Monday, February 15, 2021 9:52 AM

Math 234

Sec. 4 MW: 10:00 – 11:15

Videos: on itc.

Zoom online Meeting

recordings

ritaj + itc

Messages videos

Exams: university policy.

Final

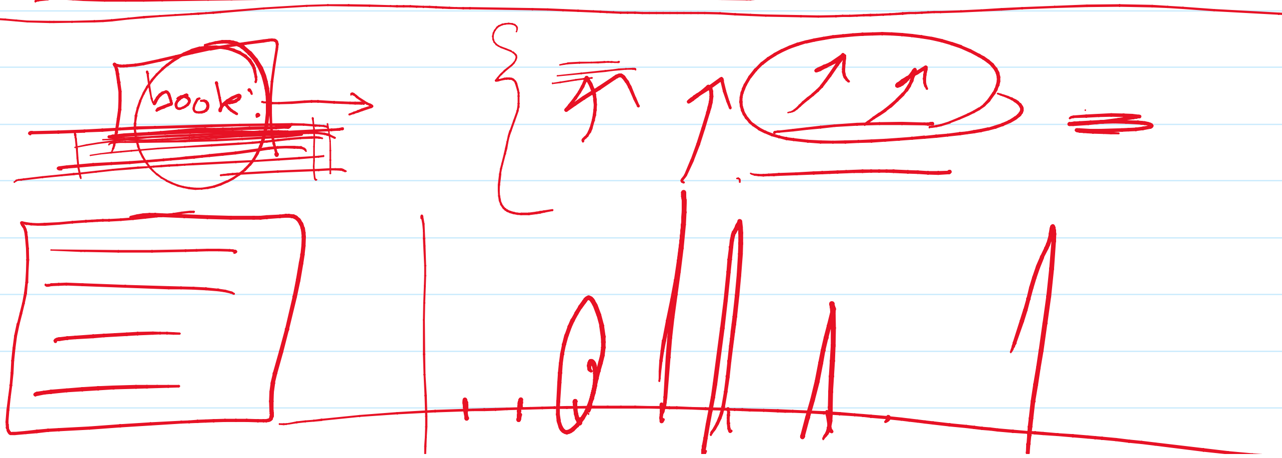
Outline:

Textbook: Introduction to linear algebra with applications, Steven J. Leon. 9th edition.

pdf on itc.

$\boxed{\text{ch: } 1, 2, 3, 4, 6.}$

book:



report :

Name + ID #

Exams : Midterm

short exams

50% - final Exam

40 — 60 %

Outline :

Calculus → techniques

$f(x)dx$

* Linear algebra → Theory 50%

→ Applications 50%

* Systems of linear equations. ch 1, 2

Matrices

Vector space. ch 3

Matrices

ch 4 : functions between vector spaces.

Ch6 Eigenvalues of a Matrix

Chapter 1 Systems of Linear Equations.

Linear → power of variables ≤ 1.

Ex: $x_1 + 2x_2 = 5$
$2x_1 + 3x_2 = 8$

$\boxed{\begin{array}{c} 2\times2\text{-system} \\ \cancel{3\times3\text{-system}} \end{array}}$

# of equations $= 2$

# of unknowns $= 2$: x_1, x_2.

solution: 2 values for x_1, x_2. that satisfy all equations.

Is $x = \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ a solution ?

$x = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ solution?

(1) $1 + 2(2) = 5 \;\underline{= 5}$

(2) $2(1) + 3(2) = 8 \;\underline{= 8}$

$\Rightarrow x = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ is a solution.

solve: $x_1 + 2x_2 = 5$
$2x_1 + 3x_2 = 8$

$-2E_1 + E_2$

$-2(1)$: $-2x_1 - 4x_2 = -10$
$2x_1 + 3x_2 = 8$

$+$: $-x_2 = -2 \Rightarrow \boxed{x_2 = 2}$

$0E_1 + E_2 \Rightarrow E_2$

$cE_1 + E_1$

$(-1)E_1 + E_1 \Rightarrow \cancel{0=0}$

From (1) $\Rightarrow$ $x_1 = 5 - 2(2) = 1$.

so the system has one solution $x = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Multiply an equation by a nonzero constant

* Multiply an equation by $\boxed{\text{a nonzero constant}}$
* Interchange two equations
* Add a multiple of one equation to another equation. (not the same equation)

$m \times n$-linear system has the form

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &= b_m \end{aligned}\right\} \to (*)$$

$m \times n$ Linear system.

m: # of equations

n: # of unknows $x_1, x_2, \dots, x_n$.

a_{ij}: coefficients, $a_{ij} \in \mathbb{R}$

b_j: constants $\in \mathbb{R}$.

* A solution to $(*)$ is an n-tuple (values of $x_1, x_2, \dots, x_n$) that satisfy all equations.

$$\boxed{x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}} \quad \left\{ x = (x_1, x_2, \dots, x_n) \right\}$$

Ex: $\left\{\begin{aligned} x_1 - x_2 + x_3 &= 2 \\ 2x_1 + x_2 - x_3 &= 4 \end{aligned}\right\}$ 2×3 - system.

unknowns x_1, x_2, x_3.

unknowns x_1, x_2, x_3.

$x=\begin{pmatrix}2\\1\\1\end{pmatrix}$ solution?

(1) ✓
(2) ✓

$\begin{pmatrix}2\\1\\1\end{pmatrix}$ is a solution.

$x=\begin{pmatrix}2\\-1\\-1\end{pmatrix}$

(1) ✓
(2) ✓

$\begin{pmatrix}2\\-1\\-1\end{pmatrix}$ is a solution.

solve: $(1)+(2) \Longrightarrow 3x_1 = 6 \Longrightarrow \boxed{x_1 = 2.}$

in (1): $2 - x_2 + x_3 = 2 \Longrightarrow -x_2 + x_3 = 0 \xrightarrow{(-1)} \boxed{x_2 - x_3 = 0}$

in (2): $4 + x_2 - x_3 = 4 \Longrightarrow x_2 - x_3 = 0 \longrightarrow x_2 - x_3 = 0$

so $x_2 - x_3 = 0 \Longrightarrow x_2 = x_3 \overset{\text{let}}{=} \alpha$

Any solution has the form $\boxed{x=\begin{pmatrix}2\\\alpha\\\alpha\end{pmatrix}}$ $\alpha \in \mathbb{R}$.

infinite # of solutions

$\begin{pmatrix}2\\3\\4\end{pmatrix}$ solution No

Ex:
$$\boxed{\begin{aligned} x_1 + x_2 &= 2\\ x_1 - x_2 &= 1\\ x_1 \quad &= 4. \end{aligned}}$$

$\boxed{3\times 2\text{- system}}$

solve: $(3) \Longrightarrow \boxed{x_1 = 4}$

in (1) $\Longrightarrow 4 + x_2 = 2 \Longrightarrow \boxed{x_2 = -2}$

$-2 \neq 3$

in (1) $\Rightarrow$ $4 + x_2 = 2$ $\Rightarrow$ $\boxed{x_2 = -2}$

in (2) $\Rightarrow$ $4 - x_2 = 1$ $\Rightarrow$ $\boxed{x_2 = 3}$

$-2 \neq 3$

so the system has no solution.

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mxn-system

- exactly one solution
- infinite # of solutions

} consistent system.

- No solution } inconsistent system.

2x2-system:

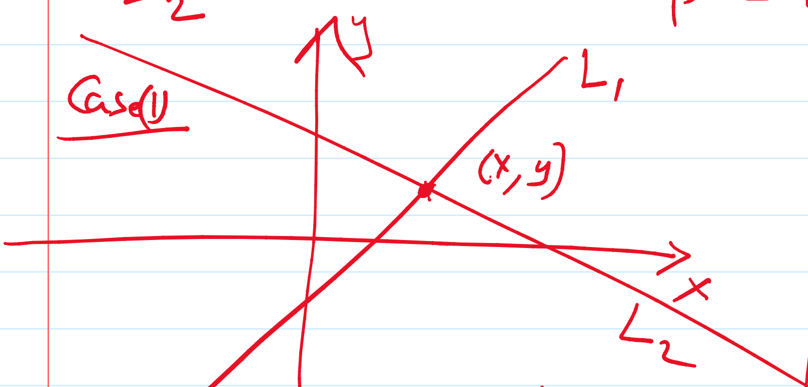
$$\boxed{ax + by = c}$$
$$\boxed{dx + ey = f}$$

unknowns x, y.

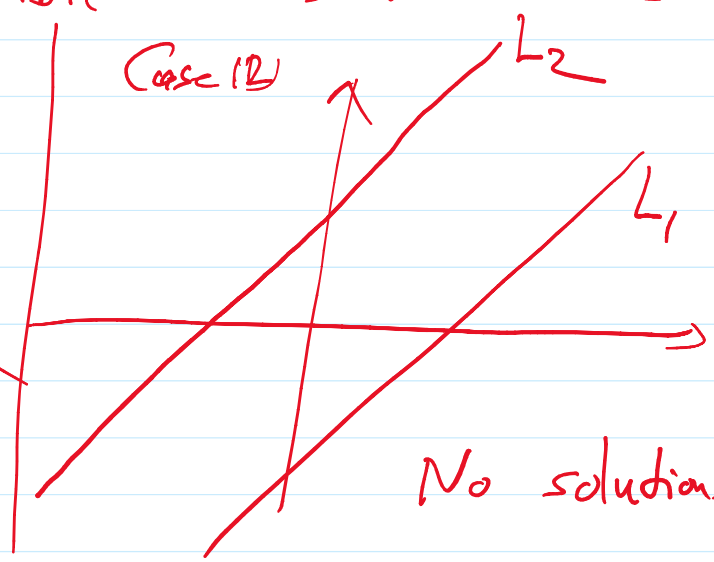
L_1 = Line

L_2: "

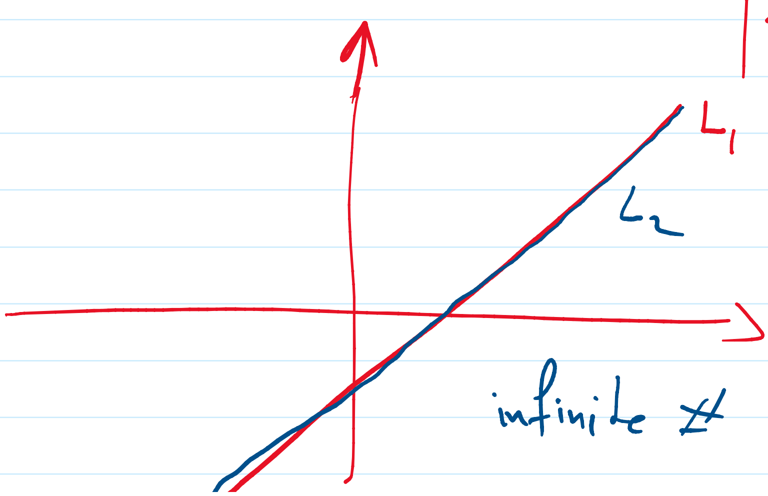
possible cases for L_1, L_2.



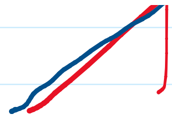
exactly one solution



No solutions.



infinite # of solutions.



infinite # of solutions.