Chapter 2: Determinants

2.1 :- The Determinant of A matrix

In Matrices:-
- if $\det(A) \neq 0 \rightsquigarrow A$ is nonsingular
- if $\det(A) = 0 \rightsquigarrow A$ is singular

$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ for 2x2 matrices :- $\det(A) = (a_{11}a_{22}) - (a_{21}a_{12})$

$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ for 3x3 matrices:- $\det(A) = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

Where :- $A_{11} = (-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$

You can choose any form (any row or column)

In General :- $A_{ij} = (-1)^{i+j} M_{ij}$

$\det(A) = a_{i1}A_{i1} + a_{i2}A_{i2} \cdots$ (row i)

$= a_{1j}A_{1j} + a_{2j}A_{2j}$ ——. (column j)

2.2 : Properties of Determinants

$$\det(EA) = \det(E)\det(A)$$

-1 — E Type I

α — E Type II

1 — E Type III

Notes

1- If A has a row or column of zeros Then $\det(A) = 0$

2- If A has two identical rows or columns Then the determinant of $A = 0$

3- $\det(A^T) = \det(A)$

4- If A is Triangular (upper or lower) Then

$*\det(A)$ = Products of elements in the diagonal

Theory :- $a_{i1}A + a_{i2}A_{j2} + a_{i3}A_{j3} \cdots = \begin{cases} \det(A) & i = j \\ 0 & : i \neq j \end{cases}$

5- $\det(AB) = \det(A)\det(B)$

6- $\det(A^{-1}) = \dfrac{1}{\det(A)}$

7- $\det(KA) = K^n \det(A)$

8 • عندما يدخل K على المصفوفة يتوزع على جميع الصفوف والأعمدة ولكن عندما يدخل على determ فانه يدخل على صف أو عمود واحد

9- $\det A = |A|$

ممكن كتابته على هذا الشكل

2.3: Additional Topics and Applications

The adjoint of A:-

$$\operatorname{adj}(A) = \begin{pmatrix} \text{cofactors} \end{pmatrix}^T$$

$$= \begin{pmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & & \vdots \\ A_{n1} & \cdots & A_{nn} \end{pmatrix}^T$$

$$= \begin{pmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ & & & \\ & & \cdots & A_{nn} \end{pmatrix}$$

$A \operatorname{adj}(A) = \det(A)\, I$ ⟵ This is really important

$$\det(\operatorname{adj}(A)) = \det(A)^{n-1}$$

* If A is nonsingular Then $\operatorname{adj}(A)$ is nonsingular
* If A is singular Then $\operatorname{adj}(A)$ is singular

Cramer's Rule

If $A_{n \times n}$ is nonsingular Then The unique solution of $Ax=b$ is Given By:-

$x_i = \dfrac{\det(A_i)}{\det(A)}$ where A_i is obtained from A by replacing The ith Column of A by b

Ex:- $A = \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}$ $b = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ 2.3:- Question 2 Outline

$$X_1 = \frac{\det(A_1)}{\det(A)} = \frac{\begin{vmatrix} 2 & 3 \\ 5 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix}} = \frac{-11}{-5} = \frac{11}{5}$$

$$X_2 = \frac{\det(A_2)}{\det(A)} = \frac{\begin{vmatrix} 2 & 2 \\ 3 & 5 \end{vmatrix}}{5} = \frac{4}{5}$$

$$X = \begin{pmatrix} \frac{11}{5} \\ \frac{4}{5} \end{pmatrix}$$