

# Chapter 2: Determinants

## 2.1 :- The Determinant of A matrix

- In Matrices:-
- if  $\det(A) \neq 0 \Rightarrow A$  is nonsingular
  - if  $\det(A) = 0 \Rightarrow A$  is singular

$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$  for  $2 \times 2$  matrices :-  $\det(A) = (a_{11}a_{22}) - (a_{21}a_{12})$

$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$  for  $3 \times 3$  matrices :-  $\det(A) = a_{11}A_{11} + a_{22}A_{12} + a_{33}A_{13}$

Where :-  $A_{11} = (-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

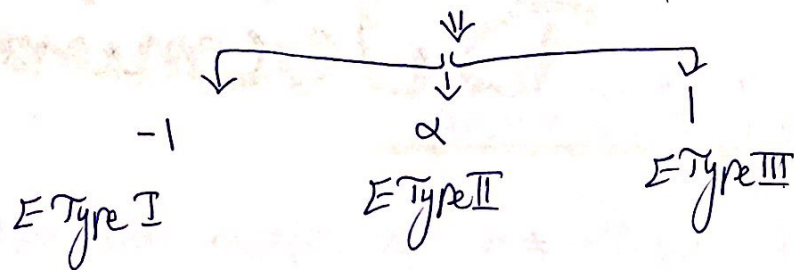
In General :-  $A_{ij} = (-1)^{i+j} M_{ij}$

You can choose any form (any row or column)

$$\det(A) = a_{i1}A_{i1} + a_{i2}A_{i2} + \dots \text{--- (row } i)$$
$$= a_{1j}A_{1j} + a_{2j}A_{2j} + \dots \text{--- (column } j)$$

## 2.2 Properties of Determinants

$$\det(EA) = \det(E) \det(A)$$



### Notes

- 1- If  $A$  has a row or column of zeros Then  $\det(A) = 0$
- 2- If  $A$  has two identical rows or columns Then the determinant of  $A = 0$
- 3-  $\det(A^T) = \det(A)$
- 4- If  $A$  is Triangular (upper or lower) Then
  - \*  $\det(A) =$  Products of elements in the diagonal

Theory :-  $a_{i1}A_{j1} + a_{i2}A_{j2} + a_{i3}A_{j3} \dots = \begin{cases} \det(A) & \text{if } i=j \\ 0 & \text{if } i \neq j \end{cases}$

5-  $\det(AB) = \det(A) \det(B)$

6-  $\det(A^{-1}) = \frac{1}{\det(A)}$

7-  $\det(KA) = K^n \det(A)$

8- عندنا يدخل  $K$  على الصفوف  
توزع على جميع الصفوف  
والاشارة  $K^n$  عندنا  
يخضع على  $\det$  فانه يدخل على صف  $n$  عدد واحد

9-  $\det(A) = |A|$   
يمكن كتابته على هذا الشكل  $\rightarrow$

## 2.3 Additional Topics and Applications

The adjoint of  $A$ :-

$$\text{adj}(A) = (\text{cofactors})^T$$

$$= \begin{pmatrix} A_{11} & \dots & A_{1n} \\ \vdots & & \vdots \\ A_{n1} & \dots & A_{nn} \end{pmatrix}^T$$

$$= \begin{pmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & A_{22} & \dots & A_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & A_{nn} \end{pmatrix}$$

$$A \text{ adj}(A) = \det(A) I \quad \leftarrow \text{This is really important}$$

$$\det(\text{adj}(A)) = \det(A)^{n-1}$$

- \* If  $A$  is nonsingular Then  $\text{adj}(A)$  is nonsingular
- \* If  $A$  is singular Then  $\text{adj}(A)$  is singular

### Cramer's Rule

If  $A_{n \times n}$  is nonsingular Then The unique solution of  $Ax=b$  is Given By:-

$$x_i = \frac{\det(A_i)}{\det(A)}$$
 where  $A_i$  is obtained from  $A$  by replacing The  $i$ th column of  $A$  by  $b$

$$\text{Ex.:- } A = \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}$$

$$b = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

2.3:- Question 2  
Outline

$$X_1 = \frac{\det(A_1)}{\det(A)} = \frac{\begin{vmatrix} 2 & 3 \\ 5 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix}} = \frac{-11}{-5} = \frac{11}{5}$$

$$X_2 = \frac{\det(A_2)}{\det(A)} = \frac{\begin{vmatrix} 2 & 2 \\ 3 & 5 \end{vmatrix}}{5} = \frac{4}{5}$$

$$X = \begin{pmatrix} \frac{11}{5} \\ \frac{4}{5} \end{pmatrix}$$