1.4 Matrix Algebra

Rules:

1- $A+B = B+A$ Commutative law

2- $A+(B+C) = (A+B)+C$ associative law

3- $(AB)C = A(BC)$

لكن ممنوع تغيير الترتيب

4- $A(B+C) = AB + AC$

5- $\alpha\beta A = \alpha(\beta A)$ α & β are scalars

6- $\alpha(AB) = (\alpha A)B = A(\alpha B)$

7- $(\alpha+\beta)A = \alpha A + \beta A$

8- $\alpha(A+B) = \alpha A + \alpha B$

9- $(A^T)^T = A$ Transpose

10- $(\alpha A)^T = \alpha (A)^T$

11- $(AB)^T = B^T A^T$

12- $(A+B)^T = A^T + B^T$ التوزيع ليس له أهمية

13- Power of a Matrix :-

If $A = (a_{ij})_{n\times n}$ We define $A^k = \underbrace{A.A.A\cdots}_{k\text{-times}}$

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Identity Matrix

$$I_n = (\alpha_{ij})_{n\times n} = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{if } i\neq j \end{cases} = \begin{pmatrix} & & i<j \\ & a_{i=j} & \\ i>j & & \end{pmatrix}$$

* When you Multiply I By any Matrix, It stays the same

Matrix Inverse

Note:
- I in matrices is like 1 in numbers

If you have Matrix A
And There exist Matrix B where $AB = BA = I$

Then:- 1- B is The inverse of A } Invertible
2- A is Nonsingular
And So is B

If Not:-
A is singular and has no inverse } Non-Invertible

- **Ways to write a system**

$A = (a_{ij})_{m \times n}$ $\quad X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$ vector

unknowns

$Ax = b$

$\begin{pmatrix} a_{11}x_1 & a_{12}x_2 & \cdots \\ a_{21}x_1 & a_{22}x_2 & \cdots \\ \vdots & \vdots & \\ a_{m1} & & a_{mn} \end{pmatrix}$

$Ax = x_1\begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} + x_2\begin{pmatrix} \vdots \\ \vdots \end{pmatrix} + \cdots = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \end{pmatrix}$

$Ax = x_1a_1 + x_2a_2 + x_3a_3 + \cdots + x_na_n = b$

Linear Combination

- You have vectors : $a_1, a_2 \cdots a_n$
- α is a scalar
- Linear Combination $\Rightarrow$ Sum of $\alpha_1a_1 + \alpha_2a_2 \cdots \alpha_na_n$

$\rightarrow$ System $Ax=b$ is Consistent if and only if $b = x_1a_1 + \cdots x_na_n$

Note :-

If $A_{n\times n}$ is non singular Then it has an **inverse** B And B. is **unique**

- If A, B are non singular :-

→ $A \pm B$ Does NOT have to be non singular

→ AB Is Non singular And $(AB)^{-1} = B^{-1}A^{-1}$

Note

- If A is non singular & $AB = AC$ Then $B = C$