How to solve systems Using Matrices

Ex:- Out-line Question 6-h

$$x_2 + x_3 + x_4 = 0$$
$$3x_1 + 3x_3 - 4x_4 = 7$$
$$x_1 + x_2 + x_3 + 2x_4 = 6$$
$$2x_1 + 3x_2 + x_3 + 3x_4 = 6$$

$$\left(\begin{array}{cccc|c} 0 & 1 & 1 & 1 & 0 \\ 3 & 0 & 3 & -4 & 7 \\ 1 & 1 & 1 & 2 & 6 \\ 2 & 3 & 1 & 3 & 6 \end{array}\right)$$

We use Elementary. Row operation

① Replace R_1 with R_3

Pivot element

$$\left(\begin{array}{cccc|c} \boxed{1} & 1 & 1 & 2 & 6 \\ 3 & 0 & 3 & -4 & 7 \\ 0 & 1 & 1 & 1 & 0 \\ 2 & 3 & 1 & 3 & 6 \end{array}\right)$$

should be zero

- You have to use it to make The elements Below it all zero

② $-3R_1 + R_2 \rightsquigarrow$ And put it in R_2

③ $-2R_1 + R_4 \rightsquigarrow$ " " " " R_4

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 6 \\ 0 & -3 & 0 & -10 & -11 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & -1 & -6 \end{array}\right)$$

pivot element

②

-3	-3	-3	-6	-18
+3	0	3	-4	7
0	-3	0	-10	-11

③

-2	-2	-2	-4	-12
+ 2	3	1	3	6
0	1	-1	-1	-6

To make it easier Replace R_2 with R_3

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 6 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & -3 & 0 & -10 & -11 \\ 0 & 1 & -1 & -1 & -6 \end{array}\right)$$

4 $3R_2 + R_3 \rightsquigarrow R_3$
5 $-R_2 + R_4 \rightsquigarrow$ 〃 〃 〃

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 6 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & \boxed{3} & -7 & -11 \\ 0 & 0 & -2 & -2 & -6 \end{array}\right)$$

Pilot element

4
$$\begin{array}{ccccc|c} & 0 & 3 & 3 & 3 & 0 \\ + & 0 & -3 & 0 & -10 & -11 \\ \hline & 0 & 0 & 3 & -7 & -11 \end{array}$$

5
$$\begin{array}{ccccc|c} & 0 & -1 & -1 & -1 & 0 \\ + & 0 & 1 & -1 & -1 & 6 \\ \hline & 0 & 0 & -2 & -2 & 6 \end{array}$$

5 $\frac{R_3}{3} \rightsquigarrow R_3$

6 $2R_3 \rightsquigarrow R_3$

$-\frac{6}{3}$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 6 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -\frac{7}{3} & -\frac{11}{3} \\ 0 & 0 & 0 & -\frac{20}{3} & -\frac{40}{3} \end{array}\right)$$

6
$$\begin{array}{ccccc|c} & 0 & 0 & 2 & -\frac{14}{3} & -\frac{22}{3} \\ + & 0 & 0 & -2 & -2 & -6 \\ \hline & 0 & 0 & 0 & -\frac{20}{3} & -\frac{40}{3} \end{array}$$

$-\frac{22}{3}+\frac{18}{3}$

Now we write the solutions:-

$\cancel{-}\frac{20}{3} x_4 = \cancel{-}\frac{40}{3}$

$\boxed{x_4 = 2}$

$x_3 - \frac{7}{3} \times 2 = -\frac{11}{3}$

$x_3 = -\frac{11}{3} + \frac{14}{3} =$

$\boxed{x_3 = 1}$

$x_2 + 1 + 2 = 0$

$\boxed{x_2 = -3}$

$x_1 + -3 + 1 + 2 \times 2 = 6$

$x_1 = 6 - 2 = 4$ $\boxed{x_1 = 4}$

So The Solution is:-

$$X = \begin{pmatrix} 4 \\ -3 \\ 1 \\ 2 \end{pmatrix}$$

How to Use Gauss elemination method:-

Ex:- Out line 1.2 Question (3) c

$$\left[\begin{array}{ccc|c} 1 & -3 & 0 & 2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Does it have a row $(0\ 0\ 0 \cdots \mid c \neq 0)$?

No :→ System Consistent

Does it have Free variables?

Yes :→ It has infinit Solutions

There is 2 leading variables : X_1, X_3

and 1 Free variable : X_2

How to turn a system into the Row echelon form:

Ex:- Out line 1.2 Question 5 d

$$3X_1 + 2X_2 - X_3 = 4$$
$$X_1 - 2X_2 + 2X_3 = 1$$
$$11X_1 + 2X_2 + X_3 = 14$$

$$\left(\begin{array}{ccc|c} 3 & 2 & -1 & 4 \\ 1 & -2 & 2 & 1 \\ 11 & 2 & 1 & 14 \end{array}\right)$$

first solve the system

① $R_2 \rightsquigarrow R_1$

pivot

$$\left(\begin{array}{ccc|c} \boxed{1} & -2 & 2 & 1 \\ 3 & 2 & -1 & 4 \\ 11 & 2 & 1 & 14 \end{array}\right)$$

0

② $-3R_1 + R_2 \rightsquigarrow R_2$

③ $-11R_1 + R_3 \rightsquigarrow R_3$

$$\left(\begin{array}{ccc|c} 1 & -2 & 2 & 1 \\ 0 & \boxed{8} & -7 & 1 \\ 0 & 24 & -21 & -3 \end{array}\right)$$

④ $-3R_2 + R_3 \rightsquigarrow R_3$

$$\left(\begin{array}{ccc|c} 1 & -2 & 2 & 1 \\ 0 & 8 & -7 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

⑤ $\frac{R_2}{8}$

$$\left(\begin{array}{ccc|c} 1 & \boxed{-2} & 2 & 1 \\ 0 & 1 & -\frac{7}{8} & \frac{1}{8} \\ 0 & 0 & 0 & 0 \end{array}\right)$$

should be zero

There are 2 leading variables X_1 . X_2

There is 1 free variable X_3

→ Now to turn it to Reduced . R. E. F :-

⑥ $2R_2 + R_1 \rightsquigarrow R_1$

$$\left(\begin{array}{ccc|c} 1 & 0 & \frac{1}{4} & \frac{5}{4} \\ 0 & 1 & -\frac{7}{8} & \frac{1}{8} \\ 0 & 0 & 0 & 0 \end{array}\right) \checkmark$$

$-\frac{7}{4} + \frac{8}{4}$

The solution

$X_3 = \alpha$

$X_2 = \frac{7}{8}\alpha + \frac{1}{8} \Rightarrow \boxed{X_2 = \frac{7}{8}\alpha + \frac{1}{8}}$

$X_1 + \frac{\alpha}{4} = \frac{5}{4} \quad \boxed{X_1 = \frac{5}{4} - \frac{\alpha}{4}}$