Birzeit University
Mathematics Department
Math 234

Second Exam | Summer Semester 2008
Student Name: .... Number:..................Section: . . . .
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Question 1 (60 points). **True of False**

(1) (. . .) If A is a nonzero 3×2 matrix such that $Ax = 0$ has none zero solutions, then $\text{rank}(A) = 1$.

(2) Let A, B be two row equivalent matrices, Mark each of the following by true or false

- (. . .) Column space of A = Column space of B
- (. . .) Row space of A = Row space of B
- (. . .) Null space of A = Null space of B

(3) (. . .) If A is a 3×5-matrix, then there exists 3 columns of A that are linearly independent.

(4) Let A be an 6×4 matrix, and $N(A) = \{0\}$. Mark each of the following by true or false

- (. . .) The system $Ax = b$ is consistent for every $b \in \mathbb{R}^n$.
- (. . .) The columns of A span $\mathbb{R}^6$.
- (. . .) Nullity of A is 4.
- (. . .) The rows of A form a spanning set for $\mathbb{R}^4$
- (. . .) The columns of A are linearly independent.

(5) Let A be a 3×5-matrix. Mark each of the following by true or false

- (. . .) The system $Ax = 0$ has only the zero solution.
- (. . .) The columns of A form a spanning set for $\mathbb{R}^3$.
- (. . .) $3 \leq \text{rank}(A) \leq 5$.
- (. . .) The columns of A are linearly independent.
- (. . .) The rows of A are linearly dependent.

(6) Let A be a 3×5 matrix and $\text{rank}(A) = 3$. Mark each of the following by true or false

- (. . .) The rows of A are linearly independent.
- (. . .) The columns of A are linearly independent.
- (. . .) The system $Ax = 0$ has only the zero solution.

(7) Let A be a 4×4 matrix, $\text{rank}(A) = 3$. Mark each of the following by true or false

- (. . .) The rows of A are linearly dependent.
- (. . .) The system $Ax = b$ is consistent for every $b \in \mathbb{R}^n$.
- (. . .) Nullity of A is 1.
- (. . .) A is nonsingular.
- (. . .) The columns of A form a basis for $\mathbb{R}^4$.

(8) (...) If A is an $n \times n$ singular matrix, then $\text{rank}(A) = n$.

(9) (...) Every spanning set for $\mathbb{R}^{2\times 2}$ contains at least 4 vectors.

(10) (...) The set $W = \{p(x) \in P_5 : \text{degree of } p(x) \text{ is even}\}$ is a subspace of P_5.

(11) (...) If U and W are subspaces of a vector space V, then $U \cap W \neq \Phi$.

(12) (...) Let $S = \{v_1, v_2, \cdots, v_r\}$ be a set of vectors in $\mathbb{R}^n$. If $r > n$, then S is not a spanning set for $\mathbb{R}^n$.

(13) (...) If $f_1, f_2, \cdots, f_n \in C^{n-1}[a, b]$ and $W[f_1, f_2, \cdots, f_n](x) = 0$, for all $x \in [a, b]$, then $f_1, f_2, \cdots, f_n$ are linearly dependent.

(14) (...) If a matrix A is row equivalent to B, then the nonzero rows of B form a basis for the row space of A.

(15) (...) If A is an $n \times n$ matrix and the row space of A is $\mathbb{R}^{1\times n}$, then the column space of A is $\mathbb{R}^n$.

(16) (...) The vector $(3, -1, 0)^T$ is in $\text{span}\{(2, -1, 3)^T, (-1, 1, 1)^T, (1, 1, 9)^T\}$

(17) (...) If A is an $m \times n$ matrix, $m > n$, then either the rows or the columns of A are linearly independent.

(18) (...) If A is a 6×3-matrix, with $\text{rank}(A) = 3$, then the system $Ax = 0$ has only the zero solution.

(19) (...) If $S = \{v_1, \cdots, v_n\}$ is a linearly independent subset of a vector space V, and v is not in $\text{Span}(S)$, then $\{v_1, \cdots, v_n, v\}$ are linearly independent.

(20) (...) If A is a nonzero 3×2 matrix such that $Ax = 0$ has nonzero solutions, then $\text{rank}(A) = 1$.

(21) (...) In $\mathbb{R}^3$ every set with more than 3 vectors can be reduced to a basis for $\mathbb{R}^3$.

(22) (...) If A is a nonsingular $n \times n$ matrix, then the columns of A are linearly independent.

(23) (...) Let V be a vector space with $\dim(V) = 4$ and S a subspace of V. If $v_1, v_2, v_3, v_4 \in V$ with $\text{span}(v_1, v_2, v_3, v_4) = S$, then v_1, v_2, v_3, v_4 are linearly independent.

(24) (...) If A is a square matrix, and the nullity of A is not zero, then the rows of A are linearly independent.

Question 2 (24 points)**.** Circle the most correct answer

(1) The transition matrix from the ordered basis $[e_1, e_2]$ to the ordered basis $[\begin{pmatrix} 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}]$ is

(a) $\begin{pmatrix} -2 & 1 \\ 3 & -1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & -1 \\ -3 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} -1 & 1 \\ 3 & -2 \end{pmatrix}$

(2) let A be a 5×3-matrix, if the row echelon form of A has 2 nonzero rows, then nullity(A) is

(a) 3
(b) 2
(c) 1
(d) 0

(3) If A is a nonzero 3×5 matrix, then

(a) $0 \leq \text{nullity}(A) \leq 3$
(b) $1 \leq \text{nullity}(A) \leq 3$
(c) $2 \leq \text{nullity}(A) \leq 4$
(d) $0 \leq \text{nullity}(A) \leq 2$

(4) If A is a singular $n \times n$-matrix, then

(a) $0 \leq \text{rank}(A) \leq n$
(b) $0 \leq \text{rank}(A) < n$
(c) $0 < \text{rank}(A) < n$
(d) $0 < \text{rank}(A) \leq n$

(5) If A is an $m \times n$-matrix, $b \in$ column space of A and the columns of A are linearly independent, then the system $Ax = b$ has

(a) no solution
(b) exactly one solution
(c) infinitely many solutions
(d) none

(6) $\dim\big(\text{span}(x^2, 3 + x^2, x^2 + 1)\big)$ is

(a) 0
(b) 1
(c) 2
(d) 3

(7) One of the following sets is a subspace of P_4

(a) $\{f(x) \in P_4 : f(0) = 1\}$
(b) $\{f(x) \in P_4 : f(1) = 1\}$
(c) $\{f(x) \in P_4 : f(1) = 0\}$
(d) $\{f(x) \in P_4 : f(x) = x^3 + bx^2 + cx, b, c \in \mathbb{R}\}$

(8) Let v_1, v_2 be linearly independent in a vector space V, $V \neq$span(v_1, v_2), then

(a) $\dim(V) \geq 2$
(b) $\dim(V) \geq 3$
(c) $\dim(V) \leq 2$
(d) $\dim(V) \leq 3$

(9) If A is an $n \times n$-matrix and for each $b \in \mathbb{R}^n$ the system $Ax = b$ is consistent, then

(a) A is nonsingular
(b) $\text{rank}(A) = n$
(c) $\text{nullity}(A) = 0$
(d) all of the above

(10) If V is a vector space and $\{v_1, v_2, \cdots, v_n\}$ is a spanning set for V and $v_{n+1} \in V$, then

(a) the set $\{v_1, v_2, \cdots, v_{n+1}\}$ is not a spanning set.
(b) $v_1, v_2, \cdots, v_{n+1}$ are linearly independent
(c) $v_1, v_2, \cdots, v_{n+1}$ are linearly dependent
(d) none

(11) $S = \{ax^3 + ax^2 + cx - b(x+1) + c | a, b, c \in \mathbb{R}\}$ is a subspace of P_4. A basis for S is

(a) $\{x^3 + x^2, x + 1, 1\}$
(b) $\{x^3 + x^2, x^2, \}$
(c) $\{x^3 + x^2, x + 1\}$
(d) $\{x^3 + x^2, x^2 + 1\}$

(12) Let A be an $m \times n$ matrix. If the columns of A span $\mathbb{R}^m$, then

(a) $n \leq m$
(b) $m \leq n$
(c) $n = m$
(d) the columns of A form a basis for $\mathbb{R}^n$.

Question 3. [5%] Let A be an $n \times n$-matrix. Prove that $\text{rank}(A) = n$ if and only if $\det(A) \neq 0$.

Question 4 (6 points)**.** Let $E = [1, x+1, x^2]$, $F = [1+x, x+x^2, 2+x^2]$ be two bases for P_3

(a) Find the transition matrix from E to F.

(b) Find the coordinate vector of $p(x) = 2x^2 - x - 1$ with respect to F.

Question 5 (6 points)**.** If $A = \begin{pmatrix} 1 & 1 & 0 & 2 & 0 \\ 1 & 2 & -1 & 0 & 1 \\ 2 & 3 & -1 & 2 & 1 \\ 4 & 5 & -1 & 6 & 1 \end{pmatrix}$

(a) Find a basis for the row space of A.

(b) Find a basis for the column space of A.

(c) Find $\text{Rank}(A)$, $\text{Nullity}(A)$.