Sep 11th, 19
Wed

* Sec 1.6 : Applications of functions in Business and Economic

مراجعة

variables costs.

$C(x)$ total cost $= mx + b$: fixed costs.

$R(x)$ Revenue f $= px$

$P(x)$ Profit f $= R(x) - C(x)$.

Ex:
insurance
rent
Salaries
(رواتب)

Total cost $C(x)$

↙ Variables costs ↘ fixed costs (b)

Variables costs: ↙ # of units Produced (x) ↘ cost per unit (m)

$\Rightarrow V.C = mx$

$C(x) = (V.C) + (F.C) = mx + b$

m: cost per unit

x: # of units Produced

b: fixed cost.

* if m = constant $\Rightarrow$ Linear function.

$\Rightarrow C(x)$ = Linear function.

So: m = slope = $\overline{MR}$ (the offecient of x) معامل x

b = y-intercept = C-intercept

$C(x)$ محور الاقتران

= fixed cost.

* $\boxed{(\text{fixed cost} = C(0))}$.

* $m = \text{slope} = \overline{MR}$ = cost per unit produced
= cost per unit " and sold

$\boxed{\overline{MR}}$: The cost of producing one additional unit at any level of production (Marginal cost).

مهم جدا

Revenue function : $R(X)$ اقتران العوائد

$R(x)$ = (Selling price per unit)(# of units).

سعر البيع لكل قطعة

$R(x) = pX$

((p) small)

$\boxed{p = MR = \text{slope} = m}$

Linear فقط عندما تكون

P(x) : Profit function اقتران الربح

$P(x) = R(X) - C(x) \rightarrow$

(P) capital

العوائد - الثمن.

معامل x : marginal profit

* $p = \overline{MR}$: The revenue of ~~producing~~ selling one addition unit at any level of Production.

marginal revenue (not cost)

$\overline{MR}$

$$P(x) = R(x) - C(x)$$

الربح الحدي * ⇒ marginal profit : The profit of producing and selling one additional unit at any level of production.

* Break even point : $P(x) = 0$

⇒ $C(x) = R(x)$ فرق الربح والخسارة

$(x, R(x))$ or $(x, C(x))$

Profit : when $P(x)$ +ve (positive)

loss : when $P(x)$ -ve (negative)

Q: 2, 3, 13, 15, 23   1.6

Page 112 :

Chapter 1
Section 6

2- Suppose a stereo receiver company has the total cost $C(x) = 210x + 3300$ and the total Revenue $R(x) = 430x$.

a)- what is the equation of the profit funct

$$P(x) = R(x) - C(x)$$
$$= 430x - 210x - 3300$$
$$= \boxed{220x - 3300}$$

$\Rightarrow P(x) = 220x - 3300$

b)- What is the profit on 500 items.

$\Rightarrow P(500) = (220 \times 500) - 3300$

$= 110000 - 3300$

$= \$106700 \Rightarrow$ (profit).

5- A Linear cost function is
$C(x) = 5x + 250$:

a). What are the slope and the C-intercept

$\Rightarrow$ slope $= 5$

C-intercept $(0, b) = C(0)$

C-int $= 250$ ~~$\Rightarrow (0, 250)$~~

شطع.

b). What is the marginal cost? and what does it mean?

$\Rightarrow$ marginal cost $(\overline{MC})$ = slope $= 5$

it means the cost of producing one additional unit at any level of production.

c). How are your answers to a and b related? (شو العلاقة بين a, b)

$\Rightarrow$ slope $= \overline{MC}$

$= \overline{MC}$

$\uparrow$

= 1 additional

* d)- What is the cost of producing one more item if 50 items are currently being produced? and what is it if 100 are ~~are~~ currently being produced.

تذكير: $C(x) = 5x + 250$

sec 1: 1 more $\Rightarrow \overline{MC}$

$\overline{MC} = \$5$ (or: $C(51) - C(50)$).

Sec 2: $\overline{MC} = \$5$ or: $(C(101) - C(100))$

$\boxed{13-}$ المعطيات $\rightarrow$ The fixed costs are \$6600 per month Materials and labow for each helmet of this model are \$35 and (خوذة) company sells this helmet $\boxed{\$60}$ for item

a)- Write the function for monthly total cost

$\Rightarrow C(x) = mx + b$ (عدد الوحدات) (f.c) (Per)

$= 35x + 6600$

b)- Write the function of total revenue

$\Rightarrow R(x) = px$ p = selling price

$= \boxed{60}x$

c)- Find $C(200)$, $R(200)$, $P(200)$ and ~~interpert~~ interpret.

وضّح

Sep 18, 19

(Sec 1.6) : تكملة الحل

$\boxed{C-)}$ $C(200) = (35 \times 200) + 6600$
$= \$ 13600$

$R(200) = 60(200) = \$12000$

$P(200) = R(200) - C(200)$
$= 12000 - 13600$
$= -1600 \Rightarrow$ loss.

✓ interpret ↵

f)- Find the marginal profit and write a sentence that explains it meaning.

$\overline{MP} =$ x معامل

$P(x) = R(x) - C(x)$
$= 60x - (35x + 6600)$
(a , b من فرع)

$P(x) = 25x - 6600$

$\overline{MP} = 25$, The Profit of producing and selling one additional unit at any level of production.

Remember: $C(0) = \text{fixed cost}$

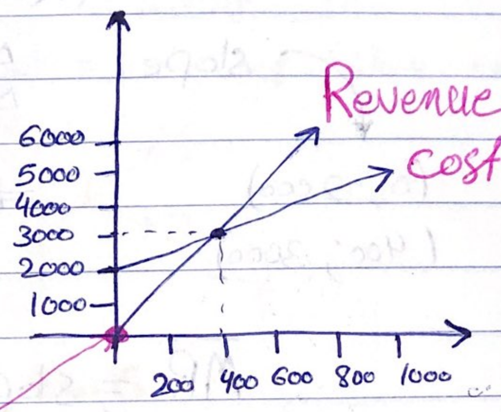
$R(0) = 0$

Ex: 15

a) Label each function correctly

(اقتران C و واحد R)

اصابنا خدد .



R: $R(0) = 0$

b) Determine the fixed cost.

A: $C(0) = \text{fixed cost}$

$C(0) = \$2000$

c) Locate the B.E point and determine the # of units sold to B.E.

B.E: $C(x) = R(x)$ $\quad (P(x) = 0)$

$\Rightarrow (400, 3000)$

→ 400 units

الكمية : محور X

imp

d)- Estimate the marginal cost and marginal revenue

A: $\overline{MC} = \text{slope } (C(x))$ (نأخذ نقطتين من اقتران ال cost)

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{3000 - 2000}{400 - 0} = 2.5$$

(0, 2000)

(400, 3000)

$$\Rightarrow \boxed{\overline{MC} = 2.5}$$

$\overline{MR} = \text{slope } (R(x))$

(نأخذ نقطتين من ال Revenue)

(0,0) , (400, 3000)

$$\text{slope} = \frac{3000 - 0}{400 - 0} = 7.5$$

$$\Rightarrow \boxed{\overline{MR} = 7.5}$$

إضافي

$$\overline{MP} = \overline{MR} - \overline{MC}$$
$$= 7.5 - 2.5$$
$$= 5$$

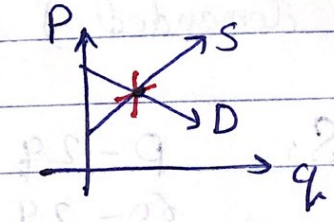
* **Law of Demand** :- as price increases, the quantity demanded will decrease. (slope: -ve)

* Law of supply :- as price increases, the quantity supplied will also increases. (slope: +ve)

قانون العرض

S: Positive

D: negative



1) equilibrium point : D = S

2) D > S ⇒ shortage "عجز"

3) D < S ⇒ Surplus "فائض"

The price will increase : السعر سيرتفع

The price will ~~[illegible]~~ fall : السعر سينخفض

← في عرض كثير

← فائض

← منخفض

السعر عشان المعروض يروح

* Tax : The demand equation doesn't change. The supply price (سعر العرض) increases by $t.

Ex $\boxed{33}$ / 115 (sec 1.6) :

If the demand for a pair of shoes is given by

$\boxed{D: \; 2p + 5q = 200}$, and the supply :

$\boxed{S: \; p - 2q = 10}$. Is there a market shortage, surplus, or equilibrium at

$p = \$60$.

A: $P = \$60$:

D: $2P + 5q = 200$

$(2 \times 60) + 5q = 200$

$5q = 80 \Rightarrow \boxed{q = 16}$

(Quantity demanded) الكمية المطلوبة

S: $P - 2q = 10$

$60 - 2q = 10$

$-2q = -50 \Rightarrow \boxed{q = \cancel{50}\ 25}$

عشان نحكم إذا في فائض أو عجز:

$\therefore$ $\boxed{S > D \Rightarrow \text{surplus}}$

Sep 25, 19
Wed

- Sec 1.6:
Demand and Supply functions

Ex: 144 find the market equilibrium point for the demand and supply functions:

D: $p = -2q + 320$ — ①

S: $p = 8q + 2$ — ②

A: Eq. point : D = S

$-2q + 320 = 8q + 2$

$10q = 318 \Rightarrow \boxed{q = 31.8}$

نعوضها في ② :

$p = 8q + 2 = (8 \times 31.8) + 2$

$\Rightarrow \boxed{p = 256.4}$

$\Rightarrow$ Eq. Point: $(31.8, 256.4)$.

* Question (خارجي)

At price of \$30 per item a company can supply 2000 units, and at price of \$35 per unit 400 more units can be supplied.

Assuming linear supply, determine the supply function.

تذكير:
السعر: Y
الكمية: X

$\overset{x_1}{2000}, \overset{y_1}{30}$; $\overset{x_2}{2400}, \overset{y_2}{35}$

$(2000, 30)$; $(2400, 35)$

(حكالي بالسؤال عند سعر 35 زادت بمقدار 400 إذن:

$2000 + 40 = 2040$)

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{35-30}{2400-2000} = \frac{5}{400} = 0.0125$$

$\Rightarrow y - y_1 = \text{slope}\,(x - x_1)$

$y - 30 = 0.0125(x - 2000)$

$\Rightarrow$ S: $y = 0.0125x + 5$

$\Rightarrow$ $\boxed{S: p = 0.0125q + 5}$ #

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* Note: Reduced by : قلّت بمقدار

Reduced to : قلّت إلى

Sec 2.3 Quadratic f (الاقترانات التربيعي)

$f(x) = ax^2 + bx + c$

$(f(x) = 0)$

1) By factoring (التحليل إلى العوامل)

2) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (القانون العام)

To graph a quadratic function :

1) x-intercepts $\Rightarrow f(x) = 0$.

2) y-intercepts $\Rightarrow f(0)$ ($\Rightarrow x = 0$).

3) vertex $\Rightarrow \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

max: معامل a $\ominus$

min: معامل a $\oplus$

Ex 2/ 151 : If a firm has the following cost and revenue functions, find the break even point :

$$C(X) = 3600 + 25X + \frac{1}{2}X^2$$
$$R(X) = \left(175 - \frac{1}{2}X\right)X$$

A: $R(X) = 175X - \frac{1}{2}X^2$

$C(X) = 3600 + 25X + \frac{1}{2}X^2$

B.E point : $R(X) = C(X) \longrightarrow (P(X) = 0)$

$$3600 + 25X + \frac{1}{2}X^2 = 175X - \frac{1}{2}X^2$$
$$+\frac{1}{2}X^2 \qquad\qquad +\cancel{\frac{1}{2}}X^2$$

$$3600 + 25X + X^2 = 175X$$
$$3600 - 150X + X^2 = 0$$
$$X^2 - 150X + 3600 = 0$$
$$(X - 120)(X - 30) = 0$$
$$X = 120 \;,\; X = 30$$

⇐ نعوّض قيم X في $R(X)$ أو $C(X)$:

when $X = 120$:

$$R(X) = 175(120) - \frac{1}{2}(120)^2$$
$$R(120) = 19800$$

when $X = 30$;

$$R(30) = 175(30) - \frac{1}{2}(30)^2$$
$$R(30) = 4800$$

#

$\Longrightarrow$ B.E points : $(30, 4800)$, $(120, 19800)$.

Oct 2^{nd}, 19

Sec 2.3: Page 151

$\boxed{6}$ If the profit function $P(x) = -1100 + 120x - x^2$ and limitations on space requires that production is less than 100. Find the break even point.

A: $P(x) = 0$ to find break even.

$-1100 + 120x - x^2 = 0$ → facturing

↳ القانون العام.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-120 \pm \sqrt{120^2 - 4(-1)(-1100)}}{2(-1)}$$

$x = 10$, 110

↳ ترفض

(حكا بالسؤال: ما بنتج أكثر من ١٠٠)

(The limitation is less than 100)

(الحد هو أقل من ١٠٠).

بيجي زين هيلد

v.imp

17 Suppose a company has fixed cost of \$ 28000 and variable cost per unit $\frac{2}{5}x + 222$ where x is the total number of units. Suppose that the selling price is $1250 - \frac{3}{5}x$.

or: demand-price

a) Find the B.E point.

$P(x) = 0$

$\hookrightarrow R(x) - C(x) = 0 \Rightarrow R(x) = C(x)$

But: $R(x) = pX$

$= \left(1250 - \frac{3}{5}x\right)X = 1250X - \frac{3}{5}X^2$

$C(x) = mx + b$

$= \left(\frac{2}{5}x + 222\right)x + 2800$

$= \frac{2}{5}x^2 + 222x + 2800$

$\Rightarrow \frac{2}{5}x^2 + 222x + 2800 = 1250x - \frac{3}{5}x^2$

$x^2 - 1028x + 2800 = 0$

$(x - 1000)(x - 28) = 0$

$x = 1000, \quad x = 28$

B.E : $(1000, \boxed{R(1000)})$	B.E : $(28, R(28))$
$R(1000) =$	$R(28)$
$\boxed{\Rightarrow (\quad\quad)}$	$\boxed{\Rightarrow (\quad\quad)}$

or $C(1000)$

Maximum ⟹ : Vertex   v.imp

b) Find the maximum revenue.

$R(x) = 1250x - \frac{3}{5}x^2$

Vertex: $\left(\frac{-b}{2a}, R\left(\frac{-b}{2a}\right)\right)$

$\frac{-b}{2a} = \frac{-1250}{2 \times \frac{-3}{5}} = 1041.6$

أو باستخدام المشتقة (القدام)

$R(1041.6) = 1250(1041.6) - \frac{3}{5}(1041.6)^2$

$= 651.04$

$\Rightarrow \boxed{(1041.6, 651.04)}$ #

c) Max. Profit.   أعلى ربح

تم ايجاده $P(x) = 1028x - x^2 - 28000$

$x = \frac{-b}{2a} = \frac{-1028}{2(-1)} = 514$

$P(514) = \boxed{236196}$ #

$\Rightarrow (514, 236196)$.

d) What price will maximize the profit?

عدد الوحدات إلي بعملنا أعلى ربح .

$p = 1250 - \frac{3}{5}x \quad = 1250 - \frac{3}{5}(514)$

$= 941.6$

Ex : 32 / 153

S: $2p - q = 50$

D: $pq = 100 + 20q$

Find the equilibrium point.

($\Rightarrow$ D=S)

A: $p = \frac{50+q}{2}$ —— ①

$p = \frac{100+20q}{q}$ —— ②

الأسهل نخلي P لحال. أو نعوض q من ① في ② ...

$\frac{50+q}{2} = \frac{100+20q}{q}$

$200 + 40q = 50q + q^2$

$q^2 + 10q - 200 = 0$

$(q - 10)(q + 20) = 0$

$\boxed{q = 10}$, $q = -20 \rightarrow$ نرفض

(عدد الوحدات ≠ سالب)

$p = \frac{100 + 20(10)}{10}$

$p = 30$ $\Rightarrow$ $\boxed{\text{Eq. point} = (\underset{q}{10}, \underset{p}{30})}$ #

مراجعة

exponential function
$(y=a^x)$
اقتران أُسّي

* $y = a^x$ → اقتران أُسّي

Ex: $f(x) = 2^x$

1) find $2^{100} = 2 \wedge 100$

(ع الآلة الحاسبة).

2) $2^{10} \cdot 2^4 = 2^{10+4} = 2^{14}$

.....

أشياء أساسية ☺

* $y = e^x$ e = Euler number

~~find~~ العدد النيبري

find e^5 :

(ع الآلة الحاسبة):

$e^5 = \boxed{\text{shift}}\ \boxed{\ln}\ \boxed{5} = 148.4$.

* $2^3 = 8 \Rightarrow \log_2 8 = 3$

← اللوغاريتم عكس الأُسّ.

* $\ln x = \log_e x$.

* $\log_2 100 = \dfrac{\ln 100}{\ln 2} = \dfrac{\log 100}{\log 2}$

imp

Oct 9, 19
Wednesday

* 5.1 + 5.2 : Logarithmic and Exponential functions.

(تكملة)

Ex. ① Solve $\ln 5^{3x-4} = \ln 11$

لمّا تكون القوة مجهولة : نأخذ $\boxed{\ln}$ للطرفين

(أو $\boxed{\text{Log}}$).

$$\ln 5^{3x-4} = \ln 11$$

$$3x-4 \ \ln 5 = \ln 11$$

$$3x-4 = \frac{\ln 11}{\ln 5}$$

$$3x-4 = 1.49 \Rightarrow 3x = 5.49$$

$$\boxed{x = 1.83}$$

Ex. ② $e^{5x+4} = 2$

$$\ln e^{5x+4} = \ln 2$$

$$5x+4 \ \underbrace{\cancel{\ln e}}_{1} = \ln 2$$

$$5x+4 = 0.69 \Rightarrow 5x = -3.3$$

$$\boxed{x = -0.66}$$

Ex ③: Solve $\log_4 (x-1) = 2$

$$4^2 = x-1$$

$$16 = x-1$$

$$\boxed{x = 17}$$

(4): $\log_2 x + \log_2 (x-1) = 1$

$\log_2 x(x-1) = 1$

$\log_2 x^2 - x = 1$

$2^1 = x^2 - x \Rightarrow x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\boxed{x = 2}$ or $x = -1$

نرفض (ما أمام لو ≠ سالب) .

(5): $\log_5 4 - \log_5 (x+1) = 0$

$\log_5 \frac{4}{x+1} = 0$

$5^0 = \frac{4}{x+1} \Rightarrow x + 1 = 4$

$\boxed{x = 3}$

أو (أستاذي حمز dr) .

$\log_5 4 = \log_5 (x+1)$

زي ما نلاحظ لو منقدر نحبة :

$4 = x + 1$

$\boxed{x = 3}$

* Sec 6.1 + 6.2 : Simple and compound Interest (فائدة)

(الربح البسيط والمركب)

Simple Interest (I):

$$\boxed{\text{Interest} = prt}$$

; p: present value

r: annual rate % (نسبة الفائدة)

t: # of years . لازم نتعامل بالسنوات

future value (S):

$S = p + I$

$$\boxed{S = p + prt} = p(1 + rt)$$

Ex 6/376 :

\$800 is invested for 5 years at an annual ~~rate~~ simple rate of 14%.

ع عشان نختلف عن ال compound

a) How much interest will be earned?

$I = prt = 800 \times 0.14 \times 5$

$\boxed{I = \$560}$ نسبة الفائدة (مش كم صار معي)

(future value) S ⇐ (نسبة الفائدة) 560 + (800) المبلغ الحالي = كم صار معي ⇐

b) future value -

$S = p + I$

$= 800 + 560 = \boxed{\$1360}$

Ex 8/376 : $1800 is invested for 9 months at an annual Simple interest rate of 15%.

a) How much interest will be earned?

$$t = \frac{9}{12} = \frac{3}{4} = 0.25$$

$$I = prt = (1800)(0.15)\left(\frac{3}{4}\right)$$

$$\boxed{I = \$202.5}$$

b) future value?

$$S = I + p = 1800 + 202.5$$

$$\boxed{S = \$2002.5}$$

Ex 21/ : If $5000 is invested at 8% simple, how long does it take to be worth $9000 → 9000 علشان يصبح

$$S = p + prt$$

$$9000 = 5000(1 + (0.08\,t))$$

$$9000 = 5000 + 400t$$

$$4000 = 400t$$

$$\boxed{t = 10 \text{ years}}$$

* Sec 6.2 : Compound Interest.

بضيفوا فائدة (نسبة معينة) ع المبلغ اللي صار معي مش ع المبلغ الأصلي
← هاد الفرق بينه وبين ال Simple Interest

* compounded annually : (مرة في السنة)

رفعناها

$$S = p(1 + r)^{t}$$

* Compounded semiannually : (مرتين في السنة)

$$S = p\left(1 + \frac{r}{2}\right)^{2t}$$

* Compounded quarterly : (٤ مرات في السنة)

$$S = p\left(1 + \frac{r}{4}\right)^{4t}$$

* compounded monthly : (شهرياً)

$$S = p\left(1 + \frac{r}{12}\right)^{12t}$$

* Compounded continuously : مدى الحياة (كل ثانية)
← ∞

$$S = p\,e^{rt}$$

Examples page 389:

$\boxed{10}$ What is the future value if \$8600 is invested for 8 years at 10% compounded semiannually

$$S = P\left(1+\frac{r}{2}\right)^{2t} = 8600\left(1+\frac{0.1}{2}\right)^{16}$$

$$S = 8600\,(1.05)^{16}$$
$$= \$18772.7$$

$\boxed{16}$ What present value amounts