12-5 4.1 linear transformations

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Ex: Find a basis and dimension of $S = \text{span}(x^2+x, x-1, x^2+1)$

spanning set for S is $\{x^2+x, x-1, x^2+1\}$ $\boxed{L.I}$ or $\boxed{L.D.}$

Solve $c_1(x^2+x) + c_2(x-1) + c_3(x^2+1) = 0$.

$$\begin{array}{ll} x^2: & c_1 + c_3 = 0 \\ x: & c_1 + c_2 = 0 \\ \text{consn}: & -c_2 + c_3 = 0 \end{array} \Bigg\} (*) \quad \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

(*) has nonzero solutions $\Rightarrow$ $x^2+x, x-1, x^2+1$ are L.D

remove one of them

$c_3 = \alpha$ free

$c_2 = \alpha$, $c_1 = -\alpha$

solutions $= \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} -\alpha \\ \alpha \\ \alpha \end{pmatrix}$, a nonzero solution $= \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

remove x^2+1 $\Longrightarrow$ $\{x^2+x, x-1\}$ is a sp. set for S.

L.I or L.D.

solve $\boxed{d_1(x^2+x) + d_2(x-1) = 0.}$ (**)

$$\begin{array}{ll} x^2: & d_1 = 0 \\ x: & d_1 + d_2 = 0 \\ \text{consns}: & -d_2 = 0 \end{array} \Bigg] \Longrightarrow d_1 = d_2 = d_3 = 0$$

(**) has only the zero solution.

$\therefore$ $\{x^2+x, x-1\}$ are L.I.

So a basis for S is $\{x^2+x, x-1\}$

$\therefore$ $\dim(S) = 2$

Ex: Find a basis and dim. of $S = \text{span}\left(x_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \end{pmatrix}, x_2 = \begin{pmatrix} 2 \\ 5 \\ -3 \\ 2 \end{pmatrix}, x_3 = \begin{pmatrix} 2 \\ 4 \\ -2 \\ 0 \end{pmatrix}, x_4 = \begin{pmatrix} 3 \\ 8 \\ -5 \\ 4 \end{pmatrix}\right)$

sp. set for S is $\{x_1, x_2, x_3, x_4\}$

Let $X = \begin{pmatrix} 1 & 2 & 2 & 3 \\ 2 & 5 & 4 & 8 \\ -1 & -3 & -2 & -5 \\ 0 & 2 & 0 & 4 \end{pmatrix}$, $\quad \boxed{C(X)} = \boxed{S}$

$= \text{span}(x_1, x_2, x_3, x_4)$

$\longrightarrow U = \begin{pmatrix} 1 & 2 & 2 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$$\longrightarrow U = \begin{pmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

basis for $S (= C(X))$ is $\left\{ \underline{x_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \end{pmatrix}}, \underline{x_2 = \begin{pmatrix} 2 \\ 5 \\ -7 \\ 2 \end{pmatrix}} \right\}$

$\boxed{4.1}$ Linear transformations:

Def: If V, W are vector spaces, a function $L: V \longrightarrow W$ is called a linear transformation if

$$\boxed{L(\alpha v + \beta u) = \alpha L(v) + \beta L(u),}$$ for all $u, v \in V$, α, β scalars.

Ex. Is $L: \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ a linear Trans.

$$L\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_1 \\ -x_2 \end{pmatrix}$$

$$L\left(\alpha\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \beta\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}\right) = L\begin{pmatrix} \alpha x_1 + \beta y_1 \\ \alpha x_2 + \beta y_2 \end{pmatrix} = \begin{pmatrix} \alpha x_1 + \beta y_1 + \alpha x_2 + \beta y_2 \\ \alpha x_1 + \beta y_1 \\ -\alpha x_2 - \beta y_2 \end{pmatrix}$$

$$= \begin{pmatrix} \alpha x_1 + \alpha x_2 \\ \alpha x_1 \\ -\alpha x_2 \end{pmatrix} + \begin{pmatrix} \beta y_1 + \beta y_2 \\ \beta y_1 \\ -\beta y_2 \end{pmatrix} = \alpha \boxed{\begin{pmatrix} x_1 + x_2 \\ x_1 \\ -x_2 \end{pmatrix}} + \beta \begin{pmatrix} y_1 + y_2 \\ y_1 \\ -y_2 \end{pmatrix}$$

$$= \alpha L\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \beta L\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

so L is a L.T.

$$L\left(2\begin{pmatrix} 1 \\ 5 \end{pmatrix} + 3\begin{pmatrix} 7 \\ 4 \end{pmatrix}\right) = 2L\begin{pmatrix} 1 \\ 5 \end{pmatrix} + 3L\begin{pmatrix} 7 \\ 4 \end{pmatrix}$$

$$= 2\begin{pmatrix} 6 \\ 1 \\ -5 \end{pmatrix} + 3\begin{pmatrix} 11 \\ 7 \\ -4 \end{pmatrix} = \begin{pmatrix} 45 \\ 23 \\ -22 \end{pmatrix}$$

Ex: Is $L: P_3 \longrightarrow P_2$ a L.T.?

$$L(p(x)) = p'(x)$$

$$L(5x^2 + 7x + 1) = 10x + 7$$

$$L(\alpha p(x) + \beta q(x)) = (\alpha p(x) + \beta q(x))' = (\alpha p(x))' + (\beta q(x))'$$

$$= \alpha p'(x) + \beta q'(x)$$

$$= \alpha L(p(x)) + \beta L(q(x))$$

$= \alpha L(\beta \ldots) + \beta L(\ldots)$

so L is a L.T.

Ex: $L: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ L.T?

$$L\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1+1 \\ x_2+2 \end{pmatrix}$$

$$L\left(\alpha\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \beta\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}\right) = L\begin{pmatrix} \alpha x_1 + \beta y_1 \\ \alpha x_2 + \beta y_2 \end{pmatrix} = \begin{pmatrix} \alpha x_1 + \beta y_1 + 1 \\ \alpha x_2 + \beta y_2 + 2 \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} \alpha x_1 + 1 \\ \alpha x_2 + 2 \end{pmatrix}} + \boxed{\begin{pmatrix} \beta y_1 \\ \beta y_2 \end{pmatrix}}$$

$\#$

$$\alpha L\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \beta L\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \alpha\begin{pmatrix} x_1+1 \\ x_2+2 \end{pmatrix} + \beta\begin{pmatrix} y_1+1 \\ y_2+2 \end{pmatrix} = \begin{pmatrix} \alpha x_1 + \alpha + \beta y_1 + \beta \\ \alpha x_2 + 2\alpha + \beta y_2 + 2\beta \end{pmatrix}$$

so L is not a L.T.

Remark: ① If $L: V \longrightarrow W$ is a L.T., then

$$\boxed{L(0_V) = 0_W}$$

Proof: $L(0_V) = L(v + (-v)) = L(1.v + (-1)v)$

$= 1.L(v) + (-1)L(v)$

$= \boxed{L(v)} - \boxed{L(v)} = 0_W$

② $L(\alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n) = L(\alpha_1 v_1) + L(\alpha_2 v_2 + \boxed{\cdots + \alpha_n v_n})$

$\vdots$

$L(\alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n) = \alpha_1 L(v_1) + \alpha_2 L(v_2) + \cdots + \alpha_n L(v_n)$

③ $L(-v) = L((-1)v) = (-1)L(v) = -L(v).$

④: $L: V \longrightarrow W$ is a L.T if it satisfies the two condition:

$$\boxed{\begin{array}{l} ① \ L(v_1 + v_2) = L(v_1) + L(v_2) \text{ and} \\ ② \ L(\alpha v) = \alpha L(v). \end{array}}$$

$$L(\alpha_1 v_1 + \alpha_2 v_2) = \alpha_1 L(v_1) + \alpha_2 L(v_2)$$

$$\boxed{L(\alpha_1 v_1 + \alpha_2 v_2) = \alpha_1 L(v_1) + \alpha_2 L(v_2)} \quad (*)$$

Ex: $L : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ is L a L.T.?

$$L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 + x_3 \end{pmatrix}$$

Use (2) conditions: (1) $L\left(\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}\right) = L\begin{pmatrix} x_1+y_1 \\ x_2+y_2 \\ x_3+y_3 \end{pmatrix}$

$$= \begin{pmatrix} x_1+y_1+x_2+y_2 \\ x_2+y_2+x_3+y_3 \end{pmatrix} = \begin{pmatrix} x_1+x_2 \\ x_2+x_3 \end{pmatrix} + \begin{pmatrix} y_1+y_2 \\ y_2+y_3 \end{pmatrix}$$

$$= L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + L\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

(2) $L\left(\alpha \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}\right) = L\begin{pmatrix} \alpha x_1 \\ \alpha x_2 \\ \alpha x_3 \end{pmatrix}$

$$= \begin{pmatrix} \alpha x_1 + \alpha x_2 \\ \alpha x_2 + \alpha x_3 \end{pmatrix} = \alpha \begin{pmatrix} x_1 + x_2 \\ x_2 + x_3 \end{pmatrix}$$

$$= \alpha L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

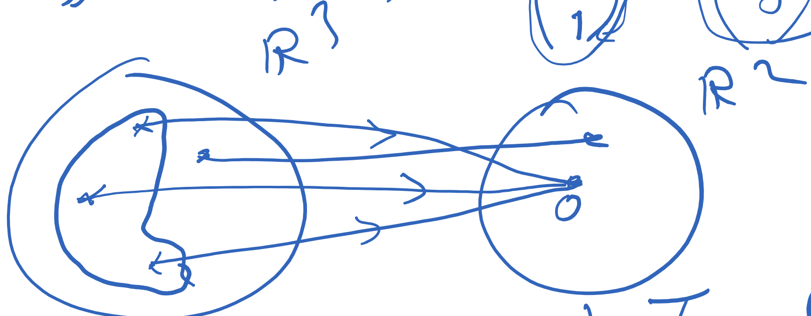
So L is a L.T.

$L : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$

$$L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 + x_3 \end{pmatrix}$$

$$L\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad L\begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$L\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad L\begin{pmatrix} -2 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$



Def: If $L : V \longrightarrow W$ is a L.T., and S a subspace of V, we define

(1) Kernel of L = $Ker(L) = \{ v \in V : L(v) = 0_W \}$

(2) $L(S) = \{ L(s) : s \in S \}$.

(3) $L(V) = \{ L(v) : v \in V \}$ = Image of L.

③ $L(V) = \{L(v) : v \in V\}$ = Image of L.
$= Im(L)$.

Th. If $L: V \to W$ is a L.T., then
① $Ker(L)$ is a subspace of V.
② If S is a subspace of V, then $L(S)$ is a subspace of W.

Proof: ① $Ker(L) = \{v \in V : L(v) = 0_W\}$

① $Ker(L) \neq \phi$ since $0_V \in Ker(L)$
$L(0_V) = 0_W$

② Let $v_1, v_2 \in Ker(L)$,
$\Longrightarrow L(v_1) = 0$, $L(v_2) = 0$.
Is $\boxed{v_1 + v_2} \in Ker(L)$? $\quad L(v_1 + v_2) \overset{L.T}{=} L(v_1) + L(v_2)$
$= 0 + 0 = 0$.
so $v_1 + v_2 \in Ker(L)$.

③ Let $v_1 \in Ker(L)$, α scalar.
$\Longrightarrow L(v_1) = 0$.
Is $\alpha v_1 \in Ker(L)$? : $\quad L(\alpha v_1) = \alpha L(v_1) = \alpha(0) = 0$
so $\alpha v_1 \in Ker(L)$.

$\Longrightarrow \boxed{Ker(L)}$ is a subspace of V.

• $Im(L) = L(V)$ subspace of W.

Ex: $L: \mathbb{R}^3 \to \mathbb{R}^2$, $L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \boxed{\begin{pmatrix} x_1 + x_2 \\ x_2 + x_3 \end{pmatrix}}$

Find a basis and dim of $Ker(L)$ and $Im(L)$.

① $Ker(L) = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 : L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$

$= \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} : \boxed{\begin{pmatrix} x_1 + x_2 \\ x_2 + x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}} \right\}$

$= \left\{ \begin{pmatrix} \alpha \\ -\alpha \\ \alpha \end{pmatrix} : \alpha \text{ is a scalar} \right\}$

$x_1 + x_2 = 0$
$x_2 + x_3 = 0$ } solve.

$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right)$

$x_3 = \alpha$ free.
$x = -\alpha$, $x = \alpha$

$$= \left\{ \alpha \boxed{\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}} : \alpha \text{ scalar} \right\}$$

$x_3 = \alpha$ free.

$x_2 = -\alpha$, $x_1 = \alpha$

so $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \alpha \\ -\alpha \\ \alpha \end{pmatrix}$.

basis for $Ker(L)$ is $\left\{ \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\}$

$\dim(Ker(L)) = 1$.

* $Im(L) = \left\{ L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} : \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 \right\}$

$$Im(L) = \left\{ \begin{pmatrix} x_1 + x_2 \\ x_2 + x_3 \end{pmatrix} : x_1, x_2, x_3 \in \mathbb{R} \right\}.$$

$$= \left\{ x_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + x_3 \begin{pmatrix} 0 \\ 1 \end{pmatrix} : x_1, x_2, x_3 \in \mathbb{R} \right\}$$

sp. set for $Im(L)$ is $\underbrace{\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}}_{\text{L.I or L.D.}}$

3 vectors in $\mathbb{R}^2$, $\dim(\mathbb{R}^2) = \boxed{2.}$

$3 > \dim(\mathbb{R}^2) = 2 \implies$ L.D.

remove one: since $\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ so remove $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

so $\underbrace{\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}}_{\text{L.D or L.I}}$ is a sp. set for $Im(L)$.

So a basis for $Im(L)$ is $\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$.

$\dim(Im(L)) = 2$.

Ex: $L : P_3 \to \mathbb{R}^2$

$$L(p(x)) = \begin{pmatrix} \int_0^1 p(x)\,dx \\ p'(0) \end{pmatrix}.$$

① Is L a L.T. (check)

② Find a basis and dim of $Ker(L)$, $Im(L)$.

* $Ker(L) = \left\{ p(x) \in P_3 : L(p(x)) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$.

$$= \left\{ p(x) = ax^2 + bx + c : \begin{pmatrix} \int_0^1 (ax^2+bx+c)\,dx \\ \ldots \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$$

$p'(x) = 2ax + b$

$p'(0) = b$

$$= \left\{ p(x) = ax^2+bx+c : \begin{pmatrix} \int_0^1 (ax^2+bx+c)\,dx \\ p'(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$$

$p'(0) = b$

$$= \left\{ p(x) = ax^2+bx+c : \begin{pmatrix} \frac{ax^3}{3} + \frac{bx^2}{2} + cx \Big|_0^1 \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$$

$$= \left\{ p(x) = ax^2+bx+c : \boxed{\begin{pmatrix} \frac{a}{3} + \frac{b}{2} + c \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}} \right\}.$$

solve: $\boxed{\begin{array}{l} \frac{a}{3} + \frac{b}{2} + c = 0 \\ b = 0. \end{array}}$

$$\left(\begin{array}{ccc|c} \frac{1}{3} & \frac{1}{2} & 1 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & \frac{3}{2} & 3 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right)$$

$c = \alpha$ free

$b = 0$, $a = -\frac{3}{2}(0) - 3\alpha$

$\boxed{a = -3\alpha.}$

$$= \left\{ p(x) = -3\alpha x^2 + 0x + \alpha : \alpha \text{ scalar} \right\}.$$

$$\text{Ker}(L) = \left\{ \alpha \left(\boxed{-3x^2+1} \right) : \alpha \text{ scalar} \right\}.$$

basis for $\text{Ker}(L)$ is $\{-3x^2+1\}$

$\dim(\text{Ker}(L)) = 1.$

$$\text{Im}(L) = \left\{ L(p(x) = ax^2+bx+c) : a,b,c \in \mathbb{R} \right\}$$

$$= \left\{ \begin{pmatrix} \int_0^1 (ax^2+bx+c)\,dx \\ p'(0) \end{pmatrix} : a,b,c \in \mathbb{R} \right\} = \left\{ \begin{pmatrix} \frac{a}{3} + \frac{b}{2} + c \\ b \end{pmatrix} : a,b,c \in \mathbb{R} \right\}$$

$$= \left\{ a \begin{pmatrix} 1/3 \\ 0 \end{pmatrix} + b \begin{pmatrix} 1/2 \\ 1 \end{pmatrix} + c \begin{pmatrix} 1 \\ 0 \end{pmatrix} : a,b,c \in \mathbb{R} \right\}.$$

sp. set for $\text{Im}(L) = \left\{ \begin{pmatrix} 1/3 \\ 0 \end{pmatrix}, \boxed{\begin{pmatrix} 1/2 \\ 1 \end{pmatrix}}, \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\}.$

L.I or L.D.

remove one:

$$c_1 \begin{pmatrix} 1/3 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 1/2 \\ 1 \end{pmatrix} + c_3 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} \frac{1}{3} & \frac{1}{2} & 1 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right) \longrightarrow \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} -3\alpha \\ 0 \\ \alpha \end{pmatrix} \quad \text{a sol} = \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$$

remove $\begin{pmatrix}1\\0\end{pmatrix}$

so $\underbrace{\left\{ \begin{pmatrix}1/3\\0\end{pmatrix}, \begin{pmatrix}1/2\\1\end{pmatrix} \right\}}_{L.I.}$ is a sp. set for $Im(L)$

so a basis for $Im(L)$ is $\left\{ \begin{pmatrix}1/3\\0\end{pmatrix}, \begin{pmatrix}1/2\\1\end{pmatrix} \right\}$

$\dim(Im(L)) = 2.$

Ex: If $L: \mathbb{R}^2 \to \mathbb{R}^2$ is a L.T.

and $L\begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}$ and $L\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}4\\5\end{pmatrix}$

Find $L\begin{pmatrix}8\\7\end{pmatrix}$.

$\begin{pmatrix}8\\7\end{pmatrix} = \overset{-7}{\alpha_1}\begin{pmatrix}1\\-1\end{pmatrix} + \overset{15}{\alpha_2}\begin{pmatrix}1\\0\end{pmatrix}$

$\alpha_1 + \alpha_2 = 8$

$-\alpha_1 = 7 \Rightarrow \alpha_1 = -7.$

$\alpha_2 = 8 + 7 = 15$

$L\begin{pmatrix}8\\7\end{pmatrix} = L\left(-7\begin{pmatrix}1\\-1\end{pmatrix} + 15\begin{pmatrix}1\\0\end{pmatrix}\right)$

$= -7L\begin{pmatrix}1\\-1\end{pmatrix} + 15L\begin{pmatrix}1\\0\end{pmatrix}$

$= -7\begin{pmatrix}2\\3\end{pmatrix} + 15\begin{pmatrix}4\\5\end{pmatrix} = \begin{pmatrix}-14+60\\-21+75\end{pmatrix} =$

$\boxed{4.1}$ / 6.1