35 Change of Basis

- ordered Basis :- $B = [V_1, V_2, \text{------}, V_n]$
- Cordinate vector of x $\quad [X]_B = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix}$

$$X = \alpha_1 V_1 + \alpha_2 V_2 + \text{-----} + \alpha_n V_n$$

Ex:- ④ 3.5:-

$$E = \left\{ \begin{pmatrix} 5 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \end{pmatrix} \right\}$$

$$x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad y = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \quad z = \begin{pmatrix} 10 \\ 7 \end{pmatrix}$$

find $[x]_E$, $[y]_E$, $[z]_E$

- هذا السؤال معناه اوجد α_1 و α_2 التي تجعل $x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ عند ضربهم بـ عناصر E وجمعهم

$$[X]_E = \alpha_1 \begin{pmatrix} 5 \\ 3 \end{pmatrix} + \alpha_2 \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{aligned} 5\alpha_1 + 3\alpha_2 &= 1 \\ 3\alpha_1 + 2\alpha_2 &= 1 \end{aligned} \rightsquigarrow \text{solve system}$$

$$\left(\begin{array}{cc|c} 5 & 3 & 1 \\ 3 & 2 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} 1 & \frac{3}{5} & \frac{1}{5} \\ 3 & 2 & 1 \end{array}\right) = \left(\begin{array}{cc|c} 1 & \frac{3}{5} & \frac{1}{5} \\ 0 & \frac{1}{5} & \frac{2}{5} \end{array}\right) \rightsquigarrow \alpha_2 = \frac{2}{\cancel{5}} \times \cancel{5} = 2$$

$$\alpha_1 + \left(\frac{3}{5}\right)(2) = \frac{1}{5}$$
$$\alpha_1 = -1$$

So $[X]_E = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

Remark: if The Basis you're Given is standard Basis Then The Coordinate vector of X is X it self

$$[X]_{SB} = X$$

standard Basis

- Change of Basis :-

You have two Bases, U, W, Given $[V]_U$

$\rightsquigarrow$ Get $[V]_W$ from $[V]_U$

$$\boxed{[V]_W = T_{U \to W} \, [V]_U}$$

$T_{U \to W}$ ← Transition Matrix from U to W

find The Transition matrix from U to W.

↙ معناها اكتب U بدلالة W

Ex: 6 35

$\rightsquigarrow V_1 = \begin{pmatrix} 4 \\ 6 \\ 7 \end{pmatrix}, \; V_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \; V_3 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$

$\rightsquigarrow U_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \; U_2 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}, \; U_3 = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$

- find Transition matrix from $\{V_1, V_2, V_3\}$ to $\{(U_1, U_2, U_3)\}$

Method one :-

اكتب V بدلالة U

$$\begin{pmatrix} 4 \\ 6 \\ 7 \end{pmatrix} = \alpha_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + \alpha_3 \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$$

Solve System :-

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 1 & 2 & 3 & 6 \\ 1 & 2 & 4 & 7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 2 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$\alpha_1 = 4 - 2 - 1 = 1$

$\rightsquigarrow \alpha_2 = 1$

$\rightsquigarrow \alpha_3 = 1$

So $V_1 = U_1 + U_2 + U_3$

$$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \beta_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta_2 \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + \beta_3 \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 1 & 2 & 3 & 1 \\ 1 & 2 & 4 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 2 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$\alpha_1 = -1$

$\alpha_2 = 1$

$\rightsquigarrow \alpha_3 = 0$

So $V_2 = -U_1 + U_2 + 0 \cdot U_3$

$$\begin{pmatrix}0\\1\\2\end{pmatrix} = \gamma_1\begin{pmatrix}1\\1\\1\end{pmatrix} + \gamma_2\begin{pmatrix}1\\2\\2\end{pmatrix} + \gamma_3\begin{pmatrix}2\\3\\4\end{pmatrix}$$

$$\left(\begin{array}{ccc|c}1 & 1 & 2 & 0\\1 & 2 & 3 & 1\\1 & 2 & 4 & 2\end{array}\right) \sim \left(\begin{array}{ccc|c}1 & 1 & 2 & 0\\0 & 1 & 1 & 1\\0 & 1 & 2 & 2\end{array}\right) \sim \left(\begin{array}{ccc|c}1 & 1 & 2 & 0\\0 & 1 & 1 & 1\\0 & 0 & 1 & 1\end{array}\right) \begin{array}{l}\rightsquigarrow \alpha_1 = -2\\ \rightsquigarrow \alpha_2 = 0\\ \rightsquigarrow \alpha_3 = 1\end{array}$$

So $v_3 = -2u_1 + 0u_2 + u_3$

So $T_{v\to u} = \begin{pmatrix}1 & -1 & -2\\1 & 1 & 0\\1 & 0 & 1\end{pmatrix}$

Method two:-

$T_{u\to w} = \underline{W^{-1}U}$

find W^{-1} (By $\dfrac{adj(W)}{|W|}$ or find R.R.E.F of $(W|I)$ I is W^{-1})