Chapter 4: linear transformation

- A function $L: V \to W$ where V, W are vector spaces is called a linear Transformation if :-

$$L(\alpha_1 V_1 + \alpha_2 V_2) = \alpha_1 L(V_1) + \alpha_2 L(V_2)$$

عملية الجمع في V عملية الجمع معرفة في W

If $L: V \to W$ is a L.T Then:-

1) $L(V_1 + V_2) = L(V_1) + L(V_2)$
2) $L(\alpha V) = \alpha L(V)$
3) $L(0_V) = 0_W$
4) $L(-V) = -L(V)$

- Kernel:- $L: V \to W$ is a L.T

$\text{Ker}(L) = \{ v \in V : L(v) = 0 \}$ ⇝ roots of L

⇝ $\text{Ker}(L)$ is a subspace of V

- Image of L (Range of L) is:- let $L: V \to W$

$\text{Im}(L) = \{ L(v) : v \in V \}$ ⇝ $\{ w \in W : \text{There exists } v \in V \text{ where } L(v) = w$

↳ subspace of W

• find range and Kernel of each of The L.T on $\mathbb{R}^3$

a) $L(x) = \begin{pmatrix} x_3 \\ x_2 \\ x_1 \end{pmatrix}$

1 Ker: $L\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} c \\ b \\ a \end{pmatrix}$

find $\underline{L(v) = 0}$

$L\begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0$ So $\rightarrow$ $\begin{pmatrix} c \\ b \\ a \end{pmatrix} = 0$

only when $c = b = a = 0$

so Kernel $= \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$

Range: $L\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} c \\ b \\ a \end{pmatrix} \in \mathbb{R}$

$= c\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + a\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

so $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$ is The Range (standard Basis for $\mathbb{R}^3$

so Range $(L(x)) = \mathbb{R}^3$