Answers to Selected Exercises

Chapter 1

1.1 1. (a) $(11, 3)$; (b) $(4, 1, 3)$; (c) $(-2, 0, 3, 1)$;
(d) $(-2, 3, 0, 3, 1)$

2. (a) $\begin{Bmatrix} 1 & -3 \\ 0 & 2 \end{Bmatrix}$; (b) $\begin{Bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{Bmatrix}$;

(c) $\begin{Bmatrix} 1 & 2 & 2 & 1 \\ 0 & 3 & 1 & -2 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 4 \end{Bmatrix}$

3. (a) One solution. The two lines intersect at the point (3, 1).
(b) No solution. The lines are parallel.
(c) Infinitely many solutions. Both equations represent the same line.
(d) No solution. Each pair of lines intersect in a point; however, there is no point that is on all three lines.

4. (a) $\left\{\begin{array}{rr|r} 1 & 1 & 4 \\ 1 & -1 & 2 \end{array}\right\}$; (c) $\left\{\begin{array}{rr|r} 2 & -1 & 3 \\ -4 & 2 & -6 \end{array}\right\}$;

(d) $\left\{\begin{array}{rr|r} 1 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & 3 & 3 \end{array}\right\}$

6. (a) $(1, -2)$; (b) $(3, 2)$; (c) $(\frac{1}{2}, \frac{2}{3})$;
(d) $(1, 1, 2)$; (e) $(-3, 1, 2)$;
(f) $(-1, 1, 1)$; (g) $(1, 1, -1)$;
(h) $(4, -3, 1, 2)$

7. (a) $(2, -1)$; (b) $(-2, 3)$

8. (a) $(-1, 2, 1)$; (b) $(3, 1, -2)$

1.2 1. Row echelon form: (a), (c), (d), (g), and (h); reduced row echelon form: (c), (d), and (g)

2. (a) Inconsistent;
(c) consistent, infinitely many solutions;
(d) consistent $(4, 5, 2)$; (e) inconsistent;
(f) consistent, $(5, 3, 2)$

3. (b) $\emptyset$;
(c) $\{(2 + 3\alpha, \alpha, -2) \mid \alpha \text{ real}\}$;
(d) $\{(5 - 2\alpha - \beta, \alpha, 4 - 3\beta, \beta) \mid \alpha, \beta \text{ real}\}$;
(e) $\{(3 - 5\alpha + 2\beta, \alpha, \beta, 6) \mid \alpha, \beta \text{ real}\}$;
(f) $\{(\alpha, 2, -1) \mid \alpha \text{ real}\}$

4. (a) x_1, x_2, x_3 are lead variables.
(c) x_1, x_3 are lead variables and x_2 is a free variable.
(e) x_1, x_4 are lead variables and x_2, x_3 are free variables.

5. (a) $(5, 1)$; (b) inconsistent; (c) $(0, 0)$;
(d) $\left\{\left(\frac{5-\alpha}{4}, \frac{1+7\alpha}{8}, \alpha\right) \middle| \alpha \text{ real}\right\}$;
(e) $\{(8 - 2\alpha, \alpha - 5, \alpha)\}$;
(f) inconsistent;
(g) inconsistent; (h) inconsistent;
(i) $(0, \frac{3}{2}, 1)$;
(j) $\{(2 - 6\alpha, 4 + \alpha, 3 - \alpha, \alpha)\}$;
(k) $\{(\frac{15}{4} - \frac{5}{8}\alpha - \beta, -\frac{1}{4} - \frac{1}{8}\alpha, \alpha, \beta)\}$;

6. (a) $(0, -1)$;
(b) $\{(\frac{3}{4} - \frac{5}{8}\alpha, -\frac{1}{4} - \frac{1}{8}\alpha, \alpha, 3) \mid \alpha \text{ is real}\}$;
(d) $\{\alpha(-\frac{4}{3}, 0, \frac{1}{3}, 1)\}$

8. $a \neq -2$

9. $\beta = 2$

10. (a) $a = 5, b = 4$; (b) $a = 5, b \neq 4$

11. (a) $(-2, 2)$; (b) $(-7, 4)$

12. (a) $(-3, 2, 1)$; (b) $(2, -2, 1)$

15. $x_1 = 280, x_2 = 230, x_3 = 350, x_4 = 590$

19. $x_1 = 2,\ x_2 = 3,\ x_3 = 12,\ x_4 = 6$

20. 6 moles N_2, 18 moles H_2, 21 moles O_2

21. All three should be equal, i.e., $x_1 = x_2 = x_3$.

22. (a) $(5, 3, -2)$; (b) $(2, 4, 2)$;
(c) $(2, 0, -2, -2, 0, 2)$

1.3 1. (a) $\begin{Bmatrix} 6 & 2 & 8 \\ -4 & 0 & 2 \\ 2 & 4 & 4 \end{Bmatrix}$;

(b) $\begin{bmatrix} 4 & 1 & 6 \\ -5 & 1 & 2 \\ 3 & -2 & 3 \end{bmatrix}$;

(c) $\begin{bmatrix} 3 & 2 & 2 \\ 5 & -3 & -1 \\ -4 & 16 & 1 \end{bmatrix}$;

(d) $\begin{bmatrix} 3 & 5 & -4 \\ 2 & -3 & 16 \\ 2 & -1 & 1 \end{bmatrix}$;

(f) $\begin{bmatrix} 5 & 5 & 8 \\ -10 & -1 & -9 \\ 15 & 4 & 6 \end{bmatrix}$;

(h) $\begin{bmatrix} 5 & -10 & 15 \\ 5 & -1 & 4 \\ 8 & -9 & 6 \end{bmatrix}$

2. (a) $\begin{bmatrix} 15 & 19 \\ 4 & 0 \end{bmatrix}$; (c) $\begin{bmatrix} 19 & 21 \\ 17 & 21 \\ 8 & 10 \end{bmatrix}$;

(f) $\begin{bmatrix} 6 & 4 & 8 & 10 \\ -3 & -2 & -4 & -5 \\ 9 & 6 & 12 & 15 \end{bmatrix}$

(b) and (e) are not possible.

3. (a) 3×3; (b) 1×2

4. (a) $\begin{bmatrix} 3 & 2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$;

(b) $\begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & -1 \\ 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$;

(c) $\begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$

9. (a) $\mathbf{b} = 2\mathbf{a}_1 + \mathbf{a}_2$

10. (a) inconsistent; (b) consistent;
(c) inconsistent

13. $\mathbf{b} = (8, -7, -1, 7)^T$

14. $\mathbf{w} = (\frac{1}{2}, \frac{1}{3}, \frac{1}{6})^T$, $\mathbf{r} = (\frac{43}{120}, \frac{45}{120}, \frac{32}{120})^T$

18. $b = a_{22} - \dfrac{a_{12}a_{21}}{a_{11}}$

1.4

7. $A = A^2 = A^3 = A^n$

8. $A^{2n} = I, A^{2n+1} = A$

13. (a) $\begin{bmatrix} 1 & -2 \\ -3 & 7 \end{bmatrix}$, (c) $\begin{bmatrix} 1 & -\frac{3}{2} \\ -1 & 2 \end{bmatrix}$

31. 4500 married, 5500 single

32. (b) 0 walks of length 2 from V_2 to V_3 and 3 walks of length 2 from V_2 to V_5;
(c) 6 walks of length 3 from V_2 to V_3 and 2 walks of length 3 from V_2 to V_5

33. (a) $A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$;

(c) 5 walks of length 3 from V_2 to V_4 and 7 walks of length 3 or less

1.5

1. (a) type I;
(b) not an elementary matrix;
(c) type III; (d) type II

3. (a) $\begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}$; (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$;

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$

4. (a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$; (b) $\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$;

(c) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

5. (a) $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$;

(b) $F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

6. (a) $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$;

(b) $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$;

(c) $E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

8. (a) $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$,

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$

9. (b) (i) $(0, -1, 1)^T$, (ii) $(-4, -2, 5)^T$,
(iii) $(0, 3, -2)^T$

10. (a) $\begin{bmatrix}0&1\\1&1\end{bmatrix}$; (b) $\begin{bmatrix}3&-5\\-1&2\end{bmatrix}$;

(c) $\begin{bmatrix}-4&3\\ \frac{3}{2}&-1\end{bmatrix}$; (d) $\begin{bmatrix}\frac{1}{3}&0\\-1&\frac{1}{3}\end{bmatrix}$;

(f) $\begin{bmatrix}3&0&-5\\0&\frac{1}{3}&0\\-1&0&2\end{bmatrix}$;

(g) $\begin{bmatrix}2&-3&3\\-\frac{3}{5}&\frac{6}{5}&-1\\-\frac{2}{5}&-\frac{1}{5}&0\end{bmatrix}$;

(h) $\begin{bmatrix}-\frac{1}{2}&-1&-\frac{1}{2}\\-2&-1&-1\\ \frac{3}{2}&1&\frac{1}{2}\end{bmatrix}$

11. (a) $\begin{bmatrix}-1&0\\4&2\end{bmatrix}$; (b) $\begin{bmatrix}-8&5\\-14&9\end{bmatrix}$

12. (a) $\begin{bmatrix}20&-5\\-34&7\end{bmatrix}$; (c) $\begin{bmatrix}0&-2\\-2&2\end{bmatrix}$

1.6 1. (b) $\begin{bmatrix}I\\A^{-1}\end{bmatrix}$; (c) $\begin{bmatrix}A^TA&A^T\\A&I\end{bmatrix}$;

(d) AA^T+I; (e) $\begin{bmatrix}I&A^{-1}\\A&I\end{bmatrix}$

3. (a) $A\mathbf{b}_1=\begin{bmatrix}3\\3\end{bmatrix}$, $A\mathbf{b}_2=\begin{bmatrix}4\\-1\end{bmatrix}$;

(b) $\begin{bmatrix}1&1\end{bmatrix}B=\begin{bmatrix}3&4\end{bmatrix}$,

$\begin{bmatrix}2&-1\end{bmatrix}B=\begin{bmatrix}3&-1\end{bmatrix}$;

(c) $AB=\begin{bmatrix}3&4\\3&-1\end{bmatrix}$

4. (a) $\left[\begin{array}{cc|cc}3&1&1&1\\3&2&1&2\\\hline 1&1&1&1\\1&2&1&1\end{array}\right]$;

(c) $\left[\begin{array}{cc|cc}2&2&2&2\\2&4&2&2\\\hline 3&1&1&1\\3&2&1&2\end{array}\right]$;

(d) $\left[\begin{array}{cc|cc}1&2&1&1\\1&1&1&1\\\hline 3&2&1&2\\3&1&1&1\end{array}\right]$

5. (b) $\left[\begin{array}{ccc|c}0&2&0&-2\\8&5&8&-5\\3&2&3&-2\\\hline 5&3&5&-3\end{array}\right]$;

(d) $\left[\begin{array}{cc}3&-3\\2&-2\\1&-1\\\hline 5&-5\\4&-4\end{array}\right]$

13. $A^2=\begin{bmatrix}B&O\\O&B\end{bmatrix}$, $A^4=\begin{bmatrix}B^2&O\\O&B^2\end{bmatrix}$

14. (a) $\begin{bmatrix}O&I\\I&O\end{bmatrix}$; (b) $\begin{bmatrix}I&O\\-B&I\end{bmatrix}$

CHAPTER TEST A

1. False 2. True 3. True 4. True 5. False
6. False 7. False 8. False 9. False 10. True
11. True 12. True 13. True 14. False
15. True

Chapter 2

2.1 1. (a) $\det(M_{21})=-8$, $\det(M_{22})=-2$, $\det(M_{23})=5$;

(b) $A_{21}=8$, $A_{22}=-2$, $A_{23}=-5$

2. (a) and (c) are nonsingular.

3. (a) 1; (b) 4; (c) 0; (d) 58;
(e) -39; (f) 0; (g) 8; (h) 20

4. (a) 2; (b) -4; (c) 0; (d) 0

5. $-x^3+ax^2+bx+c$

6. $\lambda=6$ or -1

2.2 1. (a) -24; (b) 30; (c) -1

2. (a) 10; (b) 20

3. (a), (e), and (f) are singular while (b), (c), and (d) are nonsingular.

4. $c=5$ or -3

7. (a) 20; (b) 108; (c) 160; (d) $\frac{5}{4}$

9. (a) -6; (c) 6; (e) 1

13. $\det(A)=u_{11}u_{22}u_{33}$

2.3 1. (a) $\det(A)=-7$, $\operatorname{adj}A=\begin{bmatrix}-1&-2\\-3&1\end{bmatrix}$,

$A^{-1}=\begin{bmatrix}\frac{1}{7}&\frac{2}{7}\\ \frac{3}{7}&-\frac{1}{7}\end{bmatrix}$;

(c) $\det(A)=3$, $\operatorname{adj}A=\begin{bmatrix}-3&5&2\\0&1&1\\6&-8&-5\end{bmatrix}$,

$A^{-1}=\frac{1}{3}\operatorname{adj}A$

2. (a) $(\frac{5}{7},\frac{8}{7})$; (b) $(\frac{11}{5},-\frac{4}{5})$;
(c) $(4,-2,2)$; (d) $(2,-1,2)$;
(e) $(-\frac{2}{3},\frac{2}{3},\frac{1}{3},0)$

3. $-\frac{3}{4}$

4. $(\frac{1}{2}, -\frac{3}{4}, 1)^T$
5. (a) $\det(A) = 0$, so A is singular.
(b) $\operatorname{adj} A = \begin{bmatrix} -1 & 2 & -1 \\ 2 & -4 & 2 \\ -1 & 2 & -1 \end{bmatrix}$ and
$A \operatorname{adj} A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
9. (a) $\det(\operatorname{adj}(A)) = 8$ and $\det(A) = 2$;
(b) $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & -1 & 1 \\ 0 & -6 & 2 & -2 \\ 0 & 1 & 0 & 1 \end{bmatrix}$
14. DO YOUR HOMEWORK.

CHAPTER TEST A

1. True 2. False 3. False 4. True 5. False
6. True 7. True 8. True 9. False 10. True

Chapter 3

3.1

1. (a) $\|\mathbf{x}_1\| = 10$, $\|\mathbf{x}_2\| = \sqrt{17}$;
(b) $\|\mathbf{x}_3\| = 13 < \|\mathbf{x}_1\| + \|\mathbf{x}_2\|$
2. (a) $\|\mathbf{x}_1\| = \sqrt{5}$, $\|\mathbf{x}_2\| = 3\sqrt{5}$;
(b) $\|\mathbf{x}_3\| = 4\sqrt{5} = \|\mathbf{x}_1\| + \|\mathbf{x}_2\|$
7. If $\mathbf{x} + \mathbf{y} = \mathbf{x}$ for all $\mathbf{x}$ in the vector space, then $\mathbf{0} = \mathbf{0} + \mathbf{y} = \mathbf{y}$.
8. If $\mathbf{x} + \mathbf{y} = \mathbf{x} + \mathbf{z}$, then $-\mathbf{x} + (\mathbf{x} + \mathbf{y}) = -\mathbf{x} + (\mathbf{x} + \mathbf{z})$ and the conclusion follows using axioms 1, 2, 3, and 4.
11. V is not a vector space. Axiom 6 does not hold.

3.2

1. (a) and (c) are subspaces; (b), (d), and (e) are not.
2. (b) and (c) are subspaces; (a) and (d) are not.
3. (a), (c), (e), and (f) are subspaces; (b), (d), and (g) are not.
4. (a) $\{(0, 0)^T\}$;
(b) $\operatorname{Span}((-2, 1, 0, 0)^T, (3, 0, 1, 0)^T)$;
(c) $\operatorname{Span}((1, 1, 1)^T)$;
(d) $\operatorname{Span}((-5, 0, -3, 1)^T, (-1, 1, 0, 0)^T)$
5. Only the set in part (c) is a subspace of P_4.
6. (a), (b), and (d) are subspaces.
11. (a), (c), and (e) are spanning sets.
12. (a) and (b) are spanning sets.
19. (b) and (c)

3.3

1. (a) and (e) are linearly independent; (b), (c), and (d) are linearly dependent.
2. (a) and (e) are linearly independent; (b), (c), and (d) are not.
3. (a) and (b) are all of 3-space;
(c) a plane through $(0, 0, 0)$;
(d) a line through $(0, 0, 0)$;
(e) a plane through $(0, 0, 0)$
4. (a) linearly independent;
(b) linearly independent;
(c) linearly dependent
8. (a) and (b) are linearly dependent while (c) and (d) are linearly independent.
11. When α is an odd multiple of $\pi/2$. If the graph of $y = \cos x$ is shifted to the left or right by an odd multiple of $\pi/2$, we obtain the graph of either $\sin x$ or $-\sin x$.

3.4

1. Only in parts (a) and (e) do they form a basis.
2. Only in part (a) do they form a basis.
3. (c) 2
4. 1
5. (c) 2;
(d) a plane through $(0, 0, 0)$ in 3-space
6. (b) $\{(1, 1, 1)^T\}$, dimension 1;
(c) $\{(1, 0, 1)^T, (0, 1, 1)^T\}$, dimension 2
7. basis $\{(1, 1, 0, 0)^T, (1, -1, 1, 0)^T, (0, 2, 0, 1)^T\}$
11. $\{x^2 + 2, x + 3\}$
12. (a) $\{E_{11}, E_{22}\}$; (c) $\{E_{11}, E_{21}, E_{22}\}$;
(e) $\{E_{12}, E_{21}, E_{22}\}$;
(f) $\{E_{11}, E_{22}, E_{21} + E_{12}\}$
13. 2
14. (a) 3; (b) 3; (c) 2; (d) 2
15. (a) $\{x, x^2\}$; (b) $\{x - 1, (x - 1)^2\}$;
(c) $\{x(x - 1)\}$

3.5

1. (a) $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$; (b) $\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$;
(c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
2. (a) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$; (b) $\begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$;
(c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

3. (a) $\begin{bmatrix} \frac{5}{2} & \frac{7}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$; (b) $\begin{bmatrix} 11 & 14 \\ -4 & -5 \end{bmatrix}$;

(c) $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$

4. $[\mathbf{x}]_E = (-1, 2)^T$, $[\mathbf{y}]_E = (5, -8)^T$, $[\mathbf{z}]_E = (-1, 5)^T$

5. (a) $\begin{bmatrix} 2 & 0 & -1 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$; (b) $(1, -4, 3)^T$;

(c) $(0, -1, 1)^T$; (d) $(2, 2, -1)^T$

6. (a) $\begin{bmatrix} 1 & -1 & -2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$; (b) $\begin{bmatrix} 7 \\ 5 \\ -2 \end{bmatrix}$

7. $\mathbf{w}_1 = (5, 9)^T$ and $\mathbf{w}_2 = (1, 4)^T$

8. $\mathbf{u}_1 = (0, -1)^T$ and $\mathbf{u}_2 = (1, 5)^T$

9. (a) $\begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$; (b) $\begin{bmatrix} \frac{1}{4} & -\frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} \end{bmatrix}$

10. $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

3.6

2. (a) 3; (b) 3; (c) 2

3. (a) $\mathbf{u}_2$, $\mathbf{u}_4$, $\mathbf{u}_5$ are the column vectors of U corresponding to the free variables.
$\mathbf{u}_2 = 2\mathbf{u}_1$, $\mathbf{u}_4 = 5\mathbf{u}_1 - \mathbf{u}_3$, $\mathbf{u}_5 = -3\mathbf{u}_1 + 2\mathbf{u}_3$

4. (a) consistent; (b) inconsistent;
(e) consistent

5. (a) infinitely many solutions;
(c) unique solution

8. rank of $A = 3$; $\dim N(B) = 1$;

18. (b) $n - 1$

32. If $\mathbf{x}_j$ is a solution to $A\mathbf{x} = \mathbf{e}_j$ for $j = 1, \ldots, m$ and $X = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_m)$, then $AX = I_m$.

CHAPTER TEST A

1. True 2. False 3. False 4. False 5. True
6. True 7. False 8. True 9. True 10. False
11. True 12. False 13. True 14. False
15. False

Chapter 4

4.1

1. (a) reflection about x_2 axis;
(b) reflection about the origin;
(c) reflection about the line $x_2 = x_1$;
(d) the length of the vector is halved;
(e) projection onto x_2 axis

4. $(7, 18)^T$

5. All except (c) are linear transformations from R^3 into R^2.

6. (b) and (c) are linear transformations from R^2 into R^3.

7. (a), (b), and (d) are linear transformations.

9. (a) and (c) are linear transformations from P_2 into P_3.

10. $L(e^x) = e^x - 1$ and $L(x^2) = x^3/3$.

11. (a) and (c) are linear transformations from $C[0, 1]$ into R^1.

17. (a) $\ker(L) = \{\mathbf{0}\}$, $L(R^3) = R^3$;
(c) $\ker(L) = \text{Span}(\mathbf{e}_2, \mathbf{e}_3)$, $L(R^3) = \text{Span}((1, 1, 1)^T)$

18. (a) $L(S) = \text{Span}(\mathbf{e}_2, \mathbf{e}_3)$;
(b) $L(S) = \text{Span}(\mathbf{e}_1, \mathbf{e}_2)$

19. (a) $\ker(L) = P_1$, $L(P_3) = \text{Span}(x^2, x)$;
(c) $\ker(L) = \text{Span}(x^2 - x)$, $L(P_3) = P_2$

23. The operator in part (a) is one-to-one and onto.

4.2

1. (a) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$; (c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$;

(d) $\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$; (e) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

2. (a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$; (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$;

(c) $\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

3. (a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$; (b) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$;

(c) $\begin{bmatrix} 0 & 0 & 2 \\ 3 & 1 & 0 \\ 2 & 0 & -1 \end{bmatrix}$

4. (a) $(0, 0, 0)^T$; (b) $(2, -1, -1)^T$;
(c) $(-15, 9, 6)^T$

5. (a) $\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$; (b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$;

(c) $\begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix}$; (d) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

6. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$;

7. (b) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}$

8. (a) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 0 & -2 & -1 \end{bmatrix}$;
   (b) (i) $7\mathbf{y}_1 + 6\mathbf{y}_2 - 8\mathbf{y}_3$, (ii) $3\mathbf{y}_1 + 3\mathbf{y}_2 - 3\mathbf{y}_3$, (iii) $\mathbf{y}_1 + 5\mathbf{y}_2 + 3\mathbf{y}_3$

9. (a) square; (b) (i) contraction by a factor $\frac{1}{2}$, (ii) clockwise rotation by 45°, (iii) translation 2 units to the right and 3 units down

10. (a) $\begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$;
   (b) $\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$; (d) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

13. $\begin{bmatrix} 1 & \frac{1}{2} \\ 1 & 0 \end{bmatrix}$;

14. $\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ -2 & 0 & 0 \end{bmatrix}$; (a) $\begin{bmatrix} \frac{1}{2} \\ -2 \end{bmatrix}$ (d) $\begin{bmatrix} 5 \\ -8 \end{bmatrix}$

15. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$;

18. (a) $\begin{bmatrix} -1 & -3 & 1 \\ 0 & 2 & 0 \end{bmatrix}$; (c) $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 3 \end{bmatrix}$

4.3

1. For the matrix A, see the answers to Exercise 1 of Section 4.2.
   (a) $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$; (b) $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$;
   (c) $B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$; (d) $B = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$;
   (e) $B = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

2. (a) $\begin{bmatrix} 1 & 1 \\ -1 & -3 \end{bmatrix}$; (b) $\begin{bmatrix} 1 & 0 \\ -4 & -1 \end{bmatrix}$

3. $B = A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$

   (Note: in this case the matrices A and U commute; so $B = U^{-1}AU = U^{-1}UA = A$.)

4. $V = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & -2 \\ 1 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

5. (a) $\begin{bmatrix} 0 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$; (b) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$;
   (c) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; (d) $a_1x + a_2 2^n(1 + x^2)$

6. (a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$; (b) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$;
   (c) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

CHAPTER TEST A

1. False 2. True 3. True 4. False 5. False
6. True 7. True 8. True 9. True 10. False

Chapter 5

5.1

1. (a) 0°; (b) 90°

2. (a) $\sqrt{14}$ (scalar projection), $(2, 1, 3)^T$ (vector projection);
   (b) 0, $\mathbf{0}$; (c) $\frac{14\sqrt{13}}{13}$, $(\frac{42}{13}, \frac{28}{13})^T$;
   (d) $\frac{8\sqrt{21}}{21}$, $(\frac{8}{21}, \frac{16}{21}, \frac{32}{21})^T$

3. (a) $\mathbf{p} = (3, 0)^T$, $\mathbf{x} - \mathbf{p} = (0, 4)^T$, $\mathbf{p}^T(\mathbf{x} - \mathbf{p}) = 3 \cdot 0 + 0 \cdot 4 = 0$;
   (c) $\mathbf{p} = (3, 3, 3)^T$, $\mathbf{x} - \mathbf{p} = (-1, 1, 0)^T$, $\mathbf{p}^T(\mathbf{x} - \mathbf{p}) = -1 \cdot 3 + 1 \cdot 3 + 0 \cdot 3 = 0$

5. (1.8, 3.6)

6. (1.4, 3.8)

7. 0.4

8. (a) $2x + 4y + 3z = 0$; (c) $z - 4 = 0$

9. $\frac{5}{3}$

10. $\frac{8}{7}$

20. The correlation matrix with entries rounded to two decimal places is

$$\begin{bmatrix} 1.00 & -0.04 & 0.41 \\ -0.04 & 1.00 & 0.87 \\ 0.41 & 0.87 & 1.00 \end{bmatrix}$$

5.2

1. (a) $\{(3,4)^T\}$ basis for $R(A^T)$,
$\{(-4,3)^T\}$ basis for $N(A)$,
$\{(1,2)^T\}$ basis for $R(A)$,
$\{(-2,1)^T\}$ basis for $N(A^T)$;

(d) basis for $R(A^T)$:
$\{(1,0,0,0)^T, (0,1,0,0)^T(0,0,1,1)^T\}$,
basis for $N(A)$: $\{(0,0,-1,1)^T\}$,
basis for $R(A)$:
$\{(1,0,0,1)^T, (0,1,0,1)^T(0,0,1,1)^T\}$,
basis for $N(A^T)$: $\{(1,1,1,-1)^T\}$

2. (a) $\{(1,1,0)^T, (-1,0,1)^T\}$

3. (b) The orthogonal complement is spanned by $(-5,1,3)^T$.

4. $\{(-1,2,0,1)^T, (2,-3,1,0)^T\}$ is one basis for $S^\perp$.

6. (a) $\mathbf{N} = (8,-2,1)^T$; (b) $8x - 2y + z = 7$

10. $\dim N(A) = n - r$, $\dim N(A^T) = m - r$

5.3

1. (a) $(2,1)^T$; (c) $(1.6, 0.6, 1.2)^T$

2. (1a) $\mathbf{p} = (3,1,0)^T$, $\mathbf{r} = (0,0,2)^T$
(1c) $\mathbf{p} = (3.4, 0.2, 0.6, 2.8)^T$,
$\mathbf{r} = (0.6, -0.2, 0.4, -0.8)^T$

3. (a) $\{(1-2\alpha, \alpha)^T \mid \alpha \text{ real}\}$;
(b) $\{(2-2\alpha, 1-\alpha, \alpha)^T \mid \alpha \text{ real}\}$

4. (a) $\mathbf{p} = (1,2,-1)^T$, $\mathbf{b} - \mathbf{p} = (2,0,2)^T$;
(b) $\mathbf{p} = (3,1,4)^T$, $\mathbf{p} - \mathbf{b} = (-5,-1,4)^T$

5. (a) $y = 1.8 + 2.9x$

6. $0.55 + 1.65x + 1.25x^2$

14. The least squares circle will have center $(0.58, -0.64)$ and radius 2.73 (answers rounded to two decimal places).

15. (a) $\mathbf{w} = (0.1995,\ 0.2599,\ 0.3412,\ 0.1995)^T$
(b) $\mathbf{r} = (0.2605,\ 0.2337,\ 0.2850,\ 0.2208)^T$

5.4

1. $\|\mathbf{x}\|_2 = 2$, $\|\mathbf{y}\|_2 = 6$, $\|\mathbf{x}+\mathbf{y}\|_2 = 2\sqrt{10}$

2. (a) $\theta = \frac{\pi}{4}$; $\mathbf{p} = (\frac{4}{3}, \frac{1}{3}, \frac{1}{3}, 0)^T$

3. (b) $\|\mathbf{x}\| = 1$, $\|\mathbf{y}\| = 3$

4. (a) 0; (b) 5; (c) 7; (d) $\sqrt{74}$

7. (a) 1; (b) $\frac{1}{\pi}$

8. (a) $\frac{\pi}{6}$; (b) $\mathbf{p} = \frac{3}{2}x$

11. (a) $\frac{\sqrt{10}}{2}$; (b) $\frac{\sqrt{34}}{4}$

15. (a) $\|\mathbf{x}\|_1 = 7$, $\|\mathbf{x}\|_2 = 5$, $\|\mathbf{x}\|_\infty = 4$;
(b) $\|\mathbf{x}\|_1 = 4$, $\|\mathbf{x}\|_2 = \sqrt{6}$, $\|\mathbf{x}\|_\infty = 2$;
(c) $\|\mathbf{x}\|_1 = 3$, $\|\mathbf{x}\|_2 = \sqrt{3}$, $\|\mathbf{x}\|_\infty = 1$

16. $\|\mathbf{x}-\mathbf{y}\|_1 = 5$, $\|\mathbf{x}-\mathbf{y}\|_2 = 3$, $\|\mathbf{x}-\mathbf{y}\|_\infty = 2$

28. (a) not a norm; (b) norm; (c) norm

5.5

1. (a) and (d)

2. (b) $\mathbf{x} = -\frac{\sqrt{2}}{3}\mathbf{u}_1 + \frac{5}{3}\mathbf{u}_2$,
$$\|\mathbf{x}\| = \left[\left(-\frac{\sqrt{2}}{3}\right)^2 + \left(\frac{5}{3}\right)^2\right]^{1/2} = \sqrt{3}$$

3. $\mathbf{p} = (\frac{23}{18}, \frac{41}{18}, \frac{8}{9})^T$, $\mathbf{p} - \mathbf{x} = (\frac{5}{18}, \frac{5}{18}, -\frac{10}{9})^T$

4. (b) $c_1 = y_1\cos\theta + y_2\sin\theta$,
$c_2 = -y_1\sin\theta + y_2\cos\theta$

6. (a) 15; (b) $\|\mathbf{u}\| = 3$, $\|\mathbf{v}\| = 5\sqrt{2}$; (c) $\frac{\pi}{4}$

9. (b) (i) 0, (ii) $-\frac{\pi}{2}$, (iii) 0, (iv) $\frac{\pi}{8}$

21. (b) (i) $(2,-2)^T$, (ii) $(5,2)^T$, (iii) $(3,1)^T$

22. (a) $$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix};$$

23. (b) $$Q = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

29. (b) $\|1\| = \sqrt{2}$, $\|x\| = \frac{\sqrt{6}}{3}$; (c) $l(x) = \frac{9}{7}x$

5.6

1. (a) $\left\{\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)^T, \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)^T\right\}$;
(b) $\left\{\left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)^T, \left(-\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)^T\right\}$

2. (a) $$\begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \sqrt{2} & \sqrt{2} \\ 0 & 4\sqrt{2} \end{bmatrix};$$
(b) $$\begin{bmatrix} \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} \sqrt{5} & 4\sqrt{5} \\ 0 & 3\sqrt{5} \end{bmatrix}$$

3. $\{(\frac{1}{3}, \frac{2}{3}, -\frac{2}{3})^T, (\frac{2}{3}, \frac{1}{3}, \frac{2}{3})^T, (-\frac{2}{3}, \frac{2}{3}, \frac{1}{3})^T\}$

4. $u_1(x) = \frac{1}{\sqrt{2}}$, $u_2(x) = \frac{\sqrt{6}}{2}x$,
$u_3(x) = \frac{3\sqrt{10}}{4}\left(x^2 - \frac{1}{3}\right)$

5. (a) $\left\{\frac{1}{3}(2,1,2)^T, \frac{\sqrt{2}}{6}(-1,4,-1)^T\right\}$;

(b) $Q = \begin{bmatrix} \frac{2}{3} & \frac{-\sqrt{2}}{6} \\ \frac{1}{3} & \frac{2\sqrt{2}}{3} \\ \frac{2}{3} & \frac{-\sqrt{2}}{6} \end{bmatrix}$; $R = \begin{bmatrix} 3 & \frac{5}{3} \\ 0 & \frac{\sqrt{2}}{3} \end{bmatrix}$;

(c) $\mathbf{x} = \begin{bmatrix} 9 \\ -3 \end{bmatrix}$

6. (b) $\begin{bmatrix} \frac{3}{5} & -\frac{4}{5\sqrt{2}} \\ \frac{4}{5} & \frac{3}{5\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 5 & 1 \\ 0 & 2\sqrt{2} \end{bmatrix}$;

(c) $(2.1, 5.5)^T$

7. $\left\{ \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0, 0\right)^T, \left(\frac{\sqrt{2}}{3}, \frac{\sqrt{2}}{3}, -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{6}\right)^T \right\}$

8. $\left\{ \begin{bmatrix} \frac{4}{5} \\ \frac{2}{5} \\ \frac{2}{5} \\ \frac{1}{5} \end{bmatrix}, \begin{bmatrix} \frac{1}{5} \\ -\frac{2}{5} \\ -\frac{2}{5} \\ \frac{4}{5} \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} \right\}$

5.7 1. (a) $T_4 = 8x^4 - 8x^2 + 1$, $T_5 = 16x^5 - 20x^3 + 5x$;

(b) $H_4 = 16x^4 - 48x^2 + 12$,
$H_5 = 32x^5 - 160x^3 + 120x$

2. $p_1(x) = x$, $p_2(x) = x^2 - \frac{4}{\pi} + 1$

4. $p(x) = (\sinh 1)P_0(x) + \frac{3}{e}P_1(x) + 5\left(\sinh 1 - \frac{3}{e}\right) P_2(x)$
$p(x) \approx 0.9963 + 1.1036x + 0.5367x^2$

6. (a) $U_0 = 1$, $U_1 = 2xU_2 = 4x^2 - 1$

11. $p(x) = (x-2)(x-3) + (x-1)(x-3) + 2(x-1)(x-2)$

13. $1 \cdot f\left(-\frac{1}{\sqrt{3}}\right) + 1 \cdot f\left(\frac{1}{\sqrt{3}}\right)$

14. (a) degree 3 or less; (b) the formula gives the exact answer for the first integral. The approximate value for the second integral is 1.5, while the exact answer is $\frac{\pi}{2}$.

CHAPTER TEST A

1. False 2. False 3. False 4. False 5. True
6. False 7. True 8. True 9. True 10. False

Chapter 6

6.1 1. (a) $\lambda_1 = 5$, the eigenspace is spanned by $(1, 1)^T$, $\lambda_2 = -1$, the eigenspace is spanned by $(1, -2)^T$;

(b) $\lambda_1 = 3$, the eigenspace is spanned by $(4, 3)^T$, $\lambda_2 = 2$, the eigenspace is spanned by $(1, 1)^T$;

(c) $\lambda_1 = \lambda_2 = 2$, the eigenspace is spanned by $(1, 1)^T$,

(d) $\lambda_1 = 3 + 4i$, the eigenspace is spanned by $(2i, 1)^T$, $\lambda_2 = 3 - 4i$, the eigenspace is spanned by $(-2i, 1)^T$;

(e) $\lambda_1 = 2 + i$, the eigenspace is spanned by $(1, 1+i)^T$, $\lambda_2 = 2 - i$, the eigenspace is spanned by $(1, 1-i)^T$;

(f) $\lambda_1 = \lambda_2 = \lambda_3 = 0$, the eigenspace is spanned by $(1, 0, 0)^T$;

(g) $\lambda_1 = 2$, the eigenspace is spanned by $(1, 1, 0)^T$, $\lambda_2 = 1$, the eigenspace is spanned by $(1, 0, 0)^T$, $(0, 1, -1)^T$;

(h) $\lambda_1 = 1$, the eigenspace is spanned by $(1, 0, 0)^T$, $\lambda_2 = 4$, the eigenspace is spanned by $(1, 1, 1)^T$,
$\lambda_3 = -2$, the eigenspace is spanned by $(-1, -1, 5)^T$;

(i) $\lambda_1 = 2$, the eigenspace is spanned by $(7, 3, 1)^T$, $\lambda_2 = 1$, the eigenspace is spanned by $(3, 2, 1)^T$, $\lambda_3 = 0$, the eigenspace is spanned by $(1, 1, 1)^T$;

(j) $\lambda_1 = \lambda_2 = \lambda_3 = -1$, the eigenspace is spanned by $(1, 0, 1)^T$;

(k) $\lambda_1 = \lambda_2 = 2$, the eigenspace is spanned by $\mathbf{e}_1$ and $\mathbf{e}_2$, $\lambda_3 = 3$, the eigenspace is spanned by $\mathbf{e}_3$, $\lambda_4 = 4$, the eigenspace is spanned by $\mathbf{e}_4$;

(l) $\lambda_1 = 3$, the eigenspace is spanned by $(1, 2, 0, 0)^T$, $\lambda_2 = 1$, the eigenspace is spanned by $(0, 1, 0, 0)^T$,
$\lambda_3 = \lambda_4 = 2$, the eigenspace is spanned by $(0, 0, 1, 0)^T$

10. β is an eigenvalue of B if and only if $\beta = \lambda - \alpha$ for some eigenvalue λ of A.

14. $\lambda_1 = 6$, $\lambda_2 = 2$;

24. $\lambda_1 \mathbf{x}^T\mathbf{y} = (A\mathbf{x})^T\mathbf{y} = \mathbf{x}^T A^T \mathbf{y} = \lambda_2 \mathbf{x}^T \mathbf{y}$

6.2 1. (a) $\begin{bmatrix} c_1 e^{2t} + c_2 e^{3t} \\ c_1 e^{2t} + 2c_2 e^{3t} \end{bmatrix}$;

(b) $\begin{bmatrix} -c_1 e^{-2t} - 4c_2 e^{t} \\ c_1 e^{-2t} + c_2 e^{t} \end{bmatrix}$;

(c) $\begin{bmatrix} 2c_1 + c_2 e^{5t} \\ c_1 - 2c_2 e^{5t} \end{bmatrix}$;

(d) $\begin{Bmatrix} -c_1e^t\sin t + c_2e^t\cos t \\ c_1e^t\cos t + c_2e^t\sin t \end{Bmatrix}$;

(e) $\begin{Bmatrix} -c_1e^{3t}\sin 2t + c_2e^{3t}\cos 2t \\ c_1e^{3t}\cos 2t + c_2e^{3t}\sin 2t \end{Bmatrix}$;

(f) $\begin{Bmatrix} -c_1 + c_2e^{5t} + c_3e^t \\ -3c_1 + 8c_2e^{5t} \\ c_1 + 4c_2e^{5t} \end{Bmatrix}$

2. (a) $\begin{Bmatrix} e^{-3t} + 2e^t \\ -e^{-3t} + 2e^t \end{Bmatrix}$;

(b) $\begin{Bmatrix} e^t\cos 2t + 2e^t\sin 2t \\ e^t\sin 2t - 2e^t\cos 2t \end{Bmatrix}$;

(c) $\begin{Bmatrix} -6e^t + 2e^{-t} + 6 \\ -3e^t + e^{-t} + 4 \\ -e^t + e^{-t} + 2 \end{Bmatrix}$;

(d) $\begin{Bmatrix} -2 - 3e^t + 6e^{2t} \\ 1 + 3e^t - 3e^{2t} \\ 1 + 3e^{2t} \end{Bmatrix}$

4. $y_1(t) = 15e^{-0.24t} + 25e^{-0.08t}$,
$y_2(t) = -30e^{-0.24t} + 50e^{-0.08t}$

5. (a) $\begin{Bmatrix} -2c_1e^t - 2c_2e^{-t} + c_3e^{\sqrt{2}t} + c_4e^{-\sqrt{2}t} \\ c_1e^t + c_2e^{-t} - c_3e^{\sqrt{2}t} - c_4e^{-\sqrt{2}t} \end{Bmatrix}$

(b) $\begin{Bmatrix} c_1e^{2t} + c_2e^{-2t} - c_3e^t - c_4e^{-t} \\ c_1e^{2t} - c_2e^{-2t} + c_3e^t - c_4e^{-t} \end{Bmatrix}$

6. $y_1(t) = -e^{2t} + e^{-2t} + e^t$;
$y_2(t) = -e^{2t} - e^{-2t} + 2e^t$

8. $x_1(t) = \cos t + 3\sin t + \frac{1}{\sqrt{3}}\sin\sqrt{3}t$,
$x_2(t) = \cos t + 3\sin t - \frac{1}{\sqrt{3}}\sin\sqrt{3}t$

10. (a) $m_1x_1''(t) = -kx_1 + k(x_2 - x_1)$
$m_2x_2''(t) = -k(x_2 - x_1) + k(x_3 - x_2)$
$m_3x_3''(t) = -k(x_3 - x_2) - kx_3$

(b) $\begin{Bmatrix} 0.1\cos 2\sqrt{3}t + 0.9\cos\sqrt{2}t \\ -0.2\cos 2\sqrt{3}t + 1.2\cos\sqrt{2}t \\ 0.1\cos 2\sqrt{3}t + 0.9\cos\sqrt{2}t \end{Bmatrix}$

11. $p(\lambda) = (-1)^n(\lambda^n - a_{n-1}\lambda^{n-1} - \cdots - a_1\lambda - a_0)$

6.3

8. (b) $\alpha = 2$; (c) $\alpha = 3$ or $\alpha = -1$;
(d) $\alpha = 1$; (e) $\alpha = 0$; (g) all values of α

21. The transition matrix and steady-state vector for the Markov chain are

$$\begin{Bmatrix} 0.80 & 0.30 \\ 0.20 & 0.70 \end{Bmatrix} \qquad \mathbf{x} = \begin{Bmatrix} 0.60 \\ 0.40 \end{Bmatrix}$$

In the long run we would expect 60 percent of the employees to be enrolled.

22. (a) $A = \begin{Bmatrix} 0.70 & 0.20 & 0.10 \\ 0.20 & 0.70 & 0.10 \\ 0.10 & 0.10 & 0.80 \end{Bmatrix}$

(c) The membership of all three groups will approach 100,000 as n gets large.

26. The transition matrix is

$$A = 0.85\begin{Bmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{4} \\ \frac{1}{3} & 0 & 0 & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{4} \\ \frac{1}{3} & 0 & 1 & \frac{1}{4} \end{Bmatrix} + 0.15\begin{Bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{Bmatrix}$$

30. (b) $\begin{Bmatrix} e & e \\ 0 & e \end{Bmatrix}$

31. (a) $\begin{Bmatrix} 3 - 2e & 1 - e \\ -6 + 6e & -2 + 3e \end{Bmatrix}$;

(c) $\begin{Bmatrix} e & -1 + e & -1 + e \\ 1 - e & 2 - e & 1 - e \\ -1 + e & -1 + e & e \end{Bmatrix}$

32. (a) $\begin{Bmatrix} e^{-t} \\ e^{-t} \end{Bmatrix}$; (b) $\begin{Bmatrix} -3e^t - e^{-t} \\ e^t + e^{-t} \end{Bmatrix}$;

(c) $\begin{Bmatrix} 3e^t - 2 \\ 2 - e^{-t} \\ e^{-t} \end{Bmatrix}$

6.4

1. (a) $\|\mathbf{z}\| = 6$, $\|\mathbf{w}\| = 3$, $\langle \mathbf{z}, \mathbf{w}\rangle = -4 + 4i$, $\langle \mathbf{w}, \mathbf{z}\rangle = -4 - 4i$;
(b) $\|\mathbf{z}\| = 4$, $\|\mathbf{w}\| = 7$, $\langle \mathbf{z}, \mathbf{w}\rangle = -4 + 10i$, $\langle \mathbf{w}, \mathbf{z}\rangle = -4 - 10i$

2. (b) $\mathbf{z} = 4\mathbf{z}_1 + 2\sqrt{2}\mathbf{z}_2$

3. (a) $\mathbf{u}_1^H\mathbf{z} = 4 + 2i$, $\mathbf{z}^H\mathbf{u}_1 = 4 - 2i$, $\mathbf{u}_2^H\mathbf{z} = 6 - 5i$, $\mathbf{z}^H\mathbf{u}_2 = 6 + 5i$;
(b) $\|\mathbf{z}\| = 9$

4. (b) and (f) are Hermitian while (b), (c), (e), and (f) are normal.

14. (b) $\|U\mathbf{x}\|^2 = (U\mathbf{x})^H U\mathbf{x} = \mathbf{x}^H U^H U\mathbf{x} = \mathbf{x}^H\mathbf{x} = \|\mathbf{x}\|^2$

15. U is unitary, since $U^HU = (I - 2\mathbf{u}\mathbf{u}^H)^2 = I - 4\mathbf{u}\mathbf{u}^H + 4\mathbf{u}(\mathbf{u}^H\mathbf{u})\mathbf{u}^H = I$.

24. $\lambda_1 = 1, \lambda_2 = -1,$
$\mathbf{u}_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)^T, \mathbf{u}_2 = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)^T,$

$$A = 1\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} + (-1)\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

6.5 2. (a) $\sigma_1 = \sqrt{10}, \sigma_2 = 0$;
(b) $\sigma_1 = 3, \sigma_2 = 2$;
(c) $\sigma_1 = 4, \sigma_2 = 2$;
(d) $\sigma_1 = 3, \sigma_2 = 2, \sigma_3 = 1$. The matrices U and V are not unique. The reader may check his or her answers by multiplying out $U\Sigma V^T$.

3. (b) rank of $A = 2$, $A' = \begin{bmatrix} 1.2 & -2.4 \\ -0.6 & 1.2 \end{bmatrix}$

4. The closest matrix of rank 2 is

$$\begin{bmatrix} -2 & 8 & 20 \\ 14 & 19 & 10 \\ 0 & 0 & 0 \end{bmatrix},$$

The closest matrix of rank 1 is

$$\begin{bmatrix} 6 & 12 & 12 \\ 8 & 16 & 16 \\ 0 & 0 & 0 \end{bmatrix}$$

5. (a) basis for $R(A^T)$:
$\{\mathbf{v}_1 = (\frac{2}{3}, \frac{2}{3}, \frac{1}{3})^T, \mathbf{v}_2 = (-\frac{2}{3}, \frac{1}{3}, \frac{2}{3})^T\}$;
basis for $N(A)$: $\{\mathbf{v}_3 = (\frac{1}{3}, -\frac{2}{3}, \frac{2}{3})^T\}$

6.6 1. (a) $\begin{bmatrix} 3 & -\frac{5}{2} \\ -\frac{5}{2} & 1 \end{bmatrix}$; (b) $\begin{bmatrix} 2 & \frac{1}{2} & -1 \\ \frac{1}{2} & 3 & \frac{3}{2} \\ -1 & \frac{3}{2} & 1 \end{bmatrix}$

3. (a) $Q = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, $\frac{(x')^2}{4} + \frac{(y')^2}{12} = 1$, ellipse;
(d) $Q = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$,
$\left(y' + \frac{\sqrt{2}}{2}\right)^2 = -\frac{\sqrt{2}}{2}(x' - \sqrt{2})$ or
$(y'')^2 = -\frac{\sqrt{2}}{2}x''$, parabola

6. (a) positive definite; (b) indefinite;
(d) negative definite; (e) indefinite

7. (a) minimum; (b) saddle point;
(c) saddle point; (f) local maximum

6.7 1. (a) $\det(A_1) = 2, \det(A_2) = 3$, positive definite;
(b) $\det(A_1) = 3, \det(A_2) = -10$, not positive definite;
(c) $\det(A_1) = 6, \det(A_2) = 14$, $\det(A_3) = -38$, not positive definite;
(d) $\det(A_1) = 4, \det(A_2) = 8, \det(A_3) = 13$, positive definite

2. $a_{11} = 3, a_{22}^{(1)} = 2, a_{33}^{(2)} = \frac{4}{3}$

4. (a) $\begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 0 & 9 \end{bmatrix}\begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 1 \end{bmatrix}$;
(b) $\begin{bmatrix} 1 & 0 \\ -\frac{1}{3} & 1 \end{bmatrix}\begin{bmatrix} 9 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -\frac{1}{3} \\ 0 & 1 \end{bmatrix}$;
(c) $\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{4} & -1 & 1 \end{bmatrix}\begin{bmatrix} 16 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{4} \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$;
(d) $\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & 1 & 0 \\ -\frac{2}{3} & 1 & 1 \end{bmatrix}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & \frac{1}{3} & -\frac{2}{3} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

5. (a) $\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$;
(b) $\begin{bmatrix} 3 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix}$;
(c) $\begin{bmatrix} 4 & 0 & 0 \\ 2 & \sqrt{2} & 0 \\ 1 & -\sqrt{2} & 2 \end{bmatrix}\begin{bmatrix} 4 & 2 & 1 \\ 0 & \sqrt{2} & -\sqrt{2} \\ 0 & 0 & 2 \end{bmatrix}$;
(d) $\begin{bmatrix} 3 & 0 & 0 \\ 1 & \sqrt{3} & 0 \\ -2 & \sqrt{3} & \sqrt{2} \end{bmatrix}\begin{bmatrix} 3 & 1 & -2 \\ 0 & \sqrt{3} & \sqrt{3} \\ 0 & 0 & \sqrt{2} \end{bmatrix}$

6.8 1. (a) $\lambda_1 = 4, \lambda_2 = -1, \mathbf{x}_1 = (3, 2)^T$;
(b) $\lambda_1 = 8, \lambda_2 = 3, \mathbf{x}_1 = (1, 2)^T$;
(c) $\lambda_1 = 7, \lambda_2 = 2, \lambda_3 = 0, \mathbf{x}_1 = (1, 1, 1)^T$

2. (a) $\lambda_1 = 3, \lambda_2 = -1, \mathbf{x}_1 = (3, 1)^T$;
(b) $\lambda_1 = 2 = 2\exp(0)$, $\lambda_2 = -2 = 2\exp(\pi i), \mathbf{x}_1 = (1, 1)^T$;
(c) $\lambda_1 = 2 = 2\exp(0)$, $\lambda_2 = -1 + \sqrt{3}i = 2\exp\left(\frac{2\pi i}{3}\right)$, $\lambda_3 = -1 - \sqrt{3}i = 2\exp\left(\frac{4\pi i}{3}\right)$, $\mathbf{x}_1 = (4, 2, 1)^T$

3. $x_1 = 70{,}000, x_2 = 56{,}000, x_3 = 44{,}000$

4. $x_1 = x_2 = x_3$

5. $(I - A)^{-1} = I + A + \cdots + A^{m-1}$

6. (a) $(I - A)^{-1} = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix}$;

(b) $A^2 = \begin{bmatrix} 0 & -2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$,

$A^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

7. (b) and (c) are reducible.

15. (d) $\mathbf{w} = (\frac{12}{29}, \frac{12}{29}, \frac{3}{29}, \frac{2}{29})^T$
$\approx (0.4138, 0.4138, 0.1034, 0.0690)^T$

CHAPTER TEST A

1. True 2. False 3. True 4. False 5. False
6. False 7. False 8. False 9. True 10. False
11. True 12. True 13. True 14. False
15. True

Chapter 7

7.1

1. (a) 0.231×10^4; (b) 0.326×10^2; (c) 0.128×10^{-1}; (d) 0.824×10^5
2. (a) $\epsilon = -2$; $\delta \approx -8.7 \times 10^{-4}$;
   (b) $\epsilon = 0.04$; $\delta \approx 1.2 \times 10^{-3}$;
   (c) $\epsilon = 3.0 \times 10^{-5}$; $\delta \approx 2.3 \times 10^{-3}$;
   (d) $\epsilon = -31$; $\delta \approx -3.8 \times 10^{-4}$
3. (a) $(1.0101)_2 \times 2^4$; (b) $(1.1000)_2 \times 2^{-2}$; (c) $(1.0100)_2 \times 2^3$; (d) $-(1.1010)_2 \times 2^{-4}$
4. (a) 10,420, $\epsilon = -0.0018$, $\delta \approx -1.7 \times 10^{-7}$;
   (b) 0, $\epsilon = -8$, $\delta = -1$;
   (c) 1×10^{-4}, $\epsilon = 5 \times 10^{-5}$, $\delta = 1$;
   (d) 82,190, $\epsilon = 25.7504$, $\delta \approx 3.1 \times 10^{-4}$
5. (a) 0.1043×10^6; (b) 0.1045×10^6; (c) 0.1045×10^6
8. 23
9. (a) $(1.00111000000000000000000)_2 \times 2^3$ or 9.75

7.2

1. $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$
2. (a) $(2, -1, 3)^T$; (b) $(1, -1, 3)^T$; (c) $(1, 5, 1)^T$
3. (a) n^2 multiplications and $n(n-1)$ additions;
   (b) n^3 multiplications and $n^2(n-1)$ additions;
   (c) $(AB)\mathbf{x}$ requires $n^3 + n^2$ multiplications and $n^3 - n$ additions; $A(B\mathbf{x})$ requires $2n^2$ multiplications and $2n(n-1)$ additions.
4. (b) (i) 156 multiplications and 105 additions, (ii) 47 multiplications and 24 additions, (iii) 100 multiplications and 60 additions
8. $5n - 4$ multiplications/divisions, $3n - 3$ additions/subtractions
9. (a) $[(n-j)(n-j+1)]/2$ multiplications; $[(n-j-1)(n-j)]/2$ additions;
   (c) It requires on the order of $\frac{2}{3}n^3$ additional multiplications/divisions to compute A^{-1} given the LU factorization.

7.3

1. (a) $(1, 1, -2)$;
   (b) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & 8 \\ 0 & 0 & -23 \end{bmatrix}$
2. (a) $(1, 2, 2)$; (b) $(4, -3, 0)$; (c) $(1, 1, 1)$
3. $P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ -\frac{1}{2} & -\frac{1}{3} & 1 \end{bmatrix}$,
   $U = \begin{bmatrix} 2 & 4 & -6 \\ 0 & 6 & 9 \\ 0 & 0 & 5 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 6 \\ -\frac{1}{2} \\ 1 \end{bmatrix}$
4. $P = Q = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$,
   $PAQ = LU = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 0 & 2 \end{bmatrix}$,
   $\mathbf{x} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$
5. (a) $\hat{\mathbf{c}} = P\mathbf{c} = (-4, 6)^T$, $\mathbf{y} = L^{-1}\hat{\mathbf{c}} = (-4, 8)^T$, $\mathbf{z} = U^{-1}\mathbf{y} = (-3, 4)^T$
   (b) $\mathbf{x} = Q\mathbf{z} = (4, -3)^T$
6. (b) $P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, $Q = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$,
   $L = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ \frac{1}{2} & \frac{2}{3} & 1 \end{bmatrix}$, $U = \begin{bmatrix} 8 & 6 & 2 \\ 0 & 6 & 3 \\ 0 & 0 & 2 \end{bmatrix}$
7. Error $\dfrac{-2000e}{0.6} \approx -3333e$. If $e = 0.001$, then $\delta = -\frac{2}{3}$.
8. $(1.667, 1.001)$

9. (5.002, 1.000)
10. (5.001, 1.001)

7.4 1. (a) $\|A\|_F = \sqrt{2}$, $\|A\|_\infty = 1$, $\|A\|_1 = 1$;
(b) $\|A\|_F = 5$, $\|A\|_\infty = 5$, $\|A\|_1 = 6$;
(c) $\|A\|_F = \|A\|_\infty = \|A\|_1 = 1$;
(d) $\|A\|_F = 7$, $\|A\|_\infty = 6$, $\|A\|_1 = 10$;
(e) $\|A\|_F = 9$, $\|A\|_\infty = 10$, $\|A\|_1 = 12$

2. 2

4. $\|I\|_1 = \|I\|_\infty = 1$, $\|I\|_F = \sqrt{n}$;

6. (a) 10; (b) $(-1, 1, -1)^T$

27. (a) Since for any vector $\mathbf{y}$ in $\mathbb{R}^n$ we have

$$\|\mathbf{y}\|_\infty \le \|\mathbf{y}\|_2 \le \sqrt{n}\,\|\mathbf{y}\|_\infty$$

it follows that

$$\begin{aligned}\|A\mathbf{x}\|_\infty &\le \|A\mathbf{x}\|_2 \\ &\le \|A\|_2\|\mathbf{x}\|_2 \le \sqrt{n}\,\|A\|_2\|\mathbf{x}\|_\infty\end{aligned}$$

29. $\text{cond}_\infty A = 400$

30. The solutions are $\begin{pmatrix} -0.48 \\ 0.8 \end{pmatrix}$ and $\begin{pmatrix} -2.902 \\ 2.0 \end{pmatrix}$

31. $\text{cond}_\infty(A) = 28$

33. (a) $A_n^{-1} = \begin{pmatrix} 1-n & n \\ n & -n \end{pmatrix}$;
(b) $\text{cond}_\infty A_n = 4n$;
(c) $\lim_{n\to\infty} \text{cond}_\infty A_n = \infty$;

34. $\sigma_1 = 8$, $\sigma_2 = 8$, $\sigma_3 = 4$

35. (a) $\mathbf{r} = (-0.06, 0.02)^T$ and the relative residual is 0.012;
(b) 20;
(d) $\mathbf{x} = (1, 1)^T$, $\|\mathbf{x} - \mathbf{x}'\|_\infty = 0.12$;

36. $\text{cond}_1(A) = 6$

37. 0.3

38. (a) $\|\mathbf{r}\|_\infty = 0.10$, $\text{cond}_\infty(A) = 32$;
(b) 0.64;
(c) $\mathbf{x} = (12.50, 4.26, 2.14, 1.10)^T$, $\delta = 0.04$

7.5 1. (a) $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$; (b) $\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$;
(c) $\begin{pmatrix} -\frac{4}{5} & \frac{3}{5} \\ -\frac{3}{5} & -\frac{4}{5} \end{pmatrix}$

2. (a) $\begin{pmatrix} \frac{3}{5} & 0 & \frac{4}{5} \\ 0 & 1 & 0 \\ \frac{4}{5} & 0 & -\frac{3}{5} \end{pmatrix}$;
(b) $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$;
(c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$;
(d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$

3. $H = I - \frac{1}{\beta}\mathbf{v}\mathbf{v}^T$ for the given β and $\mathbf{v}$.
(a) $\beta = 90$, $\mathbf{v} = (-10, 8, -4)^T$;
(b) $\beta = 70$, $\mathbf{v} = (10, 6, 2)^T$;
(c) $\beta = 15$, $\mathbf{v} = (-5, -3, 4)^T$

4. (a) $\beta = 90$, $\mathbf{v} = (0, 10, 4, 8)^T$;
(b) $\beta = 15$, $\mathbf{v} = (0, 0, -5, -1, 2)^T$

6. (a) $H_2H_1A = R$, where $H_i = I - \dfrac{1}{\beta_i}\mathbf{v}_i\mathbf{v}_i^T$, $i = 1, 2$, and $\beta_1 = 12$, $\beta_2 = 45$.

$$\mathbf{v}_1 = \begin{pmatrix} -4 \\ 2 \\ -2 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 0 \\ 9 \\ -3 \end{pmatrix},$$

$$R = \begin{pmatrix} 3 & \frac{19}{2} & \frac{9}{2} \\ 0 & -5 & -3 \\ 0 & 0 & 6 \end{pmatrix},$$

$$\mathbf{c} = H_2H_1\mathbf{b} = \begin{pmatrix} -\frac{5}{2} \\ -5 \\ 0 \end{pmatrix};$$

(b) $\mathbf{x} = (-4, 1, 0)^T$

7. (a) $G = \begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{pmatrix}$, $\mathbf{x} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

8. It takes three multiplications, two additions, and one square root to determine H. It takes four multiplications/divisions, one addition, and one square root to determine G. The calculation of GA requires $4n$ multiplications

and $2n$ additions, while the calculation of HA requires $3n$ multiplications/divisions and $3n$ additions.

9. (a) $n-k+1$ multiplications/divisions, $2n-2k+1$ additions;
   (b) $n(n-k+1)$ multiplications/divisions, $n(2n-2k+1)$ additions
10. (a) $4(n-k)$ multiplications/divisions, $2(n-k)$ additions;
    (b) $4n(n-k)$ multiplications, $2n(n-k)$ additions
11. (a) rotation; (b) rotation;
    (c) Givens transformation;
    (d) Givens transformation

7.6

1. (a) $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$; (b) $A_2 = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$;
   (c) $\lambda_1 = 2$, $\lambda_2 = 0$; the eigenspace corresponding to λ_1 is spanned by $\mathbf{u}_1$.
2. (a) $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 5 \\ 3 \end{bmatrix}$, $\mathbf{u}_1 = \begin{bmatrix} 0.6 \\ 1.0 \\ 0.6 \end{bmatrix}$,
   $\mathbf{v}_2 = \begin{bmatrix} 2.2 \\ 4.2 \\ 2.2 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 0.52 \\ 1.00 \\ 0.52 \end{bmatrix}$,
   $\mathbf{v}_3 = \begin{bmatrix} 2.05 \\ 4.05 \\ 2.05 \end{bmatrix}$;
   (b) $\lambda_1' = 4.05$; (c) $\lambda_1 = 4$, $\delta = 0.0125$
3. (b) A has no dominant eigenvalue.
4. $A_2 = \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}$, $A_3 = \begin{bmatrix} 3.4 & 0.2 \\ 0.2 & 0.6 \end{bmatrix}$,
   $\lambda_1 = 2+\sqrt{2} \approx 3.414$, $\lambda_2 = 2-\sqrt{2} \approx 0.586$
5. (b) $H = I - \dfrac{1}{\beta}\mathbf{v}\mathbf{v}^T$, where $\beta = \frac{1}{3}$ and $\mathbf{v} = (-\frac{1}{3}, -\frac{2}{3}, \frac{1}{3})^T$;
   (c) $\lambda_2 = 3$, $\lambda_3 = 1$, $HAH = \begin{bmatrix} 4 & 0 & 3 \\ 0 & 5 & -4 \\ 0 & 2 & -1 \end{bmatrix}$

7.7

1. (a) $(\sqrt{2}, 0)^T$; (b) $(1-3\sqrt{2}, 3\sqrt{2}, -\sqrt{2})^T$;
   (c) $(1, 0)^T$; (d) $(1-\sqrt{2}, \sqrt{2}, -\sqrt{2})^T$
2. $x_i = \dfrac{d_i b_i + e_i b_{n+i}}{d_i^2 + e_i^2}$, $i = 1, \dots, n$
4. (a) $Q = \begin{bmatrix} \frac{1}{2} & -\frac{1}{6} \\ \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{5}{6} \\ \frac{1}{2} & -\frac{1}{6} \end{bmatrix}$, $R = \begin{bmatrix} 2 & 12 \\ 0 & 6 \end{bmatrix}$
   (b) $\mathbf{x} = \begin{bmatrix} 0 & \frac{1}{3} \end{bmatrix}^T$
5. (a) $\sigma_1 = \sqrt{2+\rho^2}$, $\sigma_2 = \rho$;
   (b) $\lambda_1' = 2$, $\lambda_2' = 0$, $\sigma_1' = \sqrt{2}$, $\sigma_2' = 0$
12. $A^+ = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 \end{bmatrix}$
13. (a) $A^+ = \begin{bmatrix} \frac{1}{10} & -\frac{1}{10} \\ \frac{2}{10} & -\frac{2}{10} \end{bmatrix}$;
    (b) $A^+\mathbf{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$;
    (c) $\left\{ \mathbf{y} \,\middle|\, \mathbf{y} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \alpha \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}$
15. $\|A_1 - A_2\|_F = \rho$, $\|A_1^+ - A_2^+\|_F = 1/\rho$. As $\rho \to 0$, $\|A_1 - A_2\|_F \to 0$ and $\|A_1^+ - A_2^+\|_F \to \infty$.

CHAPTER TEST A

1. False 2. False 3. False 4. True 5. False
6. False 7. True 8. False 9. False 10. False