Lec 3:-

Review :-

$(A|b) \xrightarrow{E.R.O} (U|d)$

1) If there exists a row of the form $(0\ 0 \ldots 0|c\neq 0)$ in $(U|d)$ $\rightarrow$ This implies that the system has no solution

(called inconsistent system)

2) If not / No row of the form $(0\ 0 \ldots 0 | c\neq 0)$ then the system has solutions

(called consistent system)

• Ex (the example in the previous lecture)

$$\left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \begin{array}{l} \rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 1 \\ \rightarrow x_3 + x_4 + 2x_5 = 0 \Rightarrow x_3 = -6 - x_4 \\ \rightarrow x_5 = 3 \\ 0 = 0 \\ 0 = 0 \end{array}$$

$\Rightarrow$ This implies the system is Consistent

R_1 : $x_1 + x_2 + x_3 + x_4 + x_5 = 1$

$x_1 = 1 - x_2 - x_3 - x_4 - x_5$

$= 1 - x_2 - (-6 - x_4) - x_4 - 3$

$= 1 - x_2 + 6 + \cancel{x_4} - \cancel{x_4} - 3$

$= -2 + 6 - x_2$

$\boxed{x_1 = 4 - x_2}$ بدي أطلع x_1 بدلالة x_2 و x_3 $\Rightarrow$ any solutions have the form

$$x = \begin{pmatrix} -x_2 + 4 \\ x_2 \\ -x_4 - 6 \end{pmatrix}$$

• So you can choose any value for x_2 and x_4 (Free variables)

Ex: take $x_2 = 1$, $x_4 = 0 \Rightarrow X = \begin{pmatrix} 3 \\ 1 \\ -6 \\ 0 \\ 3 \end{pmatrix}$ is a solution

take $x_2 = 0$, $x_4 = 0 \Rightarrow X = \begin{pmatrix} 4 \\ 0 \\ -6 \\ 0 \\ 3 \end{pmatrix}$

• There are infinitely many solutions

Remark:- if there are free variables then the system has infinite number of solutions

* leading ones

the first non zero element in each non zero rows

$$\begin{array}{c} \quad x_1 \qquad\qquad x_3 \qquad x_5 \\ \left(\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 1 & & 1 \\ 0 & 0 & 1 & 1 & 2 & & 0 \\ 0 & 0 & 0 & 0 & 1 & & 3 \\ 0 & 0 & 0 & 0 & 0 & & 0 \\ 0 & 0 & 0 & 0 & 0 & & 0 \end{array}\right) \end{array}$$

The corresponding variables are called leading variables

• x_1, x_3, x_5 are leading variables

The rest of variables are free variables (x_2, x_4)

x_4, x_2 نكتب كل شيء بدلالة

$X = \begin{pmatrix} \vdots \end{pmatrix}$ solution

leading variables أي الـ

$\boxed{1.2}$ Def :- Row echelon form :-

A matrix is called in row echelon form if :- 1) The first non zero element in each non zero row is 1

Ex:

$$\begin{pmatrix} \boxed{1} & 2 & 0 & 0 \\ 0 & 0 & \boxed{1} & 2 \\ 0 & 0 & \boxed{2} & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

X

2) The number of leading zeros (the first zeros in a row) in each non zero row j must be less than the number of leading zeros in row j+1 (for all j)

3) If there are rows of zeros (zero rows) they are below non zero rows

Ex :-

$$\begin{pmatrix} 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$\boxed{1}$ ✓
$\boxed{2}$ ✓
$\boxed{3}$ ✓

This matrix is in R.E.F

Ex :-

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\boxed{1}$ X

This matrix is not in R.E.F

• If we use EoF it Becomes a new matrix and it may be R.E.F or not

Ex.:- $\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$

1 ✓
2 ✓
3 X

→ • not in R.E.F

* Gauss elemination method

Augmented matrix

$\rightarrow (A|b) \xrightarrow{E.R.O}$ Matrix in R.E.F

Yes → $(0 \; 0 \; \cdots \; 0 \,|\, c \neq 0)$ inconsistant

No → Consistant

free variables:

Yes → inf number of solutions

No → only one solution

Ex.:- $3x_1 + 2x_2 - x_3 = -2$

$-3x_1 - x_2 + x_3 = 5$

$3x_1 + 2x_2 + x_3 = 2$

$$\left(\begin{array}{ccc|c} 3 & 2 & -1 & -2 \\ -3 & -1 & 1 & 5 \\ 3 & 2 & 1 & 2 \end{array}\right)$$

it's not a R.E.F But we can make it By EOF

$$\longrightarrow \left(\begin{array}{ccc|c} 3 & 2 & -1 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 2 & 4 \end{array}\right)$$

① $R_1 + R_2$
② $-R_1 + R_3$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & \frac{2}{3} & -\frac{1}{3} & -\frac{2}{3} \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 2 & 4 \end{array}\right)$$

③ $\frac{1}{3}R_1$

$x_1 = -\frac{2}{3} - \frac{2}{3}(3) + \frac{1}{3}(2)$
$= -2$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & \frac{2}{3} & -\frac{1}{3} & -\frac{2}{3} \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

④ $\frac{1}{2}R_3$

$\rightarrow x_2 = 3$

R.E.F

$\rightarrow x_3 = 2$

The system is Consistant

3 leading variables No free variables

$\rightarrow$ There is one solution

So Sol. $X = \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$