

Name.....

Number.....

Section .....

**(Q1)** [100 points] Fill the blanks with true (T) or false (F).

- [ ] (1) If  $A^2 = I$ , then  $\det(A) = \pm 1$
- [ ] (2) If  $A$  and  $B$  are  $n \times n$  nonsingular matrices, then  $\det(A - B) = \det(A) - \det(B)$ .
- [ ] (3) If  $A$  is an  $2 \times 2$  matrix, then  $|\alpha A| = \alpha^4 |A|$ .
- [ ] (4) If  $\det(A) = 1$ , then  $A^{-1} = \text{adj} A$ .
- [ ] (5) If  $A$  and  $B$  are  $n \times n$  matrices such that  $AB$  is singular, then at least  $A$  or  $B$  is singular.
- [ ] (6) If  $A = \begin{bmatrix} 2 & 5 & 7 \\ 1 & 3 & 4 \\ 2 & 1 & 6 \end{bmatrix}$ , then the  $(2, 3)$  entry of  $A^{-1}$  is  $-\frac{1}{3}$ .
- [ ] (7) If  $A$  and  $B$  are  $2 \times 2$  matrices such that  $\det(BA) = 0$ , then  $\det(A) = 0$  and  $\det(B) = 0$ .
- [ ] (8) If  $A$  and  $B$  are  $n \times n$  matrices, then  $\det((AB)^T) = \det(A)\det(B)$ .
- [ ] (9) Row equivalent matrices have the same determinants.
- [ ] (10) If  $A$  is singular, then  $\text{adj} A$  is also singular.
- [ ] (11) Cramer's rule can be used to solve any square linear system.
- [ ] (12) If  $A$  and  $B$  are  $n \times n$  matrices and  $A$  is singular, then  $AB$  is singular.
- [ ] (13)  $\det(AB) = \det(A)\det(B)$  only when  $A$  and  $B$  are nonsingular.
- [ ] (14) If  $A$  is an  $n \times n$  matrix, then  $|A^n| = |A|^n$ .
- [ ] (15) Every diagonal matrix is nonsingular.
- [ ] (16) Every Elementary matrix is nonsingular.
- [ ] (17)  $\det(-I) = -\det(I)$ .
- [ ] (18) If  $A$  is a  $5 \times 5$  skew-symmetric matrix, then the system  $Ax = 0$  has a nontrivial solution.
- [ ] (19)  $|AB| = |BA|$  for any  $n \times n$  matrices  $A$  and  $B$ .
- [ ] (20) If  $A, B, S$  are  $n \times n$  matrices such that  $S$  is nonsingular and  $A = SBS^{-1}$ , then  $|A| = |B|$ .
- [ ] (21) If  $A$  is a  $7 \times 7$  nonsingular matrix, then the RREF of  $A$  has 7 nonzero rows.
- [ ] (22) If  $|A| = 1$ , then  $A = I$ .
- [ ] (23) If  $A = LU$  is the  $LU$  factorization of  $A$  and  $U$  is nonsingular, then  $A$  is nonsingular.
- [ ] (24) If  $A$  is a singular matrix and  $U$  is the REF of  $A$ , then  $|U| = 0$ .
- [ ] (25) If  $A$  is a square and nonsingular matrix with  $|\text{adj} A| = |A|$ , then  $A$  is  $2 \times 2$ .
- [ ] (26) If  $A$  and  $B$  are square nonzero matrices with  $AB = 0$ , then both  $A$  and  $B$  are singular.
- [ ] (27) If  $\det(A) = 0$ , the  $A$  is a zero matrix.
- [ ] (28) If the diagonal entries of a square matrix are all zero, then it is singular.

- [ ] (29) If  $A$  is a  $3 \times 3$  with  $a_1 = a_3$ , then  $\det(A) = 0$  .
- [ ] (30) If the system  $A^3x = 0$  has a nontrivial solution, then  $A$  is singular .
- [ ] (31) If  $E$  is a  $4 \times 4$  elementary matrix, then the linear system  $Ex = b$  is consistent for any  $b \in \mathbb{R}^4$  .
- [ ] (32) If  $A$  is a square matrix and one of the rows is a linear combination of the others, then  $|A| = 0$ .
- [ ] (33) If  $A$  is an  $n \times n$  matrix with  $n > 1$ , then  $|\text{adj}A| = |A|^{n-1}$  .
- [ ] (34) If  $A$  is an  $n \times n$  matrix, then  $\det(A^T A) \geq 0$ .
- [ ] (35) There is a matrix  $A$  such that  $A^{-1} = \begin{bmatrix} 4 & 2 \\ 6 & 3 \end{bmatrix}$ .
- [ ] (36) If  $A^T$  is singular, then  $A^2$  is also singular.
- [ ] (37) There exists a nonsingular matrix with two identical columns.
- [ ] (38) A matrix having a zero row cannot be row equivalent to  $I$  .
- [ ] (39) If  $E$  and  $F$  are  $2 \times 2$  elementary matrices of type I and III respectively, then  $\det(-2E^T F^{-1}) = 4$ .
- [ ] (40) If  $A$  is a nonsingular diagonal matrix, then  $A^{-1}$  is also diagonal.
- [ ] (41)  $\det(AB^T) = \det(A^T B)$  for any  $n \times n$  matrices  $A$  and  $B$ .
- [ ] (42) If  $\det(A - B) = 0$ , then  $A = B$ .
- [ ] (43) If  $\det(A - B) = 0$ , then the matrix equation  $Ax = Bx$  has a nonzero solution.
- [ ] (44) A triangular matrix is nonsingular if and only if its diagonal elements are all nonzero.
- [ ] (45) If  $A$  is a nonzero matrix with  $A^k = 0$  for some positive integer  $k$ , then  $A$  is singular.
- [ ] (46) If  $x$  and  $y$  are two distinct vectors in  $\mathbb{R}^n$  such that  $Ax = Ay$ , then  $\det(A) = 0$ .
- [ ] (47) If  $A$  and  $B$  are  $3 \times 3$  matrices with  $|A| = 2$  and  $|B| = -6$ , then  $|-3AB^{-1}| = 9$ .
- [ ] (48) If  $A$  is a nonsingular matrix, then  $\text{adj}A^{-1} = (\text{adj}A)^{-1}$ .
- [ ] (49) If  $A$  is a symmetric matrix, then  $\text{adj}A$  is also symmetric.
- [ ] (50) If  $E$  and  $F$  are  $3 \times 3$  elementary matrices of type I and  $A$  is  $3 \times 3$ , then  $|-AEF| = |A|$ .