هنه النتائج الصحيحة اذا كانت
← الشروط الـ 8 كلها مع

lec 16:

$V, +, \cdot$

- $\mathbb{R}^n$
- $\mathbb{R}^{mn}$
- P_n
- $C[a,b]$

$1.V = V$

Theory: let V be a vector space Then:-

1) $0V = \vec{0}$, for all $v \in V$
2) if $v, w \in V$ and $v + w = \vec{0}$, Then $w = -v$
3) $(-1)v = -v$, for all $v \in V$

Proof:- 1) show $0V = \vec{0}$

let $V \in V$

$V = 1.V$ ⑧ condition (8)

$= (1+0).V$

$= 1.V + 0.V = V + 0.V$ $\}$ condition (5)

$V = V + 0.V$

$\vec{0} = V + (-V)$

$= V + 0.V + (-V)$

$= 0.V + V + (-V)$ -- ① condition

$= 0.V + (V + (-V))$

$= 0.V + \vec{0}$ --- ④ condition #

$\boxed{\vec{0} = 0.V}$

. لا يجوز استنتاج شرط من شرط آخر

2) Assume $V + W = \vec{0}$ $V, W \in V$

$(-V) = (-V) + \vec{0}$

$= (-V) + (V + W)$

$= (-V + V) + W$

$= \vec{0} + W = W$ #

3) $(-1)V = -V$

$(-1)V + V = (-1)V + 1.V$ —— ⑤ Condition

$= ((-1)+1).(V)$

$= 0.V$

$= \vec{0}$

لتعرف V.space لازم انا

تعرف : V و الـ addition

و Multiplication

$(2) \Longrightarrow (-1)(V) = (-V)$

* If $\alpha V = 0$, α scalar, $V \in V$

Then $\alpha = 0$ or $V = \vec{0}$

$\mathbb{R}^n$

Ex: $V = \mathbb{R}$ * ليست V.space

$x + y = y + x$

في حالة وحدة → $\alpha . x = 0$

فقط لما تكون $\alpha = 0$ أو $x = 0$

Proof:- Question ⑨:- b

Assume $\boxed{\alpha V = 0}$ —*

And Assume $\alpha \neq 0$ (show $V = 0$)

$\frac{1}{\alpha} \in \mathbb{R}$

Multiplying * by $\frac{1}{\alpha}$

$\frac{1}{\alpha} \alpha V = \frac{1}{\alpha} 0$

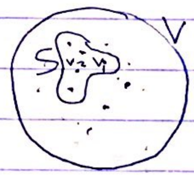
$1.V = \frac{1}{\alpha}(0)$ → scaler X zero vector

$\boxed{V = 0}$

H.W

Question 9-(a):- Show $\alpha\vec{0} = \vec{0}$, for any $\alpha \in \mathbb{R}$

vector space $\sim (V, +, \cdot)$



$\alpha v_1 \in S$
$0 v_1 = 0 \in S$

Def:- let V be a vector space
$S \subseteq V$, $S \neq \phi$ we say S is a
S is not empty
Subspace of V if

1) for all $s_1, s_2 \in S$, we have $s_1 + s_2 \in S$

2) for every $s \in S$ and α scaler,
we have $\alpha s \in S$

Ex:- V any vector space
$S = \{\vec{0}\}$, S is a sub space of V

1) $S \neq \phi$
2) let $s_1, s_2 \in S$

$\Rightarrow S_1 = 0 , \; S_2 = 0$

$S_1 + S_2 = 0 \in S$ → first Condition approved

②: let $s \in S$, α Scalar

$\Rightarrow S = 0$

Now $\alpha S = 0 \in S$ → Second Condition approved

$\underset{=\{0\}}{S}$ is a Subspace

Ex:- $S = \{ P(x) \in P_3 :- P(0) = 0 \}$

$\Rightarrow$ Show S is a Subspace of P_3

$S \neq \phi$, $p(x) = 0 \in S$

$p(x) = x^2 \in S$

- P_3 : $p(x) = ax^2 + bx + c$
- S : $p(x) = ax^2 + bx$

① let $p(x)$, $q(x) \in S$

$\rightsquigarrow P(0) = 0$, $q(0) = 0$

$P + q \in S$

$(P + q)(0) \overset{?}{=} P(0) + q(0) = 0 + 0 = 0$

$P + q \in S$

② let $p(x) \in S$

α Scaler

$\Rightarrow p(0) = 0$

$\alpha p_{(x)} \overset{?}{\in} S$

$(\alpha p)(0) = \alpha . \overset{0}{p(0)}$

$= \alpha . 0$

$= 0$

So $\alpha p(x) \in S$

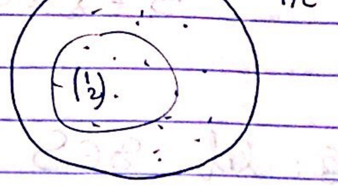
S is a Sub space of P_3

Ex:- $S = \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2 \; : x_2 = 2x_1 \right\}$

Is S a subspace?

$S \neq \phi$ (S is not empty)

$\begin{pmatrix} 0 \\ 0 \end{pmatrix} \in S$



ال S جزء من $\mathbb{R}^2$

1) let $v = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, w = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \in S$

$x_2 = 2x_1 \quad , \quad y_2 = 2y_1$

$V + W = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \cancel{\begin{pmatrix} x_1 + y_2 \\ y_2 + y_2 \end{pmatrix}} \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \end{pmatrix}$

$= \begin{pmatrix} x_1 + y_1 \\ 2x_1 + 2y_1 \end{pmatrix} = \begin{pmatrix} x_1 + y_1 \\ 2(x_1 + y_1) \end{pmatrix} \in S$

2) let $V = \begin{pmatrix} x_1 \\ 2x_1 \end{pmatrix} \in S$, α is a Scalar

$\alpha V = \alpha \begin{pmatrix} x_1 \\ 2x_1 \end{pmatrix} = \begin{pmatrix} \alpha x_1 \\ 2\alpha x_1 \end{pmatrix} \in S$

$\therefore$ S is a SubSpace of $\mathbb{R}^2$

Ex:- $S = \left\{ \begin{pmatrix} x_1 \\ 2x_1 + 1 \end{pmatrix} \in \mathbb{R}^2 : x_1 \in \mathbb{R} \right\}$

Is S a Subspace of $\mathbb{R}^2$?

$* \; S \neq \emptyset \qquad \begin{pmatrix} 0 \\ 1 \end{pmatrix} \in S$

$\begin{pmatrix} 0 \\ 0 \end{pmatrix} \notin S$

1) let $v = \begin{pmatrix} x_1 \\ 2x_1 + 1 \end{pmatrix} \qquad w = \begin{pmatrix} y_1 \\ 2y_1 + 1 \end{pmatrix}$

$$v + w = \begin{pmatrix} x_1 \\ 2x_1+1 \end{pmatrix} + \begin{pmatrix} y_1 \\ 2y_1+1 \end{pmatrix}$$

$$= \begin{pmatrix} x_1 + y_1 \\ 2x_1 + 2y_1 + 2 \end{pmatrix}$$

$$= \begin{pmatrix} x_1 + y_1 \\ 2(x_1 + y_1) + 2 \end{pmatrix}$$

Ex:- $\begin{pmatrix} 1 \\ 3 \end{pmatrix} \in S$

$\begin{pmatrix} 2 \\ 5 \end{pmatrix} \in S$

$\begin{pmatrix} 1 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 8 \end{pmatrix} \notin S$

Remark:- let S be a subset of V
if $0 \notin S$. Then S is not a
Subspace of V

Important

Proof:- let $s \in S$

Consider $0 . s = \vec{0} \notin S$
so Condition ② is not Satisfied
$\Rightarrow$ So S is not a subspace of V

Ex: $S = \{ A \in \mathbb{R}^{2\times2} : a_{21} = a_{12} \}$

IS S a subspace of $\mathbb{R}^2$

1) $S \neq \emptyset \qquad \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in S$

a_{21}

1) let $\alpha, A, B \in S$.

$\Rightarrow \quad a_{21} = -a_{12} \quad$ and $\quad b_{21} = -b_{12}$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$A+B = \begin{bmatrix} a_{11} & a_{12} \\ -a_{12} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ -b_{12} & b_{22} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ -(a_{12}+b_{12}) & a_{22}+b_{22} \end{bmatrix} \in S$$

2) let $A = \begin{bmatrix} a_{11} & a_{12} \\ -a_{12} & a_{22} \end{bmatrix} \in S$, α scalar

$$\alpha A = \begin{bmatrix} \alpha a_{11} & \alpha a_{12} \\ -(\alpha a_{12}) & a_{22} \end{bmatrix} , \in S$$

لو كانت α سالبة و a_{12} عنصر سالبة فهو موجبة تحت فيبقى الشرط صحيح

S is a subspace $\mathbb{R}^{2\times 2}$

Ex $A_{m\times n}$ any $m\times n$ matrix

$$S = \{ x \in \mathbb{R}^n ;\ Ax = 0 \}$$

= The set of all solutions to $Ax=0$

$(S = N(A))$: (null space of A)

Is $N(A)$ a Subspace of $\mathbb{R}^n$

• $N(A) \neq \emptyset \quad X = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \in N(A) \quad \begin{pmatrix} \text{solutions} \\ \text{to } Ax=0 \end{pmatrix}$

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$\Longrightarrow Av = 0$, $Aw = 0$

Consider $A(v+w) = Av + Aw$

$= 0 + 0 = 0$

$v+w \in N(A)$

* Similarly . $\alpha v \in N(A)$

Since $A(\alpha v) = \alpha (Av)$