lec 6:-

$A = (a_{ij})_{m \times n}$

$AB \neq BA$ (In general)

- **Rules:-** Take A, B, C Matrices such that the indicated operations are defined

1- $A + B = B + A$ Commutative law

2- $A + (B + C) = (A + B) + C$ associative law

3- $(AB)C = A(BC)$

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4- $A(B + C) = AB + AC$

5- $(\alpha\beta)A = \alpha(\beta A)$ α, β scalars

6- $\alpha(AB) = (\alpha A)B = A(\alpha B)$

7- $(\alpha + \beta)A = \alpha A + \beta A$

8- $\alpha(A + B) = \alpha A + \alpha B$

9- $(A^T)^T = A$ $T \to$ transpose

10- $(\alpha A)^T = \alpha(A^T)$

11)- $(AB)^T = B^T A^T$

12)- $(A + B)^T = A^T + B^T$ الترتيب ليس له أهمية

Ex. $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 6 \\ 3 & 1 \end{pmatrix}$

$$AB = \begin{pmatrix} 5 & 2 \\ 9 & 4 \end{pmatrix}$$

$$(AB)^T = \begin{pmatrix} 5 & 9 \\ 2 & 4 \end{pmatrix}$$

$(A)^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ $(B)^T = \begin{pmatrix} -1 & 3 \\ 0 & 1 \end{pmatrix}$

$$(A)^T (B)^T = \begin{pmatrix} -1 & 6 \\ -2 & 10 \end{pmatrix}$$

$$B^T A^T = \begin{pmatrix} 5 & 9 \\ 2 & 4 \end{pmatrix} = (AB)^T$$

$$(AB)^T = B^T A^T$$

* Power of a Matrix

- If $A = (a_{ij})_{n \times n}$, we define $A^k = \underbrace{A.A.A}_{k\text{-times}}$

where $k \in \mathbb{N} = \{1, 2, 3, \ldots\}$

* identity Matrix $= I_n = (\alpha_{ij})_{n \times n}$

$$\alpha_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

$$I_n = \begin{pmatrix} a_{i=j} & & i<j \\ & \ddots & \\ i>j & & \end{pmatrix}_{n \times n}$$

$$= \begin{pmatrix} 1 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 \cdots \\ \vdots & 0 & 1 & \ddots \\ 0 & \vdots & 0 & \\ 0 & 0 & 0 \cdots & 1 \end{pmatrix}$$

Ex:-

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{3 \times 3} \begin{pmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 3 \\ 1 & 4 & 2 & 3 \end{pmatrix}_{3 \times 4} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 3 \\ 1 & 1 & 2 & 3 \end{pmatrix}$$

$$I_n . B_{n \times r} = B$$

$$A_{m \times n} I_{n \times n} = A$$

A
times

• $\left(\frac{1}{r}\right) r = r\left(\frac{1}{r}\right) = 1$, $r \neq 0$ In Numbers

* let $A = (a_{ij})_{n \times n}$
Is there a Matrix $B_{n \times n}$ s.t $AB = I = BA$

Def :- let $A = (a_{ij})_{n \times n}$ if there exists a matrix $B_{n \times n}$ s.t $AB = I = BA$, we say A is nonsingular (invertible)

This B (If exists) is called inverse of A

If not (No such B), we say A is singular (non invertible)

Ex:- $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 & 5 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{pmatrix}$

$AB = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = BA$

A is non singular

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• B is inverse of A

And:- B is nonsingular

A is inverse of B

Ex.- $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$

Is A nonsingular ?

let $B = \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix}_{2\times 2}$

$AB = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$AB = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} x_1 + 2x_3 & x_2 + 2x_4 \\ 2x_1 + 4x_3 & 2x_2 + 4x_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$x_1 + 2x_3 = 1 \qquad x_2 + 2x_4 = 0$

$2x_1 + 4x_3 = 0 \qquad 2x_2 + 4x_4 = 1$

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Using G.E.M:-

$$\left(\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & 2 & 0 \\ 2 & 0 & 4 & 0 & 0 \\ 0 & 2 & 0 & 4 & 1 \end{array}\right)$$

$\Downarrow$

$$\left(\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 2 & 0 & 4 & 1 \end{array}\right)$$

System is inconsistent So No sol

There is no B s.t $AB = I \rightarrow A$ is singular (has no inverse)

* if $C_{n \times n}$, $D_{n \times n}$ s.t $CD = DC = I$

Then C and D are non singular & inverse of C is D and inverse of D is C

Remark:-

$A = (a_{ij})_{m \times n}$, $X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n \times 1}$ vector

$$AX = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix}$$

Coefficient matrix

$$= \begin{pmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{pmatrix}_{m\times 1}$$

① $(A|b) \Leftrightarrow m\times n$ system $\Leftrightarrow$ ② $Ax = b$

$\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$

unknown

$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$

$$Ax = x_1 \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} + x_2 \begin{pmatrix} a_{21} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} + \cdots + x_n \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix}$$

③ $m\times n$-system $\rightarrow Ax = x_1 a_1 + x_2 a_2 + x_3 a_3 + \cdots + x_n a_n$

3 ways to write $m\times n$ system.