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Birzeit University
Mathematics Department
Math 234

Second Exam

Summer Semester 2016/2017

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Question 1 (1.5 points each). **Mark each of the following by True or False**

(1) (T.) The vectors $\{(1,-1,1)^T, (1,-3,2)^T, (1,-2,0)^T\}$ form a basis for $\mathbb{R}^3$. $\begin{pmatrix} 1 & 1 & 1 \\ -1 & -3 & -2 \\ 1 & 2 & 0 \end{pmatrix} \to 0$

(2) (.T.) The vectors $\{t-1, t^2+2t+1, t^2+t-2\}$ form a basis for P_3.

(3) (.T.) The set $S = \{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 : x_1 + x_2 + x_3 = 0\}$ is a subspace of $\mathbb{R}^3$. $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ -1 & 1 & -2 \end{pmatrix} \to 0$

(4) (F) If A is a 4×3 matrix with $\text{rank}(A) = 3$, then the homogeneous system $Ax = 0$ has nontrivial solution. $0 \neq$ حلول

(5) (F.) If $L : P_2 \to P_1$ is the linear transformation defined by $L(ax^2 + bx + c) = (a-c)x + (b+c$ then $x^2 - x + 1$ is in $\ker(L)$. P_3

(6) (.F) It is possible to find a matrix A of size 3×5 such that $\text{nullity}(A) = 1$

(7) (.T.) If $v_1, v_2, \cdots, v_n \in V$, $\dim(V) = n$ and $\text{span}(v_1, v_2, \cdots, v_n) = V$, then $v_1, v_2, \cdots, v_n$ a linearly independent. L.I

(8) Let A be a 3×3 matrix and $\text{rank}(A) = 3$. Mark each of the following by true or false

- (..F..) The nullity of A is 3. Nonsingular
- (..T..) The system $Ax = 0$ has only the zero solution.
- (..T..) The columns of A are linearly independent.
- (..T..) The rows of A are linearly independent.
- (..T..) The columns of A form a spanning set for $\mathbb{R}^3$.

$(A+B)+(A+B)^T$
$A+B+A^T+B^T$
$A+A^T$ $\quad \alpha(A+B)^T$

(9) (T) $L : \mathbb{R}^{3\times3} \to \mathbb{R}^{3\times3}$, $L(A) = A + A^T$ is a linear transformation.

(10) (.T.) If A, B are two row equivalent $n \times n$-matrices, then $\text{rank}(A) = \text{rank}(B)$

(11) (.F) Let $L : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by $L(x, y) = (x-y, 0, 2x+3)$, then L is a linear transformati

(12) (.T.) If A is a nonzero 5×4 matrix such that $Ax = 0$ has only the zero solution, then $\text{rank}(A) =$

(13) (F) If A is an $n \times n$ nonsingular matrix, then nullity of $(A) = n$ $= 0$

(14) (.T.) If A is an $n \times n$ matrix and the row space of A is $\mathbb{R}^{1\times n}$, then the column space of A is

(15) (.T.) If A is a 5×7 matrix, then nullity of $A \geq 2$. L.I rows

(16) F If V is a vector space and $\{v_1, v_2, \cdots, v_n\}$ is a spanning set for V and $v_{n+1} \in V$, then the $\{v_1, v_2, \cdots, v_{n+1}\}$ is not a spanning set.

(17) (.F) If U, V are subspaces of a vector space W, then $U \cap V$ is also a subspace of W.

$L(\alpha x + \beta y) = \begin{pmatrix} \alpha x - \beta y \\ 0 \\ 2(\alpha x + \beta y) + 3 \end{pmatrix} \to 0$ $\quad \alpha \begin{pmatrix} x - y \\ 0 \\ 2(x+y) + 3 \end{pmatrix}$

V U X

(18) (T) If $f_1, f_2, \cdots, f_n \in C^{n-1}[a,b]$ and $W[f_1, f_2, \cdots, f_n](x) \neq 0$ for all $x \in [a,b]$, then $f_1, f_2, \cdots, f_n$ are linearly independent.

Question 2 (3 points each). Circle the most correct answer correct answer

(1) Let V be a vector space of dimension 4 and $W = \{v_1, v_2, v_3, v_4, v_5\}$ a set of nonzero vectors of V, then

L.D

(a) W is linearly dependent

(b) W is linearly independent

(c) W is a basis

(d) W is a spanning set

(2) Let $S = \left\{ p(x) = ax^2 + bx + c \in P_3 : \int_0^1 p(x)\,dx = 0 \right\}$. The dimension of S is.

$\left.\frac{ax^3}{3} + \frac{bx^2}{2} + cx \right)_0^1 = 0$

$\boxed{\frac{a}{3} + \frac{b}{2} + c = 0}$

(a) 2

(b) 3

(c) 4

(d) 1

(3) Let A be a 5×6-matrix such that $Ax = b$ is consistent for every $b \in \mathbb{R}^5$, then

rows

(a) Nullity$(A) = 0$

(b) rank$(A) = 1$

(c) rank$(A) = 5$

(d) Nullity$(A) = 6$

(4) The transition matrix from the ordered basis $[e_1, e_2]$ of $\mathbb{R}^2$ to the ordered basis $[\begin{pmatrix} 2 \\ -3 \end{pmatrix}, \begin{pmatrix} 3 \\ -5 \end{pmatrix}]$ is

(a) $\begin{pmatrix} 5 & -3 \\ 3 & -2 \end{pmatrix}$

$\begin{pmatrix} 5 & 3 \\ -3 & -2 \end{pmatrix}$

(b) $\begin{pmatrix} -5 & -3 \\ 3 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & -3 \\ 3 & -5 \end{pmatrix}$

(d) $\begin{pmatrix} -5 & 3 \\ -3 & 2 \end{pmatrix}$

(5) If A is an $n \times n$ singular matrix, then

(a) The columns of A are linearly dependent

(b) rank$(A) = n$

(c) $N(A) = \{0\}$

(d) all of the above

(6) Let $L : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear transformation defined by $L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 - x_3 \\ x_3 - x_4 \end{pmatrix}$, then $\dim(\ker(L)) =$

$\begin{pmatrix} x_1 \\ -x_1 \\ -x_1 \\ -x_1 \end{pmatrix}$

$x_1 + x_2 = 0$
$x_2 - x_3 = 0$
$x_3 - x_4 = 0$
$x_3 = x_4$
$x_2 = x_3 = x_4$
$x_1 = -x_2 = -x_3 =$

(a) 3

(b) 2

(c) 1

(d) 4

(7) Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined as $L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_3 \\ x_2 \\ x_1 \end{pmatrix}$, then $\dim(\text{Range}(L)) =$ 3

(a) 1

(b) 2

(c) 3

(d) 0

$n \geq m$

(8) Let A be an $m \times n$ matrix. If the rows of A are linearly independent $\mathbb{R}^n$, then

5×8 $m \leq n$

(a) $m \leq n$

(b) $n = m$

(c) $n \leq m$

(d) none of the above

(9) If $\{v_1, v_2, v_3\}$ is a spanning set for a vector space V and $v \in V$, then

(a) $\{v_1, v_2, v_3, v\}$ are linearly independent. ✗ $\dim(V) \leq 3$

(b) $\{v_1, v_2, v_3, v\}$ are linearly dependent. v_1, v_2, v_3, v_4 -- sp

(c) $\dim(\text{span}(v_1, v_2, v_3, v)) = 4$. $\dim < 4$

(d) $\dim(V) = 4$. L.D

(10) let A be a 4×7-matrix, if the row echelon form of A has 2 nonzero rows, then dim(column space of A) is

Nullity = 5
2 = rank = dim(C)

(a) 3

(b) 5

(c) 6

(d) 2

(11) If A is an $m \times n$-matrix, $b \in$ column space of A and the columns of A are linearly independent, then the system $Ax = b$ has

consistent one

(a) exactly one solution ✓

(b) infinitely many solutions ✗

(c) no solution ✗

(d) none of the above ✗

(12) $\dim(\operatorname{span}(x^2, 3 + x^2, x^2 + 1))$ is

(a) 1

(b) 2

(c) 3

(d) 0

$(3+x^2) = 3(x^2+1) - 2(x^2)$

$= x^2 + 3$

$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 0 & 0 \\ 0 & 3 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

(13) Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 - x_2 \\ x_1 + 2x_2 \\ x_3 \end{pmatrix}$. A basis for $\ker(L)$ is

(a) $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

(b) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

(c) $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

(d) has no basis

$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$: $x_1 - x_2 = 0$

$x_1 + 2x_2 = 0$

$x_3 = 0$

$x_1 = 0$

$x_2 = 0$

(14) If A is an $n \times n$ nonsingular matrix, then

(a) The rows of A are linearly independent ✓

(b) The columns of A are linearly dependent ✗

(c) all of the above ✓

(d) none of the above

(15) If A is a nonzero 5×2-matrix and $Ax = 0$ has infinitely many solutions, then $\operatorname{rank}(A) =$

rank ≠ 0 $\boxed{\text{rank} = 1}$

(a) 2

(b) 3

(c) 5

(d) 1

k 5

Question 3 (5+3 points). 1. Let $S = \left\{ p(x) \in P_3 : \int_0^1 p(x)\, dx = 0 \right\}$. Find a basis and dimension of S.

$S = [ax^2+bx+c : \int_0^1 ax^2+bx+c = 0]$, $a,b,c \in R$

$S = [ax^2+bx+c : [\frac{ax^3}{3} + \frac{bx^2}{2} + xc]_0^1 = 0]$

$S = [ax^2+bx+c : \frac{a}{3} + \frac{b}{2} + c = 0]$

$c = -\frac{a}{3} - \frac{b}{2}$

$S = [ax^2+bx - \frac{a}{3} - \frac{b}{2}, a,b \in R)$

$S = [a(x^2-\frac{1}{3}) + b(x-\frac{1}{2}), b,a \in R]$

$\Rightarrow$ basis for S $= [x^2-\frac{1}{3}, x$

dim $= 2$

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2. Let $L : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation, such that $L\left(\begin{pmatrix}1\\2\end{pmatrix}\right) = \begin{pmatrix}2\\3\\-1\end{pmatrix}$, and $\begin{pmatrix}-1\\1\end{pmatrix}$ e

Ker(L). Find $L\left(\begin{pmatrix}2\\3\end{pmatrix}\right)$.

$L\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}2\\3\\-1\end{pmatrix}$

$L\begin{pmatrix}-1\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$

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$\begin{pmatrix}2\\3\end{pmatrix} = \alpha_1 \begin{pmatrix}1\\2\end{pmatrix} + \alpha_2 \begin{pmatrix}-1\\1\end{pmatrix}$

$2 = \alpha_1 - \alpha_2$

$3 = 2\alpha_1 + \alpha_2$

$5 = 3\alpha_1$

$\alpha_1 = \frac{5}{3}, \alpha_2 = -\frac{1}{3}$

$\Rightarrow L\begin{pmatrix}2\\3\end{pmatrix} = L\left(\frac{5}{3}\begin{pmatrix}1\\2\end{pmatrix} - \frac{1}{3}\right.$(

$= \frac{5}{3} L\begin{pmatrix}1\\2\end{pmatrix} - \frac{1}{3}$

$= \frac{5}{3}\begin{pmatrix}2\\3\\-1\end{pmatrix} -$

$= \begin{pmatrix}10/3\\15/3\\-5/3\end{pmatrix}$

Question 4 (8 points). Let $A = \begin{pmatrix} 1 & 0 & -2 & 1 & 3 \\ -1 & 1 & 5 & -1 & -3 \\ 0 & 2 & 6 & 0 & 1 \\ \cancel{1} & \cancel{1} & \cancel{1} & \cancel{1} & \cancel{1} \end{pmatrix}$. The reduced row echelon form of A is

$U = \begin{pmatrix} 1 & 0 & -2 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$.

(a) Find a basis for $R(A)$.

basis of $R(A)$ = nonzero rows of U

$= [(1,0,-2,1,0), (0,1,3,0,0), (0,0,...$

(b) Find a basis for $C(A)$.

basis of $C(A)$ a columns of A That L.I

hav

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basis for $C(A) = \left[\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix} \right]$

(c) Find a basis for $N(A)$.

Null space for U = Null space for A

solve $U \Rightarrow Ux = 0$

solution = $N(A)$

$\begin{pmatrix} 1 & 0 & -2 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

$x_5 = 0$

$x_4 = B$

$x_3 = \alpha$

$x_2 = -3\alpha$

$x_1 = -B + 2\alpha$

solution $= \begin{pmatrix} -B + 2\alpha \\ -3\alpha \\ \alpha \\ B \\ 0 \end{pmatrix} = B \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} 2 \\ -3 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

basis for $N(A) = \left[\begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right] \longrightarrow$ nullity = 2

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(d) Find rank(A), nullity(A).

rank(A) = dim$(R(A))$ = # None zero rows = 3

Nullity(A) = # free variable = 2

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dim ≥ 3

Question 5. (1) [4points] Let V be a vector space, $v_1, v_2, v_3 \in V$ be linearly independent. $w_1 = v_1 + v_2 + v_3$, $w_2 = v_2 + v_3$, $w_3 = v_3$, show that w_1, w_2, w_3 are linearly independent.

$$c_1 W_1 + c_2 W_2 + c_3 W_3 = 0 \Longleftarrow$$ if have only 0 sol

so W_1, W_2, W_3

$$c_1(v_1+v_2+v_3) + c_2(v_2+v_3) + c_3(v_3) = 0$$

$$c_1 v_1 + c_1 v_2 + c_1 v_3 + c_2 v_2 + c_2 v_3 + c_3 v_3 = 0$$

$$c_1 v_1 + (c_1+c_2) v_2 + (c_1+c_2+c_3) v_3 = 0$$

4

Since $v_1, v_2,$

it have o

$\Rightarrow c_1+c_2+c_3=0$
$c_2+c_3=0$ $\Rightarrow 0$
$c_1=0$

so

ha

(2) [4 + 2points] Let $E = [1, x, x^2]$, $F = [1 + x, 1 + x^2, 1 + x + x^2]$ be two bases for P_3.

(a) Find the transition matrix from E to F.

(b) Find the coordinate vector of $p(x) = 2x^2 - x - 1$ with respect to F.

~~a) $T_{E\to F} = F^{-1}E$~~

a) E is standard basis

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$$T_{E\to F} = (T_{F\to E})^{-1}$$

$$(T_{F\to E})^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}^{-1} \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \Rightarrow$$

$$(T_{F\to E})^{-1} = \begin{pmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ -1 & 1 & 1 \end{pmatrix} = T_{E\to F}$$

$$[P(x)]_F = T_{E\to F}\,[P(x)]_E$$

$$[P(x)]_F = \begin{pmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ -1 & 1 & 1 \end{pmatrix}\begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 2 \end{pmatrix}$$