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## Birzeit University Mathematics Department Math 234

Math 234 هند مؤدر Second Exam Summer Semester 2018-2019 Student Name: Mchanad Moayad Shann (7 Number: 11.81401. Section: 3 Question 1 (64%). True or False (1) (......) If  $f_1, f_2, \dots, f_n \in C^{n-1}[a, b]$  and  $W[f_1, f_2, \dots, f_n](x) = 0$  for all  $x \in [a, b]$ , then  $f_1, f_2, \dots, f_n$  are linearly independent. (....) The transition matrix of two basis could be singular. (3) If U is the row echelon form or the reduced row echelon form of A, then the rank of A is equal to the number of lead 1's in U. (4) Let U be the row echelon form of A, then the nullity of A equals the number of nonzero columns of U. (5) f(x) = f(x). If A is a singular matrix, then the null space of A is  $\{0\}$ . (6) (......) If A is a  $3 \times 3$  matrix and Ax = 0 has a nontrivial solution, then the nullity of A is either 1 or 2. (7) (7...) If A is an  $n \times n$  matrix and the row space of A is  $\mathbb{R}^{1 \times n}$ , then the column space of A is (8) Let V be a vector space,  $\{v_1, v_2, \dots v_n\}$  a spanning set for V, and  $v \in V$ . Mark each of the following by true or false The vectors  $(v_1, v_2, \dots v_n, \underline{v})$  form a spanning set for V. The vectors  $[v_1, v_2, \dots v_{n-1}]$  form a spanning set for V. (10) Let  $U = \{(x,y)^T : y = x+1\}$ . Then U is a subspace of  $\mathbb{R}^2$ (11) (.1...) If A is an  $n \times n$ -matrix, and  $\det(A) \neq 0$ , then  $\operatorname{rank}(A) = n$ . (12) Let V be a vector space,  $v_1, v_2, \ldots v_n \in V$  be linearly independent, and  $v \in V$ . Mark each of the following by true or false  $\mathcal{L}$ ..) The vectors  $v_1, v_2, \dots v_{n-1}$  are linearly dependent. (18) (....) Every spanning set for  $\mathbb{R}^3$  contains at least 3 vectors. (14) (14) If A is a nonzero  $3 \times 2$  matrix such that Ax = 0 has infinitely many solutions, then 7 (15) (..., Let  $S = \{v_1, v_2, \dots, v_r\}$  be a set of vectors in  $\mathbb{R}^n$ . If r > n, then S is linearly dependent. (16) (15...) Let V be a vector space. If  $v_1, v_2, v_3, v_4 \in V$  with  $\operatorname{span}(v_1, v_2, v_3, v_4) = V$ , then

 $v_1, v_2, v_3, v_4$  are linearly independent.

- vectors is linealrly dependent.
- (19) Let A be an  $4 \times 3$  matrix, and rank(A) = 3. Mark each of the following by true or false

F The rows of A are linearly independent

.) The rows of A form a basis for  $\mathbb{R}^{1\times 3}$ 

The system Ax = b is consistent for every  $b \in \mathbb{R}^4$ .

 $\mathcal{L}$ .) The columns of A are linearly independent.

 $\mathbb{R}$ .) The columns of A form a spanning set for  $\mathbb{R}^4$ .

(.....) nullity of A is 1.

- (20) Let A be a  $3 \times 5$  matrix, and rank(A) = 3 Mark each of the following by true or false
  - n (A) = 2The columns of A form a spanning set for  $\mathbb{R}^3$ .
  - .) The rows of A are linearly independent.
  - .) The system Ax = 0 has only the zero solution.
    - ,) The columns of A are linearly dependent.
      - F..) The rows of A are linearly dependent.
    - $(\dots \square)$  nullity(A) = 0

Question 2 (32 points). Circle the most correct answer

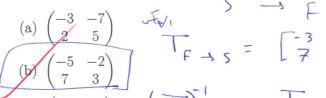
- - (d) 0
- (2) let A be a  $4 \times 7$ -matrix, if the row echelon form of A has 2 nonzero rows, then dim(column space of A) is
  - (a) 3
  - (b) 5
  - (c) 6
- (3) The coordinate vector of  $\begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$  with respect to the ordered basis  $\begin{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \end{bmatrix}$  is

  (a)  $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ (b)  $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$ (c)  $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ (d)  $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ (e)  $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ (f)  $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ (g)  $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ (h)  $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$ (b)
  - - (d)
- (4) Let V be a vector space with  $(\dim(V) = 5)$  and S a subspace of V. If  $v_1, v_2, v_3, v_4 \in S$  with  $\operatorname{span}(v_1, v_2, v_3, v_4) = S$ , then  $v_1, v_2, v_3, v_4$ dim of 5 & 4
  - (a) are linearly dependent
  - (b) form a basis for S
  - (c) is not a spanning set for V
  - (d) are linearly independent.
- (5) The coordinate vector of 2 + 6x with respect to the basis [2x, 4] is

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- (6) The dimension of the subspace  $S = \{(a-b+2c_{j}a+2b-4c_{j}-b+2c)^{T}, a, b, c \in R\}$  is
  - $\begin{pmatrix} 1 & -1 & 2 & 0 \\ 0 & -1 & 2 & 0 \end{pmatrix}$   $\Rightarrow$   $\begin{pmatrix} 0 & -1 & 0 & 2 & 0 \\ 0 & 0 & 3 & 0 & -6 & 0 \end{pmatrix}$
  - 20) ( | -1 | -2 | 0 ) A-Your = 2

    Jim of cot : Sase = 2
- (7) If A is an  $n \times n$ -matrix and for each  $b \in \mathbb{R}^n$  the system Ax = b has a unique solution, then
  - (a) rank(A) = n
  - (b)  $\operatorname{nullity}(A) = 0$
  - (a) A is nonsingular
  - (d) all of the above
- (8) If A is an  $m \times n$ -matrix,  $b \in$  column space of A and the columns of A are linearly independent, then the system Ax = b has
  - (a) exactly one solution
    - (b) infinitely many solutions (c) no solution
  - (d) none of the above
- (9) If A is a  $3 \times 4$  matrix such that nullity(A) = 1 and  $b = (1, 2, 3)^T$ , then the system Ax = b has
  - (a) at most one solution (b) exactly one solution was det.
- nonzero Z
- (c) infinitely many solutions.
- (d) no solution
- (10) The transition matrix from the ordered basis  $[e_1, e_2]$  of  $\mathbb{R}^2$  to the ordered basis  $\begin{bmatrix} -3 \\ 7 \end{bmatrix}$ ,  $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$ ] is



- (11) Let  $S = \{ax^2 + bx + a + b : a, b \in \mathbb{R}\}$ . Then a basis for S and the dimension of S are
  - $a(X^{2}+1) + b(X+1)$ (a) basis:  $x^2 + 1$ , x; dimension = 2 (b) basis:  $x^2 + 1$ , x + 1; dimension = 2

  - (c) basis:  $x^2$ , 1; dimensin = 2
  - (d) basis:  $x^2, x, 1$ ; dimension = 3

- (12) Let  $v_1, v_2, v_3$  be linearly dependent in a vector space  $V, V = \operatorname{span}(v_1, v_2 v_3)$ , then

  - (b)  $\dim(V) \le 2$ (c)  $\dim(V) \ge 3$

  - (d) none of the above
- (13) Let  $S = \{ax^2 + ax + \underline{b} : a, b \in \mathbb{R}\}$ . Then a basis and the dimension of S are

  - (b) basis:  $x^2, 1$ ; dimensin = 2
  - (c) basis:  $x^2 + x, 1$ ; dimension = 2 (d) basis:  $x^2 + 1, x$ ; dimension = 2
- (14) An  $n \times n$  matrix is singular iff
  - (a) the rows of A form a basis for  $\mathbb{R}^{1\times n}$
  - (b) the columns of A form a basis for  $\mathbb{R}^n$
  - (c)  $0 \in N(A)$
  - (d) none of the above
- (15) Let A be an  $n \times n$  matrix. Then
  - (a) The row space of A equals the null space of A
  - (b) The row space of A is contained in the column space
  - The row space of A has the same dimension as the column space
  - (d) The row space of A equals the column space of A of A
- (16) If A is a  $4 \times 3$  matrix such that nullity of A = 0, and  $b = \begin{pmatrix} 1 \\ 3 \\ 2 \\ 0 \end{pmatrix}$ , then the system Ax = b
  - (a) has exactly one solution
    - (b) is either inconsistent or has an infinite number of solutions
  - (c) is inconsistent
  - (d) is either inconsistent or has one solution

Question 3 (8 points). Let E = [1 + x, 1 - x], F = [1, x] be two ordered bases for  $P_2$ 

- 1. Find the transition matrix from E to F.
- 2. Find the coordinate vector of p(x) = -(1 + x) + 2(1 x) with respect to F.

2. Find the coordinate vector of 
$$p(x) = -(1+x) + 2(1-x)$$
 with respect to  $x$ .

(I)  $T_{E \to F} = (U^{-1}) W$ 

Such that  $x = (1 - 1) W = (1 - 1) W$ 

$$(1 - 1) = (1 - 1) W = (1 - 1) W$$

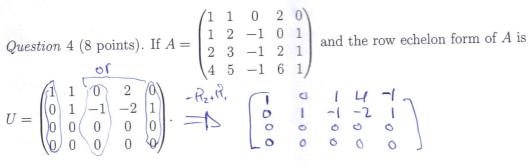
$$(2 - 1) = (1 - 1) W = (1 - 1) W = (1 - 1) W$$

$$(3 - 1) = (1 - 1) W = (1 - 1) W = (1 - 1) W$$

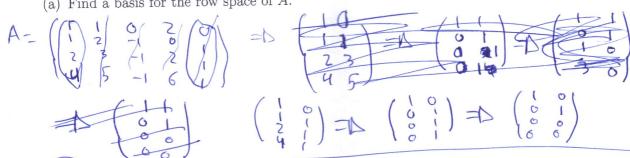
$$(4 - 1) = (1 - 1) W = (1 - 1) W = (1 - 1) W$$

$$(5 - 1) = (1 - 1) W = (1 - 1) W = (1 - 1) W = (1 - 1) W$$

$$(7 - 1) = (1 - 1) W = (1 - 1$$



(a) Find a basis for the row space of A.



(b) Find a basis for the column space of A. (17020),

(b) Find a basis for the column space of A. (12-101)

(c) Find a basis for null space 
$$A$$

$$A \times = 0$$

 $x_5 = x$ ,  $x_a = B$ ,  $x_3 = X$ 

$$X_2 = X_3 + 2 X_4 - X_5 = (X + 2 B - X) - X_2$$
 Mull Space (A) °

$$X_{1} = -2 X_{4} - X_{2}$$

$$= -2 \beta - (\chi + 2\beta - \chi) = -\chi + \chi - 4\beta$$
(1) Find Bank(4) Nullity(4)

(d) Find Rank(A), Nullity(A).