lec 8 :

Def : A matrix is called elementary of Type I (II, III) if it can be obtained from $I_{n\times n}$ by applaying row operation I (II, III) once

$$E = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \xleftarrow{R_1 \leftrightarrow R_3} I$$

type I

عشان يقوم zero

عشان يقوم واحد

$$E = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \longleftarrow \text{Not elementary}$$

$$E = \begin{pmatrix} 1 & & 0 & & 1 & 0 \\ & 1 & 0 & 1 & & \\ & & & & \ddots & \\ & 1 & & & & 1 \end{pmatrix}_{n\times n}$$

Type I

$\uparrow R_i \leftrightarrow R_j$

$$I = \begin{pmatrix} 1 & & 0 \\ & 1 & \\ & & \ddots \\ 0 & & 1 \end{pmatrix}$$

$$E = \begin{pmatrix} 1 & & & 0 \\ & 1 & & \\ & & \ddots & \\ 0 & & \alpha & \\ & & & 1 \end{pmatrix}$$

Type II

برة كله zeros

$\uparrow \alpha R_i$

I

كله واحد Diagonal

row i

$$E = \begin{pmatrix} 1 & 0 & 0 \\ c & 1 & 0 \\ & & 1 \end{pmatrix}$$

برة الـ Diagonal

مكان التغيير

Type III

jth row will change

$$I \xrightarrow{cR_i + R_j}$$

Ex $E = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$$I \xrightarrow{3R_4 + R_2}$$

Ex $E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$

$$I \xrightarrow{2R_3 + R_2}$$

$$EA = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

الضرب من اليسار

$$= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21}+a_{31} & a_{22}+2a_{32} & a_{23}+2a_{33} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$A \xrightarrow{2R_3+R_2} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21}+a_{31} & a_{22}\,a_{32} & a_{23}\,a_{33} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$\text{So:-} \quad \underline{EA} \xleftarrow{2R_3+R_2} A$$

عند ضرب الـ A من اليسار بـ E استطيع معرفة الجواب بدون القيام بعملية الضرب وذلك تطبيق الـ operation التي طبقتها على الـ I لكي أحولها إلى E وفي هذه الحالة هي $2R_3+R_2$

الضرب من اليمين

$$AE = \begin{array}{c} \begin{matrix} c_1 & c_2 & c_3 \end{matrix} \\ \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \end{array} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & 2a_{12}+a_{13} \\ a_{21} & a_{22} & 2a_{22}+a_{23} \\ a_{31} & a_{32} & 2a_{32}+a_{33} \end{pmatrix}$$

• لكي نحصل على النتيجة عليك القيام بـ :
column operation

وهي : $2C_2 + C_3$

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Remark :-

- If E is an elementary matrix, then EA (left multiplication of A by E) has the same effect as applying the row operation of E on A

$E_2 \cdot (E_1 A)$

$(A|b)$

$\text{Row oper.} \downarrow$

$(\quad)$

Theory :- If E is elementary matrix then E is non-singular and E^{-1} is elementary of the same Type

Proof :- let $E_{n \times n}$ be elementary matrix

Case ① If E is of Type I $\qquad R_i \Longleftrightarrow R_j$

$$I \xleftrightarrow{R_i \Longleftrightarrow R_j} E \xrightarrow{R_i \Longleftrightarrow R_j} I$$

Consider $E E = I$

A ψIS [illegible]

So E is non singular and $E^{-1} = E$ Same Type

Ex :- $E = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \sim$ Type I

$\therefore$ Non singular

$$E^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Case ②:- If E is elementary of Type II

$$I \xrightarrow{\alpha R_i,\ \alpha \neq 0} E$$

let F be the matrix obtained from I by $\frac{1}{\alpha} R_i$, F is elementary of Type II.

$$EF = I$$

&

$$FE = I$$

$$E \begin{pmatrix} 1 & & & 0 \\ & 1 & & \\ & & \alpha & \\ 0 & & & 1 \end{pmatrix}$$

$$F \begin{pmatrix} 1 & & 0 \\ & 1 & \\ & & \frac{1}{\alpha} \\ 0 & & \end{pmatrix}$$

$\Rightarrow$ E is non singular & $E^{-1} = F$ elem of Type II

Ex:- $E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ elementary of Type II

non singular and $E^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{4} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Case ③ If E is of Type III

R.O on $I \xrightarrow{cR_i+R_j} E = \begin{pmatrix} 1 & & 0 \\ & 1 & \\ c & & 1 \end{pmatrix} j$

• let F Be obtaind from I by R.O $-cR_i+R_j$

$$F = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ -c & & \\ 0 & & 1 \end{pmatrix}$$

$$\left.\begin{aligned} EF &= I \\ FE &= I \end{aligned}\right\}$$ E is nonsingular and $E^{-1} = F$ same Type

$$E = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\frac{1}{2} & 0 & 1 \end{pmatrix}$$

Elementary of Type III . nonsingular

$$E^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{1}{2} & 0 & 1 \end{pmatrix}$$

Ex:- E_1, E_2 are elementary $(n \times n)$

Is $E_1 E_2$ nonsingular ?

Yes They are

Elementary ? need not be elementary

I elem of Type II & III

• Equivalent Matrices

Def: If A, B are $n \times n$-matrices We say That A is equivalent to B if There are elementary matrices $E_1, E_2, E_3, \cdots, E_k$ s.t

$$A = E_k \cdots E_2 E_1 B$$

$$\left(B \xrightarrow{k \text{ row operations}} A \right)$$

هل هناك B اقوم بعمل عدد من الـ operations تعطيني A؟

اذا الجواب نعم <u>A equ B</u>

E elem $\Rightarrow$ E is row equivalent to I

Remark:

① If $A \approx B$ Then $B \approx A$

② If $A \approx B$ & $B \approx C$, Then $A \approx C$

Proof:- ① Assume $A \approxeq B$

$\rightsquigarrow$ There exists elem. matrices

$E_1 \cdots E_k$ s.t $A = \underline{E_k} \cdots E_2 E_1 B$

each E_i is elementary $\rightsquigarrow$ nonsingular

$E_k^{-1} A = E_{k-1} \cdots E_2 E_1 B$

$\xRightarrow{E_{k-1}^{-1}}$ $E_{k-1}^{-1} E_k^{-1} A = E_{k-2} \cdots E_2 E_1 B$

$\vdots$

$E_1^{-1} E_2^{-1} \cdots E_k^{-1} A = B$

• each E_i^{-1} is elementary $\Rightarrow B \approxeq A$

② Assume $A \approxeq B$ and $B \approxeq C$ $\longrightarrow E_i, F_i$

$\rightsquigarrow$ There exists elem. matrices s.t

$A = E_k \cdots E_1 B$ &

$C = F_l \cdots F_1 B$

$A =$

$B = F_l \cdots F_1 C$

$$A = \underbrace{E_k \cdots E_1 \left(F_l \cdots F_1 \right.}_{\text{element matrices}} \left. C \right)$$

$\rightsquigarrow A \approxeq C$

عبر row oper للوصول الى C من A في K+l