lec 5 :-

Ex :- $x_1 + x_2 + x_3 = 2$

$-x_1 + 2x_2 + 3x_3 = 5$

$2x_1 - x_2 - 4x_3 = 1$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ -1 & 2 & 3 & 5 \\ 2 & -1 & -4 & 1 \end{array}\right)$$

$\downarrow$ $R_1 + R_2$, $-2R_1 + R_3$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 3 & 4 & 7 \\ 0 & -3 & -6 & -3 \end{array}\right)$$

$\downarrow$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 3 & 4 & 7 \\ 0 & 0 & -2 & 4 \end{array}\right)$$

$\downarrow$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & \frac{4}{3} & \frac{7}{3} \\ 0 & 0 & 1 & -2 \end{array}\right) \rightsquigarrow 3 \text{ leading var}$$

$x_3 = \cancel{2}$

$x_2 = \frac{7}{3} + \frac{4}{3} \times 2 = \frac{15}{3} = 5$

* A matrix is set to be in reduced row echelon form (R.R.E.F) if :-

1) It is in R.E.F
2) The leading ones are the only non-zero elements in their columns

Ex ①
$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 7/3 \\ 0 & 0 & 1 & -2 \end{array}\right)$$

②
$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\boxed{1}$
$\boxed{2}$

is in R.R.E.F

③
$$\begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

④
$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$ R.E.F But not R.R.E.F

* Any matrix can be transformed to a matrix in R.R.E.F

Remember:- G.E.M: $(A|b) \xrightarrow{E.R.O} (U|d) \sim$ R.E.F

Gauss-Jordan elemination method:-

$$(A|b) \xrightarrow{E.R.O} \underbrace{(U|d)}_{\text{same as G.E.M}} \rightarrow \text{R.R.E.F}$$

Ex:- $x_1 + x_2 + x_3 + \cancel{x_4} = 1$
$-x_1 + x_2 + x_3 = 1$
$2x_1 - x_2 + 4x_3 = 2$

Use Gauss. J. E. M :-

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 1 \\ -1 & 1 & 1 & 0 & 1 \\ 2 & -1 & 4 & 0 & 2 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & -3 & 2 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & \boxed{1} & 1 & 1 \\ 0 & -3 & 2 & 0 \end{array}\right)$$

In G. J. EM
ال 1 نصفر اللي
فوقه واللي
تحت

$-R_2 + R_1$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 5 & 3 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 3/5 \end{array}\right)$$

$-R_3 + R_2$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{2}{5} \\ 0 & 0 & 1 & 3/5 \end{array}\right)$$

$x_3 = 3/5$
$x_2 = 2/5$
$x_1 = 0$

$$X = \begin{pmatrix} 0 \\ 2/5 \\ 3/5 \end{pmatrix}$$

system Cons → 3 leade
1 soluh -

$a_2 = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$

$\vec{a}_2 = \begin{pmatrix} 1 & -1 & 0 \end{pmatrix}$

$A = (a_{ij})_{m \times n}$

- operations on Matrices

1) Equality

if $A = (a_{ij})_{m \times n}$ and $B (b_{ij})_{m \times n}$

we say That $A = B$ if

$\rightsquigarrow a_{ij} = b_{ij} \quad \forall i, j$

$\rightsquigarrow$ and size, is equal

Ex :- $\begin{pmatrix} 2 & 3 & 4 \\ -1 & 0 & 1 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}_{2 \times 3}$

If $\Rightarrow$ $a = 2$, $b = 3$, $c = 4$
$d = -1$, $e = 0$, $f = 1$

1.3 + 1.4 :- Matrices

An $m \times n$ matrix has the form

columns

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{31} & a_{32} & \cdots & a_{3n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

rows

first row : $\vec{a_1}$

second row : $\vec{a_2}$

nth row : $\vec{a_3}$

$m \times n$

$a_1 \quad a_2 \quad a_3$

$$a_1 = \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix}$$

m : number of rows

n : number of columns

- ith row $\vec{a_i} = (a_{i1} \quad a_{i2} \quad \cdots \quad a_{in})$
- jth column $a_j = \begin{pmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{pmatrix}$

a_{ij}

row $\quad$ column

$$A = \begin{pmatrix} 2 & -1 & 1 \\ 1 & -1 & 0 \end{pmatrix}_{2 \times 3}$$

$a_{23} = 0$

value

ij

2) Addition (Matrix addition)

If $A = (a_{ij})_{m\times n}$, $B = (b_{ij})_{m\times n}$ (size should be the same)

We define $A \pm B = C_{m\times n}$ a matrix C

and $C_{ij} = a_{ij} \pm b_{ij} \quad \forall\, i, j$

Ex :-

$$\begin{pmatrix} 2 & -1 & 0 \\ 1 & -1 & 1 \end{pmatrix}_{2\times3} + \begin{pmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{pmatrix}_{2\times3} = \begin{pmatrix} 4 & 2 & 4 \\ 6 & 5 & 8 \end{pmatrix}_{2\times3}$$

3) Scalar Multiplication :-

If $A = (a_{ij})_{m\times n}$, α is a scalar

We define $\alpha A = (\alpha a_{ij})_{m\times n}$ تبقى الـ matrix

اضربوا في كل element موجود في matrix

Ex

$$2\begin{pmatrix} 2 & -1 & 0 \\ 1 & -1 & 1 \\ 3 & 4 & 5 \end{pmatrix} = \begin{pmatrix} 4 & -2 & 0 \\ 2 & -2 & 2 \\ 6 & 8 & 10 \end{pmatrix}$$

4) Matrix Multiplication:

If $A = (a_{ij})_{m\times n}$ & $B = (b_{ij})_{n\times r}$

- it doesn't have to be the same size for Both matrices But :-

$$m \times \boxed{n} \qquad \boxed{n} \times r$$

should Be equal

$$AB = C_{m \times r}$$

$$C_{ij} = \sum_{k=1}^{n} a_{ik}\, b_{kj}$$ يعني نضرب ونجمع

$$\rightarrow C_{ij} = a_{i1} b_{1j} + a_{i2} b_{2j} + \cdots + a_{in} b_{nj}$$

Ex =

$$A = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & 1 \end{pmatrix}_{2\times3} \qquad B = \begin{pmatrix} -1 & 1 \\ 2 & 3 \\ 4 & -1 \end{pmatrix}_{3\times2}$$

$$AB = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 2 & 3 \\ 4 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ 7 & 6 \end{pmatrix}_{2\times2}$$

$$C_{11} = (2)(-1) + (-1)(2) + (1)(4) = 0$$

$$C_{12} = (2)(1) + (-1)(3) + (1)(-1) = -2$$

$$C_{21} = (1)(-1) + (2)(2) + (1)(4) = 7$$

$$C_{22} = (1)(1) + (2)(3) + (1)(-1) = 6$$

$$\underset{3\times 2}{B}\ \underset{2\times 3}{A} = \begin{pmatrix} -1 & -1 \\ 2 & 3 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 3 & 0 \\ 7 & 4 & 5 \\ 7 & -6 & 3 \end{pmatrix}_{3\times 3}$$

Result, $AB \neq BA$

Ex:- $A = \begin{pmatrix} 1 & 1 & 3 \\ -1 & 2 & 4 \end{pmatrix}_{2\times 3}$

$B = \begin{pmatrix} 3 & -1 & 0 \\ 1 & -1 & 1 \\ 1 & 0 & 1 \end{pmatrix}_{3\times 3}$

$$\underset{2\times 3}{A}\ \underset{3\times 3}{B} = \begin{pmatrix} 7 & -2 & 4 \\ 3 & -2 & 6 \end{pmatrix}_{2\times 3}$$

$$\underset{3\times 3}{B}\ \underset{2\times 3}{A} = (\text{not defined})$$

5) Transpose of a Matrix

if $A = (a_{ij})_{m\times n}$, we define transpose of A as A^T (A^t)

$$A^T = C_{n\times m}, \quad C_{ij} = a_{ji}$$

Ex:- $\begin{pmatrix} 1 & 2 & -1 \\ 2 & 4 & 5 \end{pmatrix}^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \\ -1 & 5 \end{pmatrix}_{3\times 2}$

Ex:- $A = \begin{pmatrix} 2 & -1 & 3 \\ -1 & 4 & 5 \\ 3 & 5 & 9 \end{pmatrix}_{3\times 3}$ → Main diagonal a_{ii}

$$A^T = \begin{pmatrix} 2 & -1 & 3 \\ -1 & 4 & 5 \\ 3 & 5 & 9 \end{pmatrix}_{3\times 3} = A$$

Def:- An $n\times n$ - matrix A is called symmetric if $A^T = A$

$A = \begin{pmatrix} 0 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & 0 \end{pmatrix}$ → This diagonal Must Be all zeros

$$A^T = \begin{pmatrix} 0 & -4 & -5 \\ 4 & 0 & 6 \\ 5 & -6 & 0 \end{pmatrix} = -1 \begin{pmatrix} 0 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & -6 & 0 \end{pmatrix}$$

$$A^T = -1\,(A)$$

$$A^T = -A$$

Def: An $n\times n$ - matrix is called skew-symmetric if $A^T = -A$