Proofs of chapter 6 :-

* Eigen values are roots of $|A-\lambda I|=0$

$Ax=\lambda x \rightsquigarrow \neq 0$

$Ax-\lambda x=0$

$(A-\lambda I)x=0$ / $x\neq 0$ / it has a non zero solution

homogeneous system

So $A-\lambda I$ is singular $\Rightarrow |A-\lambda I|=0$

* If B is similar to A then A is similar to B

$A=S^{-1}BS$ $\quad \exists S$ is nonsingular

- Multiplying By S^{-1} from right

$AS^{-1}=S^{-1}B$

- Multiplying By S from left

$SAS^{-1}=B$

let $S^{-1}=D$

$D^{-1}AD=B$ $\quad$ So A is similar to B

* If A and B are similar Then They have the same characteristic Polynomials

$P_B(\lambda)=|B-\lambda I|$ $\quad$ But A is similar to B So $B=S^{-1}AS$

$=|S^{-1}AS-\lambda I|$

$=|S^{-1}AS-S^{-1}\lambda IS|$

$=|S^{-1}(AS-\lambda S)|=|S^{-1}(A-\lambda I)S|=|S^{-1}||S||A-\lambda I|$

$=|A-\lambda I|$ ✓

* The eigenvalue of a triangular matrix are The diagonal elements of the matrix

Triangular ⇝ lower or upper Triangular

lower :- $A = \begin{pmatrix} a_{11} & 0 & 0 & \cdots \\ a_{21} & a_{22} & 0 & \cdots \\ \vdots & & \cdots & \\ a_{n1} & \cdots & \cdots & a_{nn} \end{pmatrix}$

$$|A-\lambda I| = \left|\begin{pmatrix} a_{11}-\lambda & 0 & 0 & \cdots & 0 \\ a_{21} & a_{22}-\lambda & \cdots & 0 & \cdots \\ \vdots & & & & a_{nn}-\lambda \end{pmatrix}\right|$$

$$= (a_{11}-\lambda) \begin{vmatrix} a_{22}-\lambda & \cdots \\ 0 & \\ \vdots & a_{nn}-\lambda \end{vmatrix}$$

$$= (a_{11}-\lambda)(a_{22}-\lambda) \cdots (a_{nn}-\lambda)$$

$\Rightarrow a_{11} = \lambda_1,\ a_{22} = \lambda_2 \cdots a_{nn} = \lambda$ So λ is The diagonal element of each row

upper : Same idea

* $A_{n \times n}$, A is singular iff $\lambda = 0$ is an eigenvalue of A

- $|A-\lambda I| = 0$ if A is singular Then $|A| = 0$

  So $\lambda I = 0$

  λ has to be zero

- $\lambda = 0$ is an eigenvalue ⇝ A is singular

  if $\lambda = 0$ Then $|A - 0I| = 0$ ⇝ $|A| = 0$ So A is singular

* A is nonsinguler , λ is an eigenvalue of A
Then $\frac{1}{\lambda}$ is an aigen value of A^{-1}

let:- A is nonsinguler

$$Ax = \lambda x$$

Multiplying By A^{-1}

$$A^{-1}Ax = A^{-1}\lambda x$$

$$Ix = \lambda A^{-1}x$$

$$\frac{x}{\cancel{\lambda}} = \frac{\cancel{\lambda}}{\cancel{\lambda}} A^{-1}x \quad \lambda \text{ is a scaler}$$

$$\frac{1}{\lambda}x = A^{-1}x$$

So $A^{-1}x = \frac{1}{\lambda}x$ is eigenvalue for A^{-1}

* $A_{n\times n}$: $A^2 = A$ if λ is an eigenvalue for A
Then λ must be 0 or 1

$$A^2 = A$$

$$\underline{Ax = \lambda x}$$

$$A^2x = A\lambda x \rightsquigarrow = \lambda x$$

$$A = A^2 x = \lambda (Ax)$$

$$Ax = \lambda^2 x$$

$$\lambda x = \lambda^2 x$$

So $\lambda = \lambda^2$

$\rightsquigarrow \lambda(1-\lambda) = 0$

$\lambda = 0$ or $\lambda = 1$

* $A_{2\times2} \Rightarrow P(\lambda) = \lambda^2 + b\lambda + C$ show That $b = -\text{tra}(A)$ and $C = \det(A)$

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$|A - \lambda I| = \left| \begin{pmatrix} a_{11}-\lambda & a_{12} \\ a_{21} & a_{22}-\lambda \end{pmatrix} \right|$$

$$= (a_{11}-\lambda)(a_{22}-\lambda) - (a_{12}a_{21})$$

$$= a_{11}a_{22} - \lambda a_{11} - \lambda a_{22} + \lambda^2 - a_{12}a_{21}$$

$$= (a_{11}a_{22} - a_{12}a_{21}) - \lambda(a_{11}+a_{22}) + \lambda^2 = \lambda^2 + b\lambda + C$$

So $b = -\underbrace{(a_{11}+a_{22})}_{\text{Tr}(A)}$

and $C = a_{11}a_{22} - a_{12}a_{21} = \text{Det}(A)$

* $B = S^{-1}AS$ x is an eigenvector of B belonging to λ show That Sx is an eigen vector of A belonging to λ

Prove That $(A(Sx) = \lambda(Sx))$

$B = S^{-1}AS$

$Bx = \lambda x$

$S^{-1}A(Sx) = \lambda x$

Multiplying By S from left

$A(SX) = S\lambda X$

$A(SX) = \lambda(SX)$ ✓

6.3 , Diagnolization

* A is diagnolizable , $\lambda = 1, -1$

Show That $A^{-1} = A$

$A = X^{-1} D X$

$$D = \begin{pmatrix} 1 & 0 & 0 & 0 & \text{---} \\ 0 & 1 & 0 & \text{---} & \\ 0 & 0 & 1 & \text{---} & \end{pmatrix} \text{ or } \begin{pmatrix} -1 & 0 & \text{---} \\ 0 & -1 & \text{---} \\ \text{---} & \text{---} & \end{pmatrix}$$

In Both Cases $\underline{D = D^{-1}}$

$$\begin{aligned} (A^{-1}) &= (X^{-1} D X)^{-1} \\ &= (X^{-1})(X^{-1} D)^{-1} \\ &= X^{-1} D^{-1} X \\ &= X^{-1} D X = A \end{aligned}$$

So $A^{-1} = A$

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