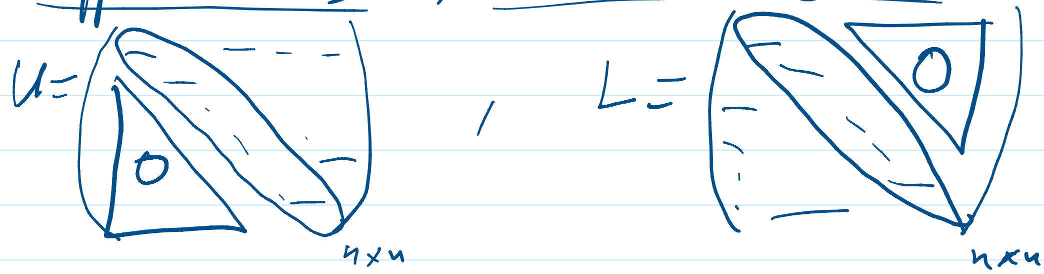
* upper triangular, lower triangular.



easy to solve.

* Given $\boxed{\text{a system } Ux=b}$, U upper triangular.

Ex. $\begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ ↑ back substitution.

$x_3 = 3$, $x_2 = -1 - 3(3) = -10$

$x_1 = 2 - 2(-10) + 3 = 25$

* Given a system $\boxed{Lx=b}$, L lower triangular.

Ex $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix}$

$\longrightarrow x_1 = 3$

$\longrightarrow 2x_1 + x_2 = 5 \Rightarrow x_2 = 5 - 2x_1 = 5 - 2(3) = -1$

$\longrightarrow x_1 + 2x_2 + 3x_3 = 1$

easy to solve.

$\Rightarrow x_3 = \dfrac{1 - 2(-1) - 3}{3} = 0$

forward substitution.

* Given $\boxed{Ax=b \text{ , and } A = LU}$, L lower triangular, U upper triangular.

solve $(LU)x = b \iff L(Ux) = b \longrightarrow$

solve $(LU)x = b \iff L(Ux) = b$ —— $(*)$

Let $\boxed{Ux = y}$ —— $(**)$

$\boxed{Ly = b}$ easy to solve for y.

In $(**)$: $\boxed{Ux = y}$ → solve for $\boxed{x}$. easy to solve.

Ex: $A = \begin{pmatrix} -2 & 1 & 2 \\ 4 & 1 & -2 \\ -6 & -3 & 4 \end{pmatrix}$, $A = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & -2 & 1 \end{pmatrix}}_{L} \underbrace{\begin{pmatrix} -2 & 1 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{pmatrix}}_{U}$ Given

solve $Ax = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ →

Let $\boxed{Ux = y}$ —— (1)

(I) solve $\boxed{Ly = b} \iff \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -2 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$y_1 = 1$, $-2y_1 + y_2 = 2 \Rightarrow y_2 = 2 + 2(1) = 4$.

$3y_1 - 2y_2 + y_3 = 3 \Rightarrow y_3 = 3 - 3(1) + 2(4) = 8$

→ $y = \begin{pmatrix} 1 \\ 4 \\ 8 \end{pmatrix}$. solution of $Ly = b$.

(II) solve $\boxed{Ux = y} \iff \begin{pmatrix} -2 & 1 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 8 \end{pmatrix}$ ↑

$x_3 = 4$, $3x_2 + 2x_3 = 4 \Rightarrow 3x_2 = 4 - 2(4) = -4$

$x_2 = \frac{-4}{3}$.

$-2x_1 + x_2 + 2x_3 = 1$

$$x_2 = -\frac{4}{3}.$$

$$-2x_1 + x_2 + 2x_3 = 1$$

$$-2x_1 = 1 + \left(\frac{4}{3}\right) - 2(4) = \frac{3+4-24}{3} = \frac{-17}{3}$$

$$\therefore \boxed{x = \begin{pmatrix} 17/6 \\ -4/3 \\ 4 \end{pmatrix}} \qquad \therefore x_1 = \frac{17}{6}$$

solution of $Ax=b$.

How to find L, U : $A = LU$. ?

Ex: $A = \begin{pmatrix} -2 & 1 & 2 \\ 4 & 1 & -2 \\ -6 & -3 & 4 \end{pmatrix}$. find L, U such that $A = LU$.

(not always possible)

LU-factorization.

* possible: If A can be reduced to an upper triangular matrix $\boxed{U}$ using only row operation III.

$$A = \begin{pmatrix} -2 & 1 & 2 \\ 4 & 1 & -2 \\ -6 & -3 & 4 \end{pmatrix} \xrightarrow[\boxed{-3R_1+R_3}]{\boxed{2R_1+R_2}} \begin{pmatrix} -2 & 1 & 2 \\ 0 & 3 & 2 \\ 0 & -6 & -2 \end{pmatrix} \xrightarrow{2R_2+R_3} \underbrace{\begin{pmatrix} -2 & 1 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{pmatrix}}_{U}$$

$cR_i + R_j$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix}_{3\times 3}$$

l_{21} : $\boxed{2R_1+R_2}$: $l_{21} = -2$,

l_{31} : $-3R_1+R_3$: $l_{31} = 3$

l_{32} : $2R_2+R_3$: $l_{32} = -2$.

$$\therefore \boxed{L = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -2 & 1 \end{pmatrix}}$$

$$\boxed{LU = A}$$

$$\boxed{LU = A}$$

$$\boxed{A} \xrightarrow{2R_1+R_2} \underbrace{E_1 A} \xrightarrow{-3R_1+R_3} \underbrace{E_2 E_1 A} \xrightarrow{2R_2+R_3} \boxed{E_3 E_2 E_1 A = U}$$

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{2R_1+R_2} E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \xrightarrow{-3R_1+R_3} E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix}, \quad \xrightarrow{2R_2+R_3} E_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}.$$

$$E_3 E_2 E_1 A = U \implies A = \boxed{E_1^{-1} E_2^{-1} E_3^{-1}} U$$
$$= \underline{L}\,\underline{U}$$

Ex: $A = \begin{pmatrix} -2 & 1 & 3 \\ 4 & -2 & 1 \\ 6 & 4 & 5 \end{pmatrix}$, find LU-factorization. (if exists)

$$A = \begin{pmatrix} \boxed{-2} & 1 & 3 \\ \boxed{4} & -2 & 1 \\ 6 & 4 & 5 \end{pmatrix} \xrightarrow[\boxed{3R_1+R_3}]{\boxed{2R_1+R_2}} \begin{pmatrix} -2 & 1 & 3 \\ 0 & \boxed{0} & 7 \\ 0 & 7 & 14 \end{pmatrix}$$

<u>Not possible</u>

Ex. $A = \begin{pmatrix} 0 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 0 \end{pmatrix}$ find LU-factorization.

not possible

Ex. $A = \begin{pmatrix} -2 & 1 & 3 \\ 4 & -2 & 1 \\ \cdots \end{pmatrix} \xrightarrow[3R_1+R_3]{2R_1+R_2} \begin{pmatrix} -2 & 1 & 3 \\ 0 & \boxed{0} & 7 \\ \cdots \end{pmatrix} \xrightarrow{~~2R_2+R_3~~} \begin{pmatrix} -2 & 1 & 3 \\ 0 & 0 & 7 \\ \cdots \end{pmatrix}$

Ex. $A = \begin{pmatrix} \cdots & & \\ 4 & -2 & 1 \\ 6 & -3 & 5 \end{pmatrix} \xrightarrow[3R_1+R_3]{2R_1+R_2} \begin{pmatrix} \cdots & & \\ 0 & 0 & 7 \\ 0 & 0 & 14 \end{pmatrix} \xrightarrow{0R_2+R_3} \begin{pmatrix} \cdots & & \\ 0 & 0 & 7 \\ 0 & 0 & 14 \end{pmatrix}$ U

$$L_1 = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix}, \quad U_1 = \begin{pmatrix} -2 & 1 & 3 \\ 0 & 0 & 7 \\ 0 & 0 & 14 \end{pmatrix}$$

$L_1U_1 = A$ check.

$\xrightarrow[-2R_2+R_3]{2R_2+R_3} \begin{pmatrix} -2 & 1 & 3 \\ 0 & 0 & 7 \\ 0 & 0 & 28 \end{pmatrix}$ U_2

$$L_2 = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & -2 & 1 \end{pmatrix}$$

$L_2U_2 = A$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 1 \\ -2 & -4 & 4 \end{pmatrix}$$

$\xrightarrow[2R_1+R_3]{2R_1+R_2} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & 10 \end{pmatrix} \xrightarrow{0R_2+R_3} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & 10 \end{pmatrix}$

U,

$$L = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$$

$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 7 \\ 0 & 1 & 10 \end{pmatrix}$ not possible.

~~ch~~

11/4 → short exam. → Quiz (20%) ITC — section

9/5 → Midterm. Meta (35%) ch1 + ch2

Monday: Quiz ~~10-15~~ Minutes.

Ch 1 {end of lecture}.