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Birzeit University
Mathematics Department
Math 234

Second Exam مهند مؤيد — Summer Semester 2018-2019

Student Name: Mohanad Moayad Shanaa Number: 1181401 Section: 3

Question 1 (64%). **True or False**

(1) (..F..) If $f_1, f_2, \cdots, f_n \in C^{n-1}[a,b]$ and $W[f_1, f_2, \cdots, f_n](x) = 0$ for all $x \in [a,b]$, then $f_1, f_2, \cdots, f_n$ are linearly independent. Singular

(2) (..F..) The transition matrix of two basis could be singular.

(3) (..T..) If U is the row echelon form or the reduced row echelon form of A, then the rank of A is equal to the number of lead 1's in U.

(4) (..F..) Let U be the row echelon form of A, then the nullity of A equals the number of nonzero columns of U.

(5) (..F..) If A is a singular matrix, then the null space of A is $\{0\}$.

(6) (..T..) If A is a 3×3 matrix and $Ax = 0$ has a nontrivial solution, then the nullity of A is either 1 or 2. Singular.

(7) (..T..) If A is an $n \times n$ matrix and the row space of A is $\mathbb{R}^{1 \times n}$, then the column space of A is $\mathbb{R}^n$.

(8) Let V be a vector space, $\{v_1, v_2, \ldots v_n\}$ a spanning set for V, and $v \in V$. Mark each of the following by true or false

- (..T..) The vectors $v_1, v_2, \ldots v_n, v$ form a spanning set for V.
- (..F..) The vectors $v_1, v_2, \ldots v_{n-1}$ form a spanning set for V.

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(9) (..T..) A basis for a nonzero vector space can't contain the zero vector.

(10) (..F..) Let $U = \{(x,y)^T : y = x+1\}$. Then U is a subspace of $\mathbb{R}^2$ $\begin{pmatrix} x \\ x+1 \end{pmatrix}$

(11) (..T..) If A is an $n \times n$-matrix, and $\det(A) \neq 0$, then $\text{rank}(A) = n$. non Singular

(12) Let V be a vector space, $v_1, v_2, \ldots v_n \in V$ be linearly independent, and $v \in V$. Mark each of the following by true or false

- (..F..) The vectors $v_1, v_2, \ldots v_n, v$ are linearly independent.
- (..F..) The vectors $v_1, v_2, \ldots v_{n-1}$ are linearly dependent.

(13) (..T..) Every spanning set for $\mathbb{R}^3$ contains at least 3 vectors.

(14) (..T..) If A is a nonzero 3×2 matrix such that $Ax = 0$ has infinitely many solutions, then $\text{rank}(A) = 1$.

(15) (..T..) Let $S = \{v_1, v_2, \cdots, v_r\}$ be a set of vectors in $\mathbb{R}^n$. If $r > n$, then S is linearly dependent.

(16) (..F..) Let V be a vector space. If $v_1, v_2, v_3, v_4 \in V$ with $\text{span}(v_1, v_2, v_3, v_4) = V$, then v_1, v_2, v_3, v_4 are linearly independent.

(17) (...F..) If A is a 3×3-matrix with nullity $(A) = 0$, then A is singular.

(18) (..T..) If V is a vector space of dimension n, then any subset of V which has more than n vectors is linealrly dependent.

(19) Let A be an 4×3 matrix, and $\text{rank}(A) = 3$. Mark each of the following by true or false

n(A) = 0

- (...F..) The rows of A are linearly independent
- (...F..) The rows of A form a basis for $\mathbb{R}^{1\times 3}$
- (..F...) The system $Ax = b$ is consistent for every $b \in \mathbb{R}^4$.
- (..T..) The columns of A are linearly independent.
- (F.....) The columns of A form a spanning set for $\mathbb{R}^4$.
- (..F..) nullity of A is 1.

(20) Let A be a 3×5 matrix, and $\text{rank}(A) = 3$ Mark each of the following by true or false

n(A) = 2

- (...T..) The columns of A form a spanning set for $\mathbb{R}^3$.
- (..T...) The rows of A are linearly independent.
- (..F...) The system $Ax = 0$ has only the zero solution.
- (...T..) The columns of A are linearly dependent.
- (...F..) The rows of A are linearly dependent.
- (...F..) $\text{nullity}(A) = 0$

Question 2 (32 points). Circle the most correct answer

(1) dim(span($\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$)) is

(a) 1
(b) 2
(c) 3
(d) 0

(2) let A be a 4×7-matrix, if the row echelon form of A has 2 nonzero rows, then dim(column space of A) is

$r(A) = 2$

(a) 3
(b) 5
(c) 6
(d) 2

(3) The coordinate vector of $\begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$ with respect to the ordered basis [$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$] is

$\begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix} = \alpha_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + \alpha_3 \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$

$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 1 & 2 & 3 & 2 \\ 1 & 2 & 4 & 5 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & 2 & 2 \end{array}\right)$

$\Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right)$

$\begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix}$

(a) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

(b) $\begin{pmatrix} -1 \\ 4 \\ -3 \end{pmatrix}$

(c) $\begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix}$

(d) $\begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$

(4) Let V be a vector space with $\dim(V) = 5$ and S a subspace of V. If $v_1, v_2, v_3, v_4 \in S$ with $\text{span}(v_1, v_2, v_3, v_4) = S$, then v_1, v_2, v_3, v_4

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dim of $S \leq 4$

(a) are linearly dependent
(b) form a basis for S
(c) is not a spanning set for V
(d) are linearly independent.

(5) The coordinate vector of $2 + 6x$ with respect to the basis $[2x, 4]$ is

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(a) $(2, 4)^T$
(b) $(4, 2)^T$
(c) $(2, 3)^T$
(d) $(3, 2)^T$

الإجابة : $(3, \frac{1}{2})^T$

(6) The dimension of the subspace $S = \{(a - b + 2c, a + 2b - 4c, -b + 2c)^T, a, b, c \in R\}$ is

(a) 1

(b) 2

(c) 3

(d) 0

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 1 & 2 & -4 & 0 \\ 0 & -1 & 2 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & 3 & -6 & 0 \\ 0 & -1 & 2 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

rank = 2

dim of col. space = 2

(7) If A is an $n \times n$-matrix and for each $b \in \mathbb{R}^n$ the system $Ax = b$ has a unique solution, then

Square   one Sol.

(a) $\text{rank}(A) = n$

(b) $\text{nullity}(A) = 0$

(c) A is nonsingular

(d) all of the above

(8) If A is an $m \times n$-matrix, $b \in$ column space of A and the columns of A are linearly independent, then the system $Ax = b$ has

(a) exactly one solution

(b) infinitely many solutions خطأ

(c) no solution خطأ

(d) none of the above

(9) If A is a 3×4 matrix such that $\text{nullity}(A) = 1$ and $b = (1, 2, 3)^T$, then the system $Ax = b$ has

rank = 3

(a) at most one solution $\nexists$

(b) exactly one solution unde. det.

(c) infinitely many solutions

(d) no solution

non zero rows

(10) The transition matrix from the ordered basis $[e_1, e_2]$ of $\mathbb{R}^2$ to the ordered basis $[\begin{pmatrix} -3 \\ 7 \end{pmatrix}, \begin{pmatrix} -2 \\ 5 \end{pmatrix}]$ is

(a) $\begin{pmatrix} -3 & -7 \\ 2 & 5 \end{pmatrix}$

(b) $\begin{pmatrix} -5 & -2 \\ 7 & 3 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & 7 \\ -2 & -5 \end{pmatrix}$

(d) $\begin{pmatrix} 5 & 2 \\ -7 & -3 \end{pmatrix}$

$S \to F$

$$T_{F \to S} = \begin{bmatrix} -3 & -2 \\ 7 & 5 \end{bmatrix}$$

$-15 + 14 = -1$

$$(T_{F \to S})^{-1} = T_{S \to F} = \frac{1}{-1}\begin{bmatrix} 5 & 2 \\ -7 & -3 \end{bmatrix} = \begin{bmatrix} -5 & -2 \\ +7 & +3 \end{bmatrix}$$

(11) Let $S = \{ax^2 + bx + a + b ; a, b \in \mathbb{R}\}$. Then a basis for S and the dimension of S are

$a(x^2+1) + b(x+1)$

(a) basis: $x^2 + 1, x$; dimension $= 2$

(b) basis: $x^2 + 1, x + 1$; dimension $= 2$

(c) basis: $x^2, 1$; dimensin $= 2$

(d) basis: $x^2, x, 1$; dimension $= 3$

(12) Let v_1, v_2, v_3 be linearly dependent in a vector space V, $V = \text{span}(v_1, v_2 v_3)$, then

(a) $\dim(V) = 3$

(b) $\dim(V) \leq 2$

(c) $\dim(V) \geq 3$

(d) none of the above

(13) Let $S = \{ax^2 + ax + b : a, b \in \mathbb{R}\}$. Then a basis and the dimension of S are

(a) basis: $x^2, x+1$; dimension $= 2$

$a(x^2 + x) + b$

(b) basis: $x^2, 1$; dimensin $= 2$

(c) basis: $x^2 + x, 1$; dimension $= 2$

(d) basis: $x^2 + 1, x$; dimension $= 2$

(14) An $n \times n$ matrix is singular iff

(a) the rows of A form a basis for $\mathbb{R}^{1\times n}$

(b) the columns of A form a basis for $\mathbb{R}^n$

(c) $0 \in N(A)$

(d) none of the above

(15) Let A be an $n \times n$ matrix. Then

(a) ~~The row space of A equals the null space of A~~

(b) The row space of A is contained in the column space

(c) The row space of A has the same dimension as the column space

(d) The row space of A equals the column space of A of A

(16) If A is a 4×3 matrix such that nullity of $A = 0$, and $b = \begin{pmatrix} 1 \\ 3 \\ 2 \\ 0 \end{pmatrix}$, then the system $Ax = b$

rank = 3

(a) has exactly one solution

(b) is either inconsistent or has an infinite number of solutions

(c) is inconsistent

(d) is either inconsistent or has one solution

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Question 3 (8 points). Let $E = [1+x, 1-x]$, $F = [1, x]$ be two ordered bases for P_2

1. Find the transition matrix from E to F.
2. Find the coordinate vector of $p(x) = -(1+x) + 2(1-x)$ with respect to F.

(1) $T_{E\to F} = (U^{-1})\, W$

Such that: $W \in E$, $U \in F$.

$W = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

? $U^{-1} \Rightarrow \left(\begin{array}{cc|cc} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cc|cc} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right)$

$U^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \frac{1}{\det U} \operatorname{adj}(U)$ Check

$T_{E\to F} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

(1) $\boxed{T_{E\to F} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}}$

(2) $[-(1+x) + 2(1-x)]_F = ??$

$\Rightarrow [-(1+x) + 2(1-x)]_E = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$[-(1+x) + 2(1-x)]_F = T_{E\to F} \cdot [-(1+x) + 2(1-x)]_E$

$= \underset{2\times 2}{\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} -1 \\ 2 \end{bmatrix}} = \begin{bmatrix} -1+2 \\ -1-2 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

(2) $\cancel{[p(x)]}$ $\boxed{[-(1+x) + 2(1-x)]_F = \begin{pmatrix} 1 \\ -3 \end{pmatrix}}$

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Question 4 (8 points). If $A = \begin{pmatrix} 1 & 1 & 0 & 2 & 0 \\ 1 & 2 & -1 & 0 & 1 \\ 2 & 3 & -1 & 2 & 1 \\ 4 & 5 & -1 & 6 & 1 \end{pmatrix}$ and the row echelon form of A is

or

$$U = \begin{pmatrix} 1 & 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \xrightarrow{-R_2+R_1} \begin{bmatrix} 1 & 0 & 1 & 4 & -1 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find a basis for the row space of A.

$$A = \begin{pmatrix} 1 & 1 & 0 & 2 & 0 \\ 1 & 2 & -1 & 0 & 1 \\ 2 & 3 & -1 & 2 & 1 \\ 4 & 5 & -1 & 6 & 1 \end{pmatrix} \Rightarrow \cancel{\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 2 & 3 \\ 4 & 5 \end{pmatrix}} \Rightarrow \cancel{\cdots}$$

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 2 & 1 \\ 4 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

The basis for the row space of A: $(1\ 1\ 0\ 2\ 0)$, $(1\ 2\ -1\ 0\ 1)$

from $C(A)$

(b) Find a basis for the column space of A.

$$C(A) = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

The one's Column.

(c) Find a basis for null space A

$Ax = 0 \Rightarrow$ نحولها U $\Rightarrow \left(\begin{array}{ccccc|c} 1 & 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$ The Sol.

$x_5 = \alpha, \quad x_4 = \beta, \quad x_3 = \gamma$

$x_2 = x_3 + 2x_4 - x_5 = \boxed{\gamma + 2\beta - \alpha} = x_2$

$x_1 = -2x_4 - x_2$

$= -2\beta - (\gamma + 2\beta - \alpha) = -\gamma + \alpha - 4\beta$

Null Space (A):

$$\begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -4 \\ 2 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

(d) Find Rank(A), Nullity(A).

Rank (A) = 2

Nullity (A) = 3

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