3.6:- Row Space and Column Space

$A = \begin{pmatrix} \vdots & \text{--- n colum} \\ \vdots & \\ \text{m row} & \end{pmatrix}$

Column space for A:-

$= \text{Span}(a_1, \cdots, a_n)$

$= \text{subspace of } \mathbb{R}^m$ U, R.R.E.F matrix of A

$= C(A)$

$\dim C(A) \leq \{m, n\}$

A, B are row equivalent Then $C(A) \neq C(B)$

Basis for $C(A)$:- corresponding columns That has leading ones in U

Row Space for A:-

$= \text{Span}(\vec{a_1}, \cdots, \vec{a_m})$

$= \text{subspace of } \mathbb{R}^{1\times n}$

$= R(A)$

$\dim R(A) \leq \min\{m, n\}$

If A is Row equivalent to B Then $R(A) = R(B)$

" U is The RREF of A Then $R(A) = R(U)$

Basis for $R(A) \Rightarrow$ nonzero rows in U (which is a Basis for U too)

Null space for A

$N(A) = \{\text{solutions of } Ax = 0\}$

. To get The basis for it

Solve $\left(\begin{array}{c|c} A & 0 \end{array}\right)$ Then write $N(A)$ as a linear combination like $\alpha_1 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \alpha_2 \begin{pmatrix} 5 \\ 6 \\ 7 \end{pmatrix}$ always l.I

here The basis is $\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 5 \\ 6 \\ 7 \end{pmatrix} \right\}$

Rank of A:-

$\text{rank}(A) = \dim(R(A)) = \dim(C(A))$

$\text{rank}(A) \leq \min\{m, n\}$

dim $N(A)$ = number of The free variables

dim $R(A)$ = number of nonzero rows in U

= number of leading ones

= " " " variables

dim $C(A)$ = number of leading ones

Rank (A) + Nullity $(A) = n$

dim $(R(A)$ $(C(A)$ — dim $(N(A))$

- The Relation between System $Ax=b$ and $R(A), C(A)$

$Ax=b$ Consistent for every $b \in \mathbb{R}^m$

$\Longleftrightarrow$ b can be written as a l.C

$\Longleftrightarrow$ $b \in C(A)$

$\Longleftrightarrow$ columns of A form a sp.set for $\mathbb{R}^m$

$\Longleftrightarrow$ columns of A form a sp.set of $\mathbb{R}^m$ $(C(A))$

$\Longleftrightarrow$ has at most one sol columns of A are L.I

$*$ $A_{n \times n}$ is nonsingular $\rightleftarrows$ Columns of A form a sp.set for $\mathbb{R}^n$

$A_{m\times n}$ Matrix

- Columns form a sp.set
  ↳ $n \geq m$
- Columns are L.I
  ↳ $n \leq m$
- Colums form a basis
  ↳ $n = m$

$A_{n\times n}$ nonsingular:-

$\Leftrightarrow$ columns of A are L.I and spset of $C(A)$

$\Leftrightarrow$ $\dim(C(A)) = n = \operatorname{rank}(A)$

$\Leftrightarrow$ nullity $(A) = 0$

$\Leftrightarrow$ rows of A are L.I

$\Leftrightarrow$ columns of A form a sp.set of $\mathbb{R}^n$

$\Leftrightarrow$ ~ ~ ~ ~ basis for $\mathbb{R}^n$

$\Leftrightarrow$ rows form a sp.set for $\mathbb{R}^{1\times n}$

$\Leftrightarrow$ ~ ~ ~ basis for $\mathbb{R}^{1\times n}$

$\Leftrightarrow$ $N(A) = \{0\}$

$\Leftrightarrow$ $Ax = b$ has only a Unique Solution for every $b \in \mathbb{R}^n$

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