular matrix, then

- (a) $N(A) = \{0\}$
- (b) The columns of A are linearly dependent
- (c) rank(A) = n
- (d) all of the above

$Ax = 0$ Consider

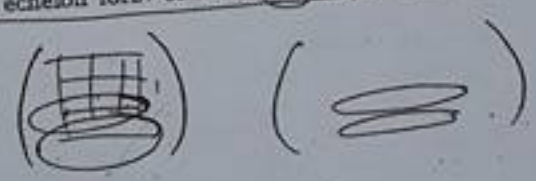
(4) $\dim(\text{span}(\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}, \begin{pmatrix} 3 \\ 6 \\ 6 \end{pmatrix}))$ is

- (a) 0
- (b) 1
- (c) 2
- (d) 3



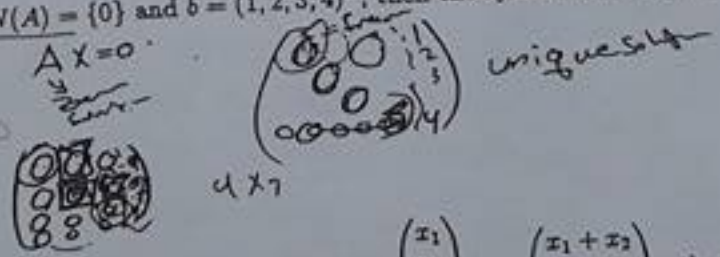
(5) Let A be a 4×7 -matrix, if the row echelon form of A has 2 nonzero rows, then $\dim(\text{column space of } A)$ is

- (a) 2
- (b) 3
- (c) 5
- (d) 6



(6) If A is a 4×3 matrix such that $N(A) = \{0\}$ and $b = (1, 2, 3, 4)^T$, then the system $Ax = b$ has

- (a) no solution
- (b) at most one solution
- (c) exactly one solution
- (d) infinitely many solutions

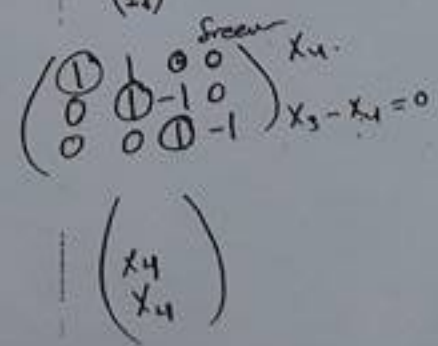


(7) Let $L: \mathbb{R}^4 \rightarrow \mathbb{R}^3$ be the linear transformation defined by $L \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 - x_3 \\ x_3 - x_4 \end{pmatrix}$, then

$\dim(\ker(L)) =$

- (a) 4
- (b) 3
- (c) 2
- (d) 1

$$\begin{aligned} x_1 + x_2 &= 0 \\ x_2 - x_3 &= 0 \\ x_3 - x_4 &= 0 \end{aligned}$$



$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$