Mathematics Department

Math 234

Second Exam — Summer 2009/2010

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Question 1 (21 points). Mark each of the following by True of False

(1) (T) Let $S = \{v_1, v_2, \cdots, v_r\}$ be a set of vectors in $\mathbf{R}^n$. If $r > n$, then S is linearly dependent.

(2) (F) If A is a nonzero 5×4 matrix such that $Ax = 0$ has only the zero solution, then $\operatorname{rank}(A) = 4$.

(3) (F) A subset of an n-dimensional vector space that contains $n+1$ elements is a spanning set.

(4) (F) Elementary row operations change the nullspace of a matrix.

(5) (F) If A is an $n \times n$ matrix, $\det(A) \neq 0$, then $\operatorname{rank}(A) \neq n$.

(6) (T) If A is a 4×3 matrix with $\operatorname{rank}(A) = 3$, then the homogeneous system $Ax = 0$ has a nontrivial solution.

(7) (T) If $L : P_2 \to P_1$ is the linear transformation defined by $L(ax^2+bx+c) = (a-c)x + (b+c)$, then $x^2 - x + 1$ is in $\ker(L)$.

(8) (F) Let $L : \mathbf{R}^2 \to \mathbf{R}^3$ be defined by $L(x, y) = (x - y, 0, 2x + 3)$, then L is a linear transformation.

(9) (T) If U, V are subspaces of a vector space W, then $U \cap V$ is also a subspace of W.

(10) (F) If the columns of $A_{n \times n}$ are linearly dependent and $b \in \mathbf{R}^n$, then the system $Ax = b$ is inconsistent.

(11) (F) Let $L : V \to W$ be a linear transformation and $\{v_1, v_2, \cdots, v_n\}$ be a basis for V, then $\{L(v_1), L(v_2), \cdots, L(v_n)\}$ is a basis for W.

(12) (F) It is possible to find a matrix A of size 3×5 such that $\operatorname{nullity}(A) = 1$

(13) (T) If A, B are two rwo equivalent $n \times n$-matrices, then $\operatorname{rank}(A) = \operatorname{rank}(B)$

(14) (T) $L : \mathbf{R}^{3\times 3} \to \mathbf{R}^{3\times 3}$, $L(A) = A + A^T$ is a linear transformation.

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Question 2 (42 points). Circle the most correct answer

(1) Let A be an $m \times n$ matrix. If the rows of A span $\mathbf{R}^n$, then

(a) $n \le m$

(b) $m \le n$

(c) $n = m$

(d) none

$n \le n$

$L(x^2 - x + 1) = 0 + 2$

$a = 1$

$b = -1$

$c = 1$

$m \ge n$

$L(\alpha(x_1, y_1) + \beta(z, w))$

$(\alpha y + \beta w), 0$

$2(\alpha x + \beta y)$

(2) Let A be a 5×6-matrix such that $Ax = b$ is consistent for every $b \in \mathbf{R}^5$, then

(a) Nullity$(A) = 6$

(b) Nullity$(A) = 0$

(c) rank$(A) = 1$

(d) rank$(A) = 5$

5x6

(3) If A is an $n \times n$ singular matrix, then

not invertible

(a) $N(A) = \{0\}$

(b) The columns of A are linearly dependent

(c) rank$(A) = n$

(d) all of the above

AX = 0 Consider

(4) dim(span($\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix}$)) is

$\begin{pmatrix} 1 & 2 & 3 \\ -2 & -2 & 3 \\ 2 & 4 & 6 \end{pmatrix} \xrightarrow[-2R]{2R_1+R_2} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 9 \\ 0 & 0 & 0 \end{pmatrix}$ 2

(a) 0

(b) 1

(c) 2

(d) 3

(5) let A be a 4×7-matrix, if the row echelon form of A has 2 nonzero rows, then dim(column space of A) is

(a) 2

(b) 3

(c) 5

(d) 6

(6) If A is a 4×3 matrix such that $N(A) = \{0\}$ and $b = (1, 2, 3, 4)^T$, then the system $Ax = b$ has

AX = 0

unique soln

(a) no solution

(b) at most one solution

(c) exactly one solution

(d) infinitely many solutions

4x7

(7) Let $L : \mathbf{R}^4 \to \mathbf{R}^3$ be the linear transformation defined by $L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 - x_3 \\ x_3 - x_4 \end{pmatrix}$, then

dim(ker(L)) =

$x_1 + x_2 = 0$

$x_2 - x_3 = 0$

$x_3 - x_4 = 0$

free x_4

$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$ $x_3 - x_4 = 0$

(a) 4

(b) 3

(c) 2

(d) 1

$\begin{pmatrix} x_4 \\ x_4 \end{pmatrix}$