# Chapter 1 :- Matrices & systems of Equations :-

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## 1.1 :- systems of linear Equations

$m \times n$ - linear systems :-

num of equations  num of unknowns

$$a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n = b_1$$
$$a_{21} x_1 + a_{22} x_2 + \cdots + a_{2n} x_n = b_2$$
$$\vdots$$
$$a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mn} x_n = b_n$$

→ Solution of the system :-

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_3 \end{pmatrix}$$, where X statisfies all equations

• If Two systems with the same unknowns have the same solutions They are called Equivalent systems.

• **Elementary row operations :-** Applying These operation on a system produces are Equivalent systems

1 :- Interchange two equations
2 :- Multiply an Equation by a non-zero Constant
3 :- Replace an equation By it's sum with a Multiple of another equation

- Augmented Matrix of a system :-

Each row represents an equation
Each column represents an unknown

$$\left(\begin{array}{ccccc|c} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} & b_2 \\ \vdots & & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} & b_3 \end{array}\right)$$

**Note**
- If There is a row where: $(0, 0 \cdots \mid c \neq 0)$ non-zero constant

There is no solution
→ System inconsistent

if not There is a solution
→ System Consistent

- Pivot Element :- هو العنصر الذي استعمله لأصفر باقي العناصر التي تقع تحته ( الشرح موجود في الملف الثاني )

- variables :-
  - leading variables :-
    - The first non-zero element in each non-zero row
  - Free variables :-
    - The rest of variables

Ex :-

$$\begin{array}{c} \begin{array}{ccccc} x_1 & & x_3 & & \end{array} \\ \left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \end{array} \rightarrow x_5$$

$x_1, x_3, x_5 \rightarrow$ leading
$x_4, x_2 \rightarrow$ free

solution $x = (\quad)$

لازم ما يظهر الـ leading var.

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# 1.2 :- Row Echelon form R.E.F

Conditions :-

$\boxed{1}$ :- The first non-zero element in each non zero row is 1

$\boxed{2}$ :- The number of leading zeros (The first zeros in a row) in each non zero row $\underline{j}$ must be less than the number of leading zeros in row $\underline{j+1}$ (for all j)

$\boxed{3}$ :- If there are rows of zeros (zero rows) They are below non-zero rows

Ex :-

$$\begin{array}{c} 1 \leftarrow \\ 2 \leftarrow \\ \rightarrow \\ \rightarrow \end{array} \begin{pmatrix} \boxed{0} & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$\boxed{1}$ ✓
$\boxed{2}$ ✓
$\boxed{3}$ ✓

## Gauss Elemination Method :-

- (A|b) is an Augmented matrix
- (U|d) is The Resultant Matrix of E.R.O on (A|b) ← Elementary row operations

$$(A|b) \xrightarrow{E.R.O} (U|d) \left( \text{Matrix in R.E.F} \right)$$

Does it have a (0, 0 ⋯ | c≠0)

No → It is a $\underline{\text{Consistent}}$ System

Yes → It is an $\underline{\text{inconsistent}}$ system

Is There a free variable (Consistent System)

No → only one solution

Yes → inf. number of solution

• Types of systems

**Underdetermined system**

- $m < n$
- It has either :-
  - → No solutions (incon.s)
  - → infinite Solutions (con.s)

**overdetermined system**

- $m > n$
- It can have No solution, one solution or infinit solutions

**Square system**

- $m = n$

Same as The previous

**Homogenuous system**

- $b_1 = b_2 = b_3 \dots = 0$
- always Consistant
- If it has a one solution it must be The Trivial solution $x = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$

Th • It has a non-trivial solution if $n > m$

Theory :- Any homogenuous and underdeterminante system has an infinite number of solutions (has non-zero solution).

# Reduced Row Echelon form (R.R.E.F)

A matrix is set to be in reduced row echelon form (R.R.E.F) if:-

1) It is in R.E.F
2) The leading ones are the only non-zero elements in their columns

→ Gauss-Jordan Elemination Method :-

$$(A|b) \xrightarrow{E.R.O} (U|d) \rightarrow R.R.EF$$