lec 23

$$\left.\begin{aligned} B &= [v_1, v_2, \ldots, v_n] \\ T &= [w_1, w_2, \ldots, w_n] \end{aligned}\right\} \text{basis for } V$$

$v \in V$

$$[v]_B = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix}, \quad [v]_w = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_n \end{pmatrix}$$

$$v = \beta_1 w_1 + \beta_2 w_2 + \ldots \beta_n w_n$$

$$v = \alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n$$

$$[v]_B = T_{w \to B} \, [v]_w$$

- elements of w in terms of elements of in B

$$T_{w \to B} = \left([w_1]_B \; [w_2]_B \; \ldots \; [w_n]_B \right)_{n \times n}$$

always Non Singular

$$T_{B \to W} = \left(T_{W \to B}\right)^{-1}$$

Ex: find the transition matrix from $U = [x^2+x-1,\ x-1,\ -3]$ to $W = [x,\ x^2,\ 1]$

If $p(x) = 7x^2 - 3x + 5$ find $[p(x)]_U$

$T_{U \to W}$ = U in terms of W

$$x^2+x-1 = 1 \cdot x + 1 \cdot x^2 + (-1)(1)$$
$$x-1 = 1 \cdot x + 0 \cdot x^2 + (-1)(1)$$

$$T_{U \to W} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ -1 & -1 & -3 \end{pmatrix}$$

$$-3 = 0 \cdot x + 0 \cdot x^2 + (-3)(1)$$

Non singular

$P(x) = 7x^2 - 3x + 5$

$[P(x)]_u =$ مم الترتيب صحيح

We have $[P_x]_w = \begin{pmatrix} -3 \\ 7 \\ 5 \end{pmatrix}$

$[P_{(x)}]_u = T_{w \to u} \; [P_{(x)}]_w$

$[P(x)]_u = \left(T_{u \to w} \right)^{-1} \; [P(x)]_w$

$= \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \begin{pmatrix} -3 \\ 7 \\ 5 \end{pmatrix}$

. Method II : $[P(x)]_u$: $7x^2 - 3x + 5$

$[P(x)]_u$: $7x^2 - 3x + 5 = d_1(x^2 + x - 1) + d_2(x-$
$+ d_3(-3)$

$\Downarrow$

$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$ Solve the system $\Rightarrow$

x^2: $\boxed{7 = d_1}$

x: $-3 = d_1 + d_2 \Rightarrow \boxed{d_2 = -3-7 = -10}$

coeff: $5 = -d_1 - d_2 - 3d_3$

$5 = -7 + 10 - 3d_3$

$\dfrac{5+7-10}{-3} = d_3$

$\boxed{d_3 = \dfrac{-2}{3}}$

So $[7x^2 - 3x + 5]_u = \begin{pmatrix} 7 \\ -10 \\ \frac{-2}{3} \end{pmatrix}$

To check Your answer

$P(x) = 7(x^2 + x - 1) - 10(x-1) - \frac{2}{3}(-3)$

$= 7x^2 + 7x - 7 - 10x + 10 + 2$

$= 7x^2 - 3x + 5$

Ex. $U = [v_1, v_2, \dots, v_n]$

$W = [w_1, w_2, \dots, w_n]$

basis for $\mathbb{R}^n$. find $T_{u\to w}$

$T_{u\to s} = (u_1\, u_2 \dots u_n) = U$ $\quad$ $[v]_s = v$

$$\begin{tikzcd} U \arrow[r] \arrow[dr, "T_{u\to w}"'] & S = [e_1, e_2, \dots, e_n] \arrow[d, shift left, "W^{-1}"] \\ & W \arrow[u, shift left, "W"] \end{tikzcd}$$

$[v]_u$ $\quad$ $[v]_w$

Because S is standard

$T_{u\to w} =$

$[v]_w = T_{u\to w}\,[v]_u \dashrightarrow$ 1

$[v]_s = T_{u\to s}\,[v]_u \dashrightarrow$ 2

$[v]_w = T_{s\to w}\,[v]_s \dashrightarrow$ 3

substitute 2 in 3

$[v]_w = (T_{s\to w})(T_{u\to s})\,[v]_u$

$= W^{-1} U\,[v]_u$

$$\boxed{T_{u\to w} = W^{-1} U}$$

Ex: find the Transition Matrix from

$$U = \left[v_1 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right]$$

to

$$W = \left[w_1 = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, w_2 = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}, w_3 = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \right]$$

$$T_{u \to w} = W^{-1} U$$

$$= \begin{pmatrix} 2 & -1 & 1 \\ 3 & 1 & 4 \\ 4 & 5 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 1 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

3.5 is finished

3.6: row: $(- - - \dots)_{1 \times n}$

$A_{m \times n}$

m rows $\in \mathbb{R}^{1 \times n}$

n Columns $\in \mathbb{R}^m$

* $\text{Span}(\vec{a}_1, \vec{a}_2, \vec{a}_3, \dots, \vec{a}_n)$

subspace of $\mathbb{R}^{1 \times n}$

$V_1, \dots, V_s \in V$

$\text{Span}(V_1, \dots V_s)$ is a subspace of V

$= \{\alpha_1 V_1 + \alpha_2 V_2 + \dots + \alpha_n V_n\}$

Remember: $N(A) = \{x \in \mathbb{R}^n : Ax = 0\}$

- Column space of $A = \text{span}(a_1, \dots, a_n)$

subspace of $\mathbb{R}^m$

$= C(A)$

- Row space of $A = \text{span}(a_1, \dots, a_n)$

subspace of $\mathbb{R}^{1 \times n}$

$= R(A)$

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Ex: basis and dimensions of $N(A)$ where $A = \begin{pmatrix} 1 & -1 & 1 & 2 \\ -1 & 1 & 1 & 3 \end{pmatrix}$

$$\longrightarrow \begin{pmatrix} 1 & -1 & 1 & 2 \\ 0 & 0 & 2 & 5 \end{pmatrix}$$

- $X_2 = \alpha$, $X_4 = \beta$ free

$X_3 = -\frac{5}{2}$

$X_1 = \alpha + \frac{5}{2}\beta - 2\beta = \alpha + \frac{\beta}{2}$

$$N(A) = \left\{ \begin{pmatrix} \alpha + \frac{1}{2}\beta \\ \alpha \\ -\frac{5}{2}\beta \\ \beta \end{pmatrix} : \beta \in \mathbb{R} \right\}$$

$$= \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} \frac{1}{2} \\ 0 \\ -5/2 \\ 1 \end{pmatrix} \quad \alpha, \beta \in \mathbb{R} \right\}$$

Sp. set for $N(A)$ is $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1/2 \\ 0 \\ -5/2 \\ 1 \end{pmatrix} \right\}$

$\rightsquigarrow$ L.I

So Basis of $N(A)$ is $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{2} \\ 0 \\ -\frac{5}{2} \\ 1 \end{pmatrix} \right\}$

$\dim(N(A)) = 2$

always L.I

Remember :- $S = \left\{ \begin{pmatrix} a+b+2c \\ b-c \\ a+b \\ c \end{pmatrix} : a,b,c \in \mathbb{R} \right\}$

$= \left\{ a \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + b \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + c \begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \end{pmatrix} \right\}$

• is a sp set But

May or May not be L.I

You have to check

$A_{m \times n}$

$R(A) = \text{Span (rows of } A)$

$\dim(R(A)) \leq m$

and span(rows of A) is a subspace of $\mathbb{R}^{1 \times n}$

$\dim(R(A)) \leq n$

$\dim(R(A)) \leq \min\{m, n\}$

Ex: $A_{5\times3}$ ← rows are L.D

$\underline{\dim \leq 3}$. So we have to remove at least two rows

. 5 rows in $\mathbb{R}^{1\times3}$, $\dim(\mathbb{R}^{1\times3}) = 3$
$5>3$ so rows are L.D

* $R(A)$
* If $A_{m\times n}$ is row equivalent to $B_{m\times n}$ Then $R(A) = R(B)$

rows of B are linear combination of row of A (row operations)

$\Rightarrow$ set of l.c of A is the same as B

* Given $A_{m \times n}$

$\hookrightarrow$ let U be the R.R.E.F of A

– $R(A) = R(U)$

nonzero rows of U are L.I and sp.s.f for R(A)

a basis for R(U) is the nonzero rows of U also they form a basis for R(A)