BIRZEIT UNIVERSITY

Mathematics Department
MATH 2351

Handout # 3 Prepared by Mohammad Madiah
Sections 5.1, 5.2, 6.1 and 6.2

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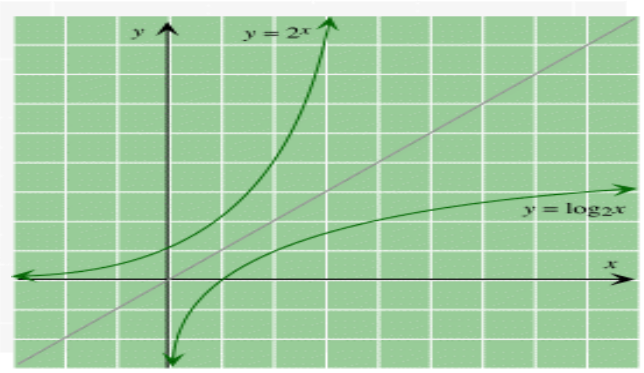
Exponential and logarithmic functions

- **Exponential functions** are functions written in the form $y=a^x$, where a is the **base**, a is positive and $a\neq 1$, and x is a real number.
- The domain of the exponential function, the values for which x can equal, are all real number. The range however, is all positive numbers.
- For a>1, the function $y=y_0a^{kx}$ is called the general exponential function
  a. k > 0 means exponential **growth**.
  b. k < 0 means exponential **decay.**
  c. **Special function:** $f(x)=y_0e^{kx}$
- For a > 0, x > 0, the function $y=\log_a x$ is called the **logarithmic function.**

$$y=\log_a x \Leftrightarrow a^y=x$$

- **Special function:** $f(x)=\ln x$.
- $\ln x=\log_e x$



Some Properties of exponential

For any real numbers a and b and positive integers m and n

1. $a^m a^n=a^{m+n}$
2. For $a\neq 0$, $\dfrac{a^m}{a^n}=\begin{cases} a^{m-n} & m>n \\ 1 & m=n \\ \dfrac{1}{a^{n-m}} & m<n \end{cases}$
3. $(ab)^m=a^m b^m$
4. $\left(\dfrac{a}{b}\right)^m=\dfrac{a^m}{b^m}\ (b\neq 0)$

5. $(a^m)^n = a^{mn}$
6. $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$ (if n even, $a \geq 0$

Some Properties logarithms

1. $\log_a 1 = 0$
2. $\log_a a = 1$
3. $\log_a a^x = x$
4. $a^{\log_a x} = x$
5. $\log_a MN = \log_a M + \log_a N$
6. $\log_a \frac{M}{N} = \log_a M - \log_a N$
7. $\log_a x^n = n \log_a x$
8. $\log_a x = \log_a y \Rightarrow x = y$
9. $\log_{10} x = \log x$ **(common logarithm)**
10. $\log_e x = \ln x$ **(natural logarithm)**
11. $\log_b a = \frac{\log_m a}{\log_m b} = \frac{\log a}{\log b} = \frac{\ln a}{\ln b}$ **(change of basis)**

Using Calculator

- Power Key ^

$3^{1.45}$**: 3 ^ 1.45 = 4.92**

- Logarithms Keys

1. log key : Use base **10**
Log 15: log 15 = 1.18

2. ln key : Use base **e**
Ln 15: ln 15 = 2.71

- Exponential Keys

3. 10^x key : Shift + **log**

$10^{1.5}$**:** Shift + **log 1.5 = 31.62**

4. e^x key : shift + **ln**

$e^{1.5}$**:** Shift + **ln 1.5 = 4.48**

Simple and Compound Interest

- If **$P** is invested at an interest rate of **r** per year, then the simple interest, and the future value **S** after **t** years are

  $S = P + I$, where $I = Prt$

- If **$P** is invested at an interest rate of **r** per year, compound annually, the future value **S** after **t** years is

  $$S = P(1+r)^t$$

- If **$P** is invested for t years at a nominal interest rate **r**, compound **m** times per year, the future value **S** is

  $$S = P\left(1+\frac{r}{m}\right)^{mt}$$

- If **$P** is invested for t years at a nominal interest rate of **r** compound **continuously**, the future value **S** is

  $$S = Pe^{rt}$$