lec 19:-

Ex :- $f_1(x) = x^2$, $f_2(x) = x|x|$ on $[-1, 1]$

$W[x^2, x|x|](x) = 0 \rightsquigarrow$ Test fails

Solve:- $c_1 x^2 + c_2 x|x| = 0 \quad \forall x \in [-1, 1]$

$x = -1 \quad \left.\begin{array}{l} c_1 - c_2 = 0 \\ c_1 + c_2 = 0 \end{array}\right\} \Rightarrow c_1 = c_2 = 0$

infinit system

But it has only the zero solution

هناك قيم لانهائية لـ system
من -1 , 1
لكن اذا عوضنا بعدد صحيح الحلول
الحل المشترك الوحيد هو 0
انه لدينا حل واحد فقط

3 هذه الفترة

Ex: $f_1(x) = x^2 \qquad f_2(x) = x|x| = x^2 \rightsquigarrow$ over $[0, 1]$

$W[x^2, x|x|]_{(x)} = 0$

Solve $\quad c_1 x^2 + c_2 x|x| = 0$

$x = 1 \quad c_1 + c_2 = 0$

$x = \frac{1}{2} \quad \frac{c_1}{4} + \frac{c_2}{4} = 0$

$c_1 f_1 + c_2 f_2 = 0 \Rightarrow f_1(x) = \frac{-c_2}{c_1} f_2(x)$

V_1, V_2 are linearly depen

Bcz one of them can be written as a scaler multi of the other

So The two functions are L.D

Theory:-

- If $\{V_1, V_2, \ldots, V_n\}$ is a spanning set for V and one of Them can be written as a linear combination of the other $(n-1)$ vectors Then These $n-1$ vectors form a spanning set for V
- If $V_1, \ldots, V_n \in V$ one of Them can be written as a l.c of the other $(n-1)$ vectors iff There exists scalers $c_1, \ldots, c_n$ not all zeros s.t:-

$$c_1V_1 + c_2V_2 + \ldots + c_nV_n = 0$$

$V_1, \ldots, V_n$ are L.D

Theory:-

If $V_1, V_2, \ldots V_n$ $V_n \in \mathbb{R}^n$

Then $V_1, \ldots, V_n$ are L.I iff The matrix $X = (V_1\ V_2 \ldots V_n)_{n \times n}$ is non singuler.

Ex: $V_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $V_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $V_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

L.I or L.D ? $\in \mathbb{R}^3$

$X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $|X| = 1$ so It's nonsingu-

That means They are L.I

Proof:- Assume $V_1, V_2, \ldots, V_n$ are L.I
let:-

Direction 1:- $A = (V_1, V_2, \ldots, V_n)$
Solve $AX = 0$
$\hookrightarrow x_1 a_1 + x_2 a_2 + \ldots + x_n a_n = 0$
$\hookrightarrow x_1 V_1 + x_2 V_2 + \ldots + x_n V_n = 0$

So it has only the zero solution

Since $V_1, \ldots, V_n$ are L.I
$\Longrightarrow AX = 0$ has only The zero sol

$\Longrightarrow A$ is nonsingular

Direction 2:- Assume $A = (V_1, V_2, \ldots, V_n)$
is nonsingular

The system $C_1 V_1 + C_2 V_2 + \ldots + C_n V_n = 0$ — *

is equivalent to $AC = 0$

$C = \begin{pmatrix} c_1 \\ | \\ c_n \end{pmatrix}$ but A is nonsingular.

$Ac = 0$ has only the zero solution
(*) has only the zero solution

$V_1, \text{---}, V_n$ are L.I

---

* $V_1, \text{---}, V_n \in V$

subspace of V ⟶ span $(V_1, \text{---}, V_n)$

let $w \in$ span $(V_1, \text{---}, V_n)$
we can write w uniquely as a linear combination of $V_1, \text{---}, V_n$

$\Longleftrightarrow V_1, \text{---}, V_n$ are linearly ind.

Proof:- Assume $w \in$ span $(V_1, \text{---}, V_n)$ can be written uniquely as a L.C of $V_1, \text{---}, V_n$

Show $(V_1, \text{---}, V_n)$ are L.I

by contradiction : Assume $V_1, \text{---}, V_n$ L.D

$\exists\ c_1, \text{---}, c_n$ not all zeros
s.t $c_1V_1 + c_2V_2 + \text{---} + c_nV_n = 0$

Now $\omega = \alpha_1 V_1 + \alpha_2 V_2 + \text{———} + \alpha_n V_n$

α_i : Scalers Unique

Now $\omega + 0 = \alpha_1 V_1 + \alpha_2 V_2 + \text{———} + \alpha_n V_n + c_1 V_1 + c_2 V_2 + \text{———} + c_n V_n$

$= (\alpha_1 + c_1) V_1 + \text{———} + (\alpha_n + c_n) V_n$

$\Longleftrightarrow$ Uniqueness $\alpha_1 + c_1 = \alpha_1 \Longrightarrow c_1 = 0$

$\alpha_2 + c_2 = \alpha_2 \Longrightarrow c_2 = 0$

$\vdots$

$\alpha_n + c_n = \alpha_n \Rightarrow c_n = 0$

A Contradiction :- So $V_1, \text{—}, V_n$ are L.I

Assume $V_1, \text{———}, V_n$ are L.I (Show unique)

Assume $\omega = \alpha_1 V_1 + \alpha_2 V_2 + \text{———} + \alpha_n V_n$

$= \beta_1 V_1 + \beta_2 V_2 + \text{———} + \beta_n V_n$

Show $(\alpha_1 = \beta_1, \text{———}, \beta_n = \alpha_n)$

$0 = \omega - \omega$

$0 = (\alpha_1 - \beta_1) V_1 + \text{———} + (\alpha_n - \beta_n) V_n$

Scaler

but $V_1, \text{———}, V_n$ are L.I

$$\left.\begin{aligned}\alpha_1 - \beta_1 &= 0 \\ &\vdots \\ \alpha_n - \beta_n &= 0\end{aligned}\right\} \quad \begin{aligned}\alpha_1 &= \beta_1 \\ &\vdots \\ \alpha_n &= \beta_n\end{aligned}$$

U, W subspaces of V show $U \cap W$ is a subspace of V:-

- $U \cap W \neq \phi$ since $0 \in U \cap W$
- let $u_1, u_2 \in U \cap W$

$\rightarrow u_1, u_2 \in U$ and $u_1, u_2 \in W$

So $u_1 + u_2 \in U$ Because (U is a subspace
$u_1 + u_2 \in W$ $\quad$ W " ")

$\rightarrow u_1 + u_2 \in U \cap W$

$u \in U \cap W$ $\quad \alpha$ scaler

$\Rightarrow u \in U$ and $u \in W$

$\alpha u \in U$ $\quad$ (U sub
$\alpha u \in W$ $\quad$ (W ")

So $\alpha u \in U \cap W$ so it is a subspace

$U \cup W$ (union of Two)

- subspaces need not to be a subspace.

Ex :- $U = \left\{ \begin{pmatrix} x \\ 0 \end{pmatrix}, x \in \mathbb{R}^2 \right\}$ is a s.p of $\mathbb{R}^2$

$W = \left\{ \begin{pmatrix} 0 \\ y \end{pmatrix}, y \in \mathbb{R}^2 \right\}$ is a s.p of $\mathbb{R}^2$

$U \cup W = \left\{ \begin{pmatrix} x \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ y \end{pmatrix} : y, x \in \mathbb{R}^2 \right\}$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ but $\begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \notin U \cup W$

$\in U \cup W$

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