

1.4 Matrix Algebra

Rules:

- 1- $A+B = B+A$ Commutative law
- 2- $A+(B+C) = (A+B)+C$ associative law
- 3- $(AB)C = A(BC)$

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- 4- $A(B+C) = AB+AC$
- 5- $\alpha\beta A = \alpha(\beta A)$ α & β are scalars
- 6- $\alpha(AB) = (\alpha A)B = A(\alpha B)$
- 7- $(\alpha+\beta)A = \alpha A + \beta A$
- 8- $\alpha(A+B) = \alpha A + \alpha B$
- 9- $(A^T)^T = A$ \rightarrow Transpose
- 10- $(\alpha A)^T = \alpha(A)^T$
- 11- $(AB)^T = B^T A^T$
- 12- $(A+B)^T = A^T + B^T$

الترتيب ليس له أهمية

13- Power of a Matrix :-
of $A = (a_{ij})_{m \times n}$

We define $A^R = \underbrace{A.A.A \dots}_{R \text{-times}}$
عدد ضربات موجب \rightarrow

Identity Matrix

$$I_n = (a_{ij})_{n \times n} = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{if } i \neq j \end{cases} = \begin{pmatrix} & & i < j \\ & a_{i=j} & \\ & & i > j \end{pmatrix}$$

* When you multiply I By any Matrix, It stays the same

Matrix Inverse

Note: I in matrices is like 1 in numbers

If you have Matrix A
And there exist Matrix B where $AB = BA = I$

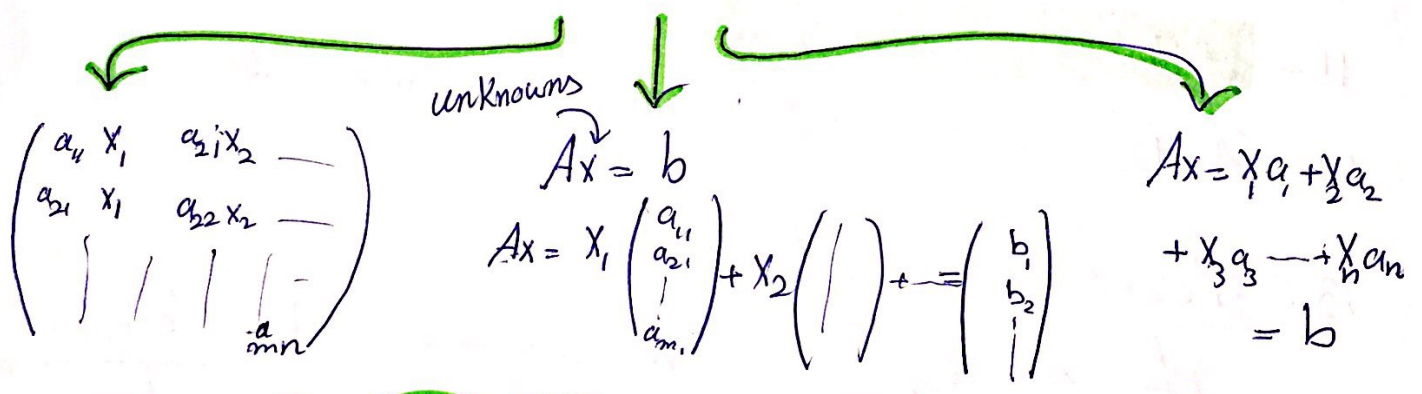
Then - 1 - B is The inverse of A } Invertible
2 - A is Nonsingular
And so is B

If Not -
A is singular and has no inverse } Non-Invertible

Ways to write a system

$$A = (a_{ij})_{m \times n} \quad X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

vector



Linear Combination

- You have vectors a_1, a_2, \dots, a_n
- α is a scalar
- Linear Combination \Rightarrow sum of $\alpha_1 a_1 + \alpha_2 a_2 + \dots + \alpha_n a_n$

\rightarrow System $Ax=b$ is consistent if and only if $b = x_1 a_1 + \dots + x_n a_n$

Note

If $A_{n \times n}$ is nonsingular Then it has an inverse B
And B is unique

• If A, B are nonsingular :-

→ $A \pm B$ Does NOT have to be nonsingular

→ AB Is Nonsingular And $(AB)^{-1} = B^{-1}A^{-1}$

Note

• If A is nonsingular & $AB = AC$ Then $B = C$