lec 10 :-

Gauss Jordan :

Square System :-

↳ $A_{n \times n}$ is nonsingular

$$Ax = b \rightarrow x = A^{-1}b$$

↳ $A_{n \times n}$ is singular $\quad Ax = b$ → inconsistent / → inf # of solutions

LU - factorization Method

- A matrix is called upper triangular if $a_{ij} = 0$ for all $i > j$
- A is called lower triangular if $a_{ij} = 0$ for all $i < j$
- A is called triangular if it is upper or lower triangular
- A matrix is called diagonal if $a_{ij} = 0$, for all $i \neq j$

Ex:- $A = \begin{pmatrix} & & i<j \\ & i=j & \\ i>j & & \end{pmatrix}_{n\times n}$

↳ $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & -1 \\ 0 & 0 & 0 \end{pmatrix}$ upper triangular

↳ $B = \begin{pmatrix} 0 & 0 & 0 \\ 1 & -1 & 0 \\ 5 & 2 & -1 \end{pmatrix}$ lower triangular

↳ $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ upper triangular & lower triangular & diagonal

* If A is a diagonal matrix, then A is upper & lower triangular

والعكس صحيح

$$O = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$ upper triangular
lower triangular
diagonal

$$A = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 2 \\ 3 & 2 & 0 \end{pmatrix}$$ Not upper or lower
neither diagonal

* Given $Ax = b$, A is upper triangular

Ex:
$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & -1 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \begin{array}{l} \to x_1 = \checkmark \\ \to x_2 = \checkmark \\ \to x_3 = \checkmark \end{array}$$

$Ax = b$ easy to solve (Back substitut)

Given $Lx = b$, L lower triangular

$$\begin{pmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ -1 & 3 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$$ (forward substitut...)

Given $Ax=b$ & $A=LU$,

$\rightsquigarrow$ L lower triangular

$\rightsquigarrow$ U upper triangular

Solving $Ax=b$

$$L(\underbrace{UX}_{n\times n})=b$$

let $Ux=y$ $\longrightarrow$ *

$\underline{L}y=b$ $\rightsquigarrow$ easy to solve

find y

Now solve $Ux=y$

easy to solve to find X

Ex:- $A=\begin{pmatrix} 2 & 4 & 2 \\ 1 & 5 & 2 \\ 4 & -1 & 9 \end{pmatrix}$, Solve $Ax=\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

low. tri. upp. tri

Given $A=LU=\begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 2 & -3 & 1 \end{pmatrix}\begin{pmatrix} 2 & 4 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 8 \end{pmatrix}$

② Solve $Ux = y$

$$\begin{pmatrix} 2 & 4 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3/2 \\ 11/2 \end{pmatrix}$$

$$8x_3 = \frac{11}{2}$$

$$\boxed{x_3 = \frac{11}{16}}$$

$$3x_2 + x_3 = \frac{3}{2}$$

$$3x_2 + \frac{11}{16} = \frac{3}{2}$$

$$3x_2 = \frac{3\times 8}{2\times 8} - \frac{11}{16}$$

$$x_2 = \frac{24-11}{16} = \frac{13}{16} \times \frac{1}{3} = \boxed{\frac{13}{48} = x_2}$$

$$2x_1 + 4x_2 + 2x_3 = 1$$

$$2x_1 + \frac{2\times 13}{2\times 12} + \frac{3\times 11}{3\times 8} = 1$$

$$26 + 33$$

$$2x_1 = 1 - \frac{59}{24}$$

$$2x_1 = \frac{24-59}{24} = -\frac{35}{24}$$

$$x_1 = \frac{-35}{48}$$

- How to find L, U where $LU = A$?

→ If A can be transformed to an upper triangular matrix using row operation III only, Then we can find L, U s.t $A = LU$

Ex:- $A = \begin{pmatrix} 2 & 4 & 2 \\ 1 & 5 & 2 \\ 4 & -1 & 9 \end{pmatrix}$

$\xrightarrow[-2R_1+R_3]{-\frac{1}{2}R_1+R_2} \begin{pmatrix} 2 & 4 & 2 \\ 0 & 3 & 1 \\ 0 & -9 & 5 \end{pmatrix}$

$\xrightarrow{3R_2+R_3} \begin{pmatrix} 2 & 4 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 8 \end{pmatrix}$

$-\frac{1}{2}$, -2, 3

U

$L = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2}? & 1 & 0 \\ 2? & -3? & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 2 & -3 & 0 \end{pmatrix}$

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$$A \begin{pmatrix} \text{—} & & \\ \textcircled{0} & & \end{pmatrix}$$

بعطي L

zero
مكانها

$$A = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \\ 1 & -1 & 1 \end{pmatrix}$$

Ex = $A = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \\ 1 & -1 & 1 \end{pmatrix}$ find L, U

$$\xrightarrow{-R_1 + R_3} \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \\ 0 & 0 & -1 \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \\ 0 & 0 & -1 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \\ 1 & -1 & 2 \end{pmatrix}$$

① Solve $\begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \\ 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$

② Solve $\begin{pmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \\ 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

① $\left(\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 0 & 3 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$ Consistent

inf # of solutn

② $\left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 3 & 4 & 2 \\ 1 & -1 & 2 & 3 \end{array}\right)$

$\downarrow$

$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 1 \\ 0 & 3 & 4 & 2 \\ 0 & 0 & 0 & 2 \end{array}\right)$ InConsistent system