

Name.....

Number.....

Section .....

**(Q1)** [60 points] Fill the blanks with true (T) or false (F).

- [ ] (1) If  $E$  an elementary matrix of type II, then it is both nonsingular and symmetric.
- [ ] (2) If  $A$  and  $B$  are  $n \times n$  symmetric matrices, then the matrix  $AB + BA$  is also symmetric.
- [ ] (3) If  $A$  is an  $n \times n$  singular matrix, then the system  $Ax = b$  has infinitely many solutions.
- [ ] (4) If  $E$  is an elementary matrix of type III, then  $E^{-1} = E$ .
- [ ] (5) If  $A$  and  $B$  are symmetric matrices, then  $AB$  is also symmetric.
- [ ] (6) If  $A^2 = I$ , then  $A^{-1} = A$ .
- [ ] (7) The product of two elementary matrices is an elementary matrix.
- [ ] (8) Any  $m \times n$  linear system  $Ax = 0$  has a nontrivial solution if  $m > n$ .
- [ ] (9) If  $A$  is a nonsingular matrix, then  $A^T$  is nonsingular.
- [ ] (10) The sum of two triangular matrices is a triangular matrix.
- [ ] (11) If  $E$  is an elementary matrix, then  $E^T$  is also elementary of the same type.
- [ ] (12) If  $A$  is a singular matrix, then the system  $Ax = 0$  has infinite number of solutions.
- [ ] (13) If  $A$  is a singular matrix and  $U$  is the *RREF* of  $A$ , then  $U$  must have at least one zero row.
- [ ] (14) Any invertible matrix is a product of elementary matrices.
- [ ] (15) If  $A$  is symmetric and nonsingular, then  $A^{-1}$  is symmetric.
- [ ] (16) All  $5 \times 5$  nonsingular matrices are row equivalent.
- [ ] (17) If  $A$  is a square matrix and the system  $Ax = 0$  has a nontrivial solution, then  $A$  is nonsingular.
- [ ] (18) If  $A$  is an  $n \times n$  nonsingular matrix, then  $A^3$  is nonsingular.
- [ ] (19) If  $A$  is a nonsingular matrix and  $\alpha$  a nonzero scalar, then  $(\alpha A)^{-1} = \alpha A^{-1}$ .
- [ ] (20) If  $A$  and  $B$  are  $n \times n$  diagonal matrices, then  $AB = BA$ .
- [ ] (21) If  $A$  is a  $3 \times 3$  matrix with  $a_1 = a_2 = a_3$ , then  $Ax = 0$  has infinitely many solutions.
- [ ] (22) If  $A$  and  $B$  are nonsingular  $n \times n$  matrices, then  $A + B$  is also nonsingular.
- [ ] (23) If  $A$  is both symmetric and skew-symmetric, then  $A$  is a zero matrix.
- [ ] (24) If the system  $Ax = b$  is consistent, then  $b$  is a linear combination of the columns of  $A$ .
- [ ] (25) A square matrix  $A$  is nonsingular iff its *RREF* is the identity matrix.
- [ ] (26) If  $b$  can be written as a linear combination of the columns of a singular matrix  $A$ , then the system  $Ax = b$  has infinitely many solutions.
- [ ] (27) If  $A, B, C$  are  $n \times n$  nonsingular matrices, then  $A^2 - B^2 = (A - B)(A + B)$ .
- [ ] (28) If  $b$  is any column of the matrix  $A$ , then the system  $Ax = b$  is consistent.
- [ ] (29) The sum of a symmetric and skew-symmetric matrices is skew-symmetric.

- [ ] (30) Let  $A$  be nonsingular. If  $A$  is skew-symmetric, then  $A^{-1}$  is skew-symmetric.
- [ ] (31) Let  $A$  be nonsingular. If  $A$  is upper triangular, then  $A^{-1}$  is upper triangular.
- [ ] (32) Let  $A$  be nonsingular. If  $A$  is diagonal, then  $A^{-1}$  is diagonal.
- [ ] (33) If  $A$  is a  $3 \times 3$  matrix and  $(2, 3, -1)^T$  is a solution to  $Ax = 0$ , then  $(-6, -9, 3)^T$  is also a solution.
- [ ] (34) If the square system  $Ax = b$  has more than one solution, then  $A$  is singular.
- [ ] (35) If  $A$  is a  $4 \times 4$  nonsingular matrix, then  $AA^T$  is both symmetric and nonsingular.
- [ ] (36) If  $A$  is a  $4 \times 4$  matrix and  $Ax = 0$  has only the zero solution, then  $A$  is row equivalent to  $I$ .
- [ ] (37) If  $A$  is a nonsingular matrix, then  $(A^T)^T = (A^{-1})^{-1}$ .
- [ ] (38) Every linear system with eight unknowns in three equations is consistent.
- [ ] (39) If the augmented matrix of a  $3 \times 2$  system is row equivalent to  $I$ , then this system is inconsistent.
- [ ] (40) The identity matrix is row equivalent to any elementary matrix of the same size.