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Birzeit University
Math. Dept.
Math. 234-A

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Q1 (20 points) Answer the following statements by true or false:

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(1) The set $S = \{v_1, \dots, v_n\}$ is a spanning set of a vector space V if every vector in V is a linear combination of the vectors of S. T

(2) The transition matrix of two basis is nonsingular. T

(3) rank(A) = number of columns of $A - N(A)$. T

(4) If $v_1, v_2, \dots, v_n$ span a vector space V and v is a linear combination of $v_2, \dots, v_n$, then $V = \mathrm{Span}\{v_2, \dots, v_n\}$. T

non zero (5) If two vectors in a vector space V are linearly dependent, then each one of them is a scalar multiple of the other. F

(6) The vectors $(4,2,3)^T, (2,3,1)^T, (2,-5,3)^T, (2,-1,3)^T$ are linearly dependent. T

(7) If 3 vectors span a vector space V, then $dim(V) = 3$. F

(8) If two matrices are row equivalent, they must have the same column space. F

(9) If b is in the column space of a matrix A, then $Ax = b$ is consistent. T

(10) If V is a vector space with dimension $n > 0$, then any set of [illegible] or more vectors in V are linearly dependent. F

Q2 :(45 points) Choose the correct answer.

(1) An $n \times n$ matrix A is invertible if
   (a) $\dim(C(A)) = \dim(R(A))$
   (b) $rank(A) = n$
   (c) $\dim(C(A)) = 0$
   (d) $Nullity(A) = n$
   (e) None

(2) Suppose that $T : V \to W$ is a linear transformation whose 2×2 standard matrix A, and $rank(A) = 2$. Then
   (a) $Ker T = \{0\}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$e_1 \cdot e_2$

(b) $Im T = W$
(c) $nullity(A) = 0$
(d) all of the above

(3) An $n \times n$ matrix A is invertible if

(a) The columns of A are linearly independent
(b) The columns of A span R^n
(c) $nullity(A) = 0$
(d) all of the above

(4) A basis for the vector space spanned by $\{1 - x - x^2, 1 + x + x^2, 2 - x, 1 - x\}$ from this set of vectors is

(a) $1 - x - x^2, 1 + x + x^2, 2 - x$
(b) $1 - x - x^2, 1 + x + x^2$
(c) $1 - x - x^2, 1 + x + x^2, 2 - x, 1 - x$
(d) $1 - x - x^2, 1 - x$
(e) $1 - x - x^2, 2 - x$

(5) The dimension of the null space of $\begin{pmatrix} 1 & 1 & 2 & 1 & 4 \\ 2 & -1 & 2 & -1 & 6 \\ 3 & 0 & 4 & 1 & 10 \end{pmatrix}$ is

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

(6) A basis for the row space of $\begin{pmatrix} 1 & 1 & 2 & 1 & 4 \\ 2 & -1 & 2 & -1 & 6 \\ 3 & 0 & 4 & 0 & 10 \end{pmatrix}$ is

(a) $(1,1,2,1,4), (2,-1,2,-1,6)$
(b) $(1,1,2,1,4), (2,-1,2,-1,6), (3,0,4,0,10)$
(c) $(1,1,2,1,4)$
(d) $(1,2,3), (1,-1,0)$
(e) $(1,1,2,1,4), (0,-2,-2,-3,-2), (0,0,0,0,0)$

(7) Let $T : R^3 \to R^3$ be a linear transformation and suppose that $T(e_1) = (1,2,-1)$, $T(e_2) = (1,-2,-1)$, $T(e_3) = (-1,2,-1)$ then $T(1,2,1) =$

(a) $(2,0,-4)$
(b) $(1,2,-3)$
(c) $(1,2,-1)$
(d) $(2,-4,-2)$
(e) *None*

(8) The transition matrix from the basis $\{2, 2 - x\}$ to $\{1, x\}$ is

(a) $\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 2 & 2 \\ 0 & -1 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & -1 \\ 2 & 2 \end{pmatrix}$

(d) $\begin{pmatrix} -1/2 & 0 \\ -1/2 & 1 \end{pmatrix}$

(e) *None*

(9) Let A be an arbitrary $n \times n$ matrix. Then the rank of A

(a) equals the dimension of the column space of A

(b) equals the dimension of the null space of A

(c) equals n

(d) None

$(A^t = A)$

(10) Let A be an arbitrary $n \times n$ matrix. Then

(a) The row space of A equals the column space of A

(b) The row space of A equals the null space of A

(c) The row space of A is contained in the column space

(d) The row space of A has the same dimension as the column space of A

(e) None

(11) For any vector space V,

(a) If V is finite-dimensional, then V is a subspace of R^n for some positive integer n

(b) If V is infinite-dimensional, then every infinite subset of V is linearly independent

(c) If V is finite-dimensional, then no finite subset of V is linearly dependent

(d) If V is finite-dimensional, then no infinite subset of V is linearly independent

(e) None

(12) One of the following is not a linear transformation

(a) $T : R^2 \to R^3$ defined by $T(x, y) = (x, y, x - y)$

(b) $T : R^2 \to R^3$ defined by $T(x, y) = (x, y, 0)$

[redacted] $T : R^2 \to R^3$ defined by $T(x, y) = (x, y, x - 1)$

[redacted] $T : R^2 \to R^3$ defined by $T(x, y) = (0, 0, 0)$

(13) The dimension of the subspace $S = \{(a + b + 2c, a + 2b + 4c, b + 2c)^T, a, b, c \in R\}$ is

(a) 0

(b) 1

(c) 2

(d) 3

$a \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + b\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + c\begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$

(14) One of the following set of vectors are linearly independent

(a) $(1, 1, 2, 1, 4), (2, 2, 4, 2, 8)$

(b) $(1, 1, 2, 1, 4), (2, -1, 2, -1, 6), (0, 0, 0, 0, 0)$

[redacted] $1, x + 1$

$c_1 (a + b + 2c) + c_2 (a + 2b + 4c) + c_3 (0b + 2c) = 0$

$(c_1 + c_2) a + (c_1 + 2c_2 + c_3) b + (2c_1 + 4c_2 + 2c_3) c = 0$

$c_1 = -c_2$

$c_1 + 2c_2 + c_3 = 0 \Rightarrow -c_2 + 2c_2 + c_3 = 0 \quad c_2 + c_3 = 0 \Rightarrow c_2 = -c_3$

$2c_1 + 4c_2 + 2c_3 = 0 \Rightarrow -2c_2 + 4c_2 - 2c_2 = 0$

(d) (1, 2, 3), (0, 1, 0), (0, 0, 1)

(e) (1, 1), (0, −2), (1, 5)

(15) One of the following is ~~not a~~ subspace in the corresponding space

(a) $S = \{f \in C(R) : f(0) = 1\}, V = C(R)$

(b) $S = \{A \in R^{2\times2} : a_{11} = 1\}, V = R^{2\times2}$

(c) $S = \{A \in R^{2\times2} : |A| = 0\}, V = R^{2\times2}$

(d) $S = \{f \in P_{10} : f(0) = 0\}, V = P_{10}$

(e) $S = \{v = (x, y) \in R^2 : x + y = 1\}, V = R^2$

Q3:(20 points) Let $T : P_3 \to R^{2\times2}$ be a linear transformation defined by $T(ax^2 + bx + c) = \begin{pmatrix} a+b & b \\ a-b-c & b+c \end{pmatrix}$

$P_3 \to P_2$

(a) Find the matrix representation of T with respect to the standard basis of P_3, $R^{2\times2}$

$A = \begin{pmatrix} L(1) & L(x) & L(x^2) \end{pmatrix}$

$L(1) = \begin{pmatrix} 0 & 0 \\ -1 & 1 \end{pmatrix}$

$L(x) = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

$L(x^2) = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

(b) Find a basis for Ker T

img = 3

$f(x) \in P_3 : L(f(x)) = 0$

$\begin{pmatrix} a+b & b \\ a-b-c & b+c \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

dim ker

$a+b=0$

$b=0 \Rightarrow a+0=0 \Rightarrow a=0$

$a-b-c=0$ $\quad b=0$

$b+c=0$ $\quad \Rightarrow c=0$ $\quad f(x) = 0x^2+0x+0$

dim Ker f = 0

(c) Find a basis for Im T

dim Im + dim ker = $\boxed{P3}$

dim ImT = 3 $\Rightarrow$ $P_3 = ax^2+bx+c$ Ex $(2x^2+x+1)$

$a\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} + b\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} + c\begin{pmatrix} 0 & 0 \\ -1 & 1 \end{pmatrix}$

(d) Find all polynomials $f \in P_2$ such that $T(f) = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

$a=1$
$a+b=1$
$b=0$
$a-b-c=1$
$c=0$
$b+c=1$

$b=0 \Rightarrow a=1$

$a+b=1$

$b=0 \Rightarrow a-b-c=1 \Rightarrow c=0$

$b=0 \Rightarrow b+c=1$

$c=1$ X

the solution is $\{\}$

Q4 (15 points):(a) Let A, B be two square $n \times n$ matrices such that $AB = 0$. Show that $rank(B) \le Nullity(A)$

need to show $C(B) \subseteq N(A)$

$b_j \in C(A)$

$Ab_j = 0$ b_j is a solution of $AX = 0$

so $C(B) \subseteq N(A)$

and since $C(B) \subseteq N(A)$

and $\dim(C(B)) =$ Rank B

$\dim(N(A)) =$ Nullity A

so

rank $(B) \le$ nullity (A)

$N(A) \Rightarrow A_{not}$

$AB = 0$

$A b_j = (0)$

(b) Let A a 5×6, B a 6×7 none zero matrices such that $AB = 0$. Show that $AX = 0$ has infinite solutions

rank $B \le$ nullity (A)

so since B is 6×7 matrix Rank $B \neq 0$ ∴ Rank $7 > B \geq 0$

so so nulity (which is num free Variable)

is bigger than Rank (B) so it couldnt be zero

$Ax = 0$ couldnt have uniqe solution and it will

since $b \in C(A)$ $Ax = 0$ will have infinitly many solutions

(c) Let $T : V \to W$ be a linear transformation. If W is a subspace of V, prove that $KerT$ is a subspace of V.

1) ker T is non empry since $0_V \in$ ker since $L(0_V) = 0_W$

2) let $V_1 \in$ ker $L(V_1) = 0$, $V_2 \in$ ker(T), $L(V_2) = 0 \Rightarrow$

$L(V_1 + V_2) \overset{??}{\in}$ ker T

since T is LT $\Rightarrow L(V_1 + V_2) = L(V_1) + L(V_2)$

$= 0 + 0 = 0$ $\in$ kerT

3) $\alpha \in R$ $V_1 \in$ kerT $L(\alpha V_1) = \alpha L(V_1) = \alpha(0) = 0 \Rightarrow$

$0 \in$ kerT

so it is supsbace.