lec2 :- Chapter 1

1.1 linear systems of equations

↳ $m \times n$ - linear system :- where :-

m :- num of equations

n :- " " unknowns : x_1, x_2 -

General form:- كل الحدود يجب أن تكون X ليست مرفوعة لقوة

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$

Ex:- $x_1 + x_2 + x_3 = 1$
$2x_1 - x_2 + 2x_3 = 2$

2×3 - system

↳ We need to find a solution for the system :-

$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$, values

- for unknowns $x_1 \ldots x_n$ that satisfies all equations

Def:- equivalent systems :- لم نضع شرط على عدد equations

Two systems with the same unknowns are called equivalent if they have the

the same set of ~~equations~~ solutions.

Ex:- $x_1 + x_2 = 3$ ---- (1)

$2x_2 = 6$ ---- (2)

from (2) $\underline{x_2 = 3}$

In (1) $x_1 + 3 = 3$ $\underline{x_1 = 0}$

Solution $= X = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$

In the matrix equations are raws

Def. Elementary row operations :-

1- Elementary operation one (I) :-

Interchange two equations ← rows

2- Multiply an equation (a raw) By a non zero Constant

3- Replace an equation (raw) By it's sum with a multiple of another equations (row)

3 equivalent systems

$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ 2x_1 - x_2 + 3x_3 = 2 \end{cases}$$

Multi. (1) By -2

$$\begin{cases} -2x_1 - 2x_2 - 2x_3 = -2 \\ 2x_1 - x_2 + 3x_3 = 2 \end{cases}$$

Add (1) to 2

$$\begin{cases} -3x_2 + x_3 = 0 \\ -2x_1 - 2x_2 - 2x_3 = -2 \end{cases}$$

Def: Applying elementary row operation on a system produces an equivalent system

Ex :-

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 5 \\ \boxed{-x_1} + 2x_2 - x_3 - 2x_4 &= 6 \\ \boxed{2x_1} - \boxed{x_2} + x_3 - 3x_4 &= 1 \\ \boxed{x_1} - \boxed{x_2} - \boxed{x_3} + x_4 &= 0 \end{aligned}$$

4x4 system

Def: Augmented matrix of a system :-

$$\left(\begin{array}{ccccc|c} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} & b_2 \\ \vdots & & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} & b_m \end{array}\right)$$

Pivot element
هو الذي استعمله لكي أصفر كل الباقي

→ for the example above :-

Pivot element

$$(A|b) = \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ -1 & 2 & -1 & -2 & 6 \\ 2 & -1 & 2 & -3 & 1 \\ 1 & -1 & -1 & 1 & 0 \end{array}\right)$$

should be zeros

• اذا كانت المعادلة الأولى فيها معامل $x_1 = 0$ نستعمل أول operation ونبدل معادلتين

اذا كانو كلهم [illegible] x_1

① $(R_1 + R_2) \rightsquigarrow R_2$ اضع الناتج محل R_2

② $((-2) \times R_1 + R_3) \rightsquigarrow R_3$ اضع الناتج محل R_3

③ $-R_1 + R_4 \rightsquigarrow R_4$ ضع الناتج R_4

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 3 & 0 & -1 & 11 \\ 0 & -3 & 0 & -5 & -9 \\ 0 & -2 & -2 & 0 & -5 \end{array}\right)$$

Pivot element Now

Now:

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & \boxed{3} & 0 & 1 & 11 \\ 0 & -3 & 0 & -5 & -9 \\ 0 & -2 & -2 & 0 & -5 \end{array}\right)$$

$$\Downarrow$$

① $\frac{1}{3}R_2$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & \boxed{1} & 0 & -\frac{1}{3} & \frac{11}{3} \\ 0 & -3 & 0 & -5 & -9 \\ 0 & -2 & -2 & 0 & -5 \end{array}\right)$$

$$\Downarrow$$

② $(3R_2 + R_3) \rightsquigarrow R_3$ بدل [illegible]

③ $(2R_2 + R_4) \rightsquigarrow R_4$ " "

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & 0 & -\frac{1}{3} & \frac{11}{3} \\ 0 & 0 & \boxed{0} & -6 & 2 \\ 0 & 0 & -2 & -\frac{2}{3} & \frac{7}{3} \end{array}\right)$$

The Pivot element Now

④ R_4 & R_3 بدل

⑤ $-\frac{1}{2}R_3$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & 0 & -\frac{1}{3} & \frac{11}{3} \\ 0 & 0 & \boxed{1} & \frac{1}{3} & -\frac{7}{6} \\ 0 & 0 & 0 & -6 & 2 \end{array}\right)$$

pivot.E

• from the matrix, we get an easier system so we solve it:-

R_4 :- $X_4 = -\frac{1}{3}$

R_3 :- $X_3 + \frac{X_4}{3} = -\frac{7}{6}$

$X_3 - \frac{1}{9} = -\frac{7}{6}$

$X_3 = \frac{3\times -7}{3\times 6} + \frac{2\times 1}{2\times 9} = \cancel{-7\times 9} \quad \cancel{\frac{-57}{18}} \quad \frac{-21+2}{18} = \frac{-19}{18}$

R_2 :- $X_2 = \frac{11}{3} + \frac{1}{3}\left(\frac{-1}{3}\right)$

$\frac{3\times 11}{3\times 3} - \frac{1}{9} = \frac{32}{9} =$

R_1 :- $X_1 = \frac{5}{1\times 18} - \frac{32}{9} + \frac{19}{18} + \frac{1}{3}$

$= \frac{90}{18} - \frac{64}{18} + \frac{19}{18} + \frac{6}{18}$

$= \frac{115}{18} - \frac{64}{18} = \frac{51}{18}$

The solution:- $X = \begin{pmatrix} \omega \\ 32/9 \\ -19/18 \\ -1/3 \end{pmatrix}$

Ex :- $x_1 + x_2 + x_3 + x_4 + x_5 = 1$

$-x_1 - x_2 + x_5 = -1$

$-2x_1 - 2x_2 + 3x_5 = 1$

$x_3 + x_4 + 3x_5 = 3$

$x_1 + x_2 + 2x_3 + 2x_4 + 4x_5 = 4$

$$\left(\begin{array}{ccccc|c} \boxed{1} & 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & 0 & 0 & 1 & -1 \\ -2 & -2 & 0 & 0 & 3 & 1 \\ 0 & 0 & 1 & 1 & 3 & 3 \\ 1 & 1 & 2 & 2 & 4 & 4 \end{array}\right)$$

① $R_1 + R_2$

⇓ ② $2R_1 + R_3$ ③ $-R_1 + R_5$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & \boxed{0} & \boxed{1} & 1 & 2 & 0 \\ 0 & 0 & 2 & 2 & 5 & 3 \\ 0 & 0 & 1 & 1 & 3 & 3 \\ 0 & 0 & 1 & 1 & 3 & 3 \end{array}\right)$$

⇓ ④ $-2R_2 + R_3$ ⑥ $-R_2 + R_5$

⑤ $-R_2 + R_4$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{array}\right)$$

⑦ $-R_3 + R_4$

⇓ ⑧ $-R_3 + R_5$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \begin{array}{l} \\ \rightsquigarrow \\ \rightsquigarrow x_5 = 3 \\ \\ \\ \end{array}$$

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or more

• Is there a row of the form?

$(0 \quad 0 \quad \text{———} \quad 0 \mid c \neq 0)$ - No

If yes :- $0x_1 + 0x_2 + 0x_2 + \cdots + 0x_n = (c \neq 0)$

has no solutions

$\Longrightarrow$ This implies the system has no solution

If not $\Longrightarrow$ There are solutions