

3.6:- Row Space and Column Space

$$A = \begin{pmatrix} \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} \begin{matrix} n \text{ column} \\ m \text{ row} \end{matrix}$$

Column Space for A:-

- = Span (a_1, \dots, a_n)
- = subspace of \mathbb{R}^m
- = $C(A)$

U, R.R.EF matrix of A

$$\dim C(A) \leq \{m, n\}$$

A, B are row equivalent Then $C(A) \neq C(B)$

Basis for $C(A)$:- columns that has leading ones in U

Row Space for A:-

- = Span $(\vec{a}_1, \dots, \vec{a}_m)$
- = subspace of $\mathbb{R}^{1 \times n}$
- = $R(A)$

$$\dim R(A) \leq \min\{m, n\}$$

If A is Row equivalent to B Then $R(A) = R(B)$

U is the RREF of A Then $R(A) = R(U)$

Basis for $R(A) \Rightarrow$ nonzero rows in U (which is a Basis for U too)

Null space for A

$N(A) = \{ \text{solutions of } Ax=0 \}$

To get The basis for it

Solve $\left(A \mid 0 \right)$ Then write $N(A)$ as a linear combination always l.i

Combination like $\alpha \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \alpha_2 \begin{pmatrix} 5 \\ 6 \\ 7 \end{pmatrix}$

here The basis is $\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 5 \\ 6 \\ 7 \end{pmatrix} \right\}$

Rank of A :-

$$\text{rank}(A) = \dim(R(A)) = \dim(C(A))$$

$$\text{rank}(A) \leq \min\{m, n\}$$

$\dim N(A)$ = number of The free variables

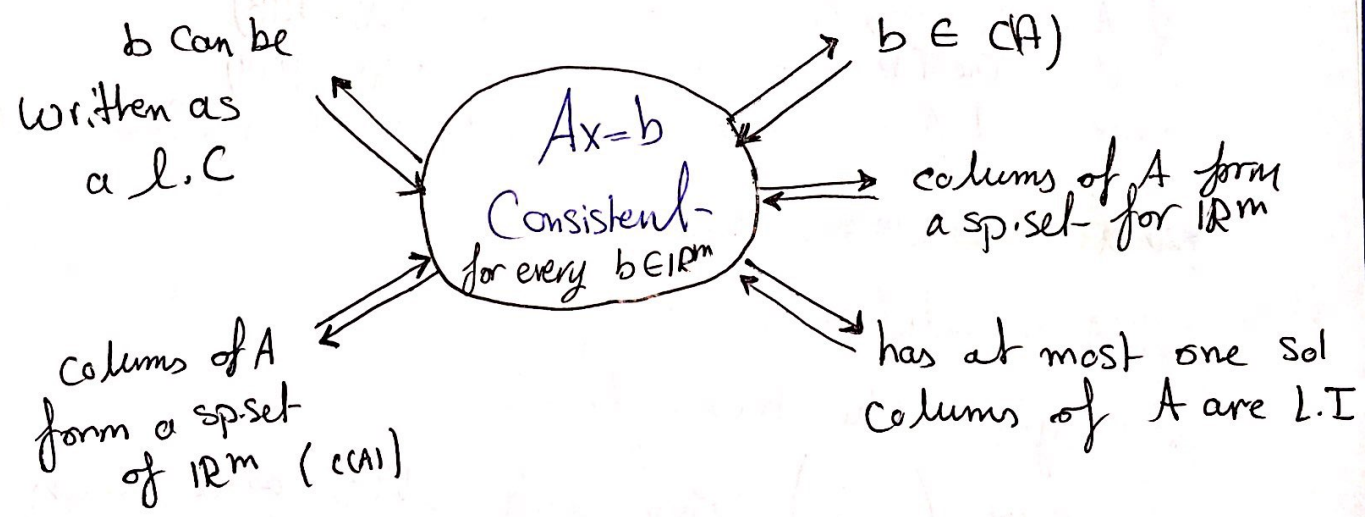
$\dim R(A)$ = number of nonzero rows in U
= number of leading ones
= $\sim \sim \sim \sim$ variables

$\dim C(A)$ = number of leading ones

$$\text{Rank}(A) + \text{Nullity}(A) = n$$

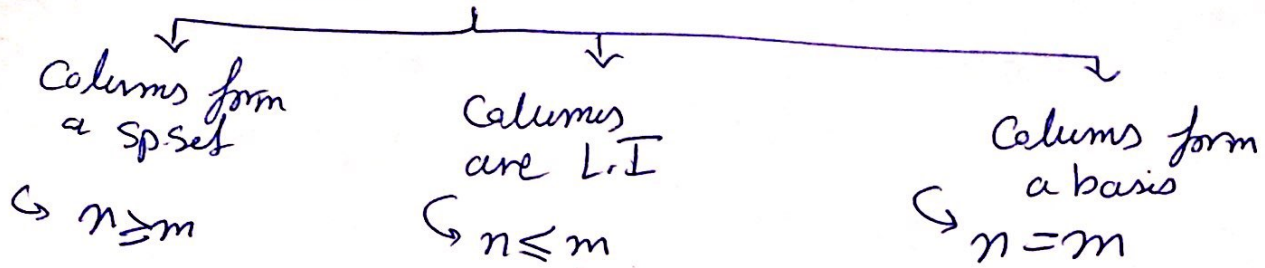
$\dim(R(A))$ \uparrow $\dim(N(A))$
 $(C(A))$

• The Relation between system $Ax=b$ and $R(A), C(A)$



* $A_{n \times n}$ is nonsingular \iff columns of A form a sp.set for \mathbb{R}^n

$A_{m \times n}$ Matrix



$A_{n \times n}$ nonsingular: -

- \hookrightarrow columns of A are L.I and spset of $C(A)$
- $\hookrightarrow \dim(C(A)) = n = \text{rank}(A)$
- $\hookrightarrow \text{nullity}(A) = 0$
- \hookrightarrow rows of A are L.I
- \hookrightarrow columns of A form a spset of \mathbb{R}^n
- $\hookrightarrow \sim \sim \sim \sim$ basis for \mathbb{R}^n
- \hookrightarrow rows form a spset for $\mathbb{R}^{1 \times n}$
- $\hookrightarrow \sim \sim \sim$ basis for $\mathbb{R}^{1 \times n}$
- $\hookrightarrow N(A) = \{0\}$
- $\hookrightarrow Ax = b$ has only a Unique Solution for every $b \in \mathbb{R}^n$