22 Find $\frac{dy}{dx}$ if $y = \ln\left(\frac{3x+2}{x^2-5}\right)^{1/4}$

$$y = \frac{1}{4}\ln\left(\frac{3x+2}{x^2-5}\right) = \frac{1}{4}\left(\ln(3x+2) - \ln(x^2-5)\right)$$

$$y' = \frac{1}{4}\left(\frac{3}{3x+2} - \frac{2x}{x^2-5}\right).$$

---

32 $y = (\ln x)^{-1}$

انتباه :

$\ln x^{-1} = -1 . \ln x$

بس هون : ال (1-) لكل ال ln ← بزبطش

ننزل السالب واحد قبلهم :

اجت بامتحان الفاينل الصيفي الفايت

$$\Rightarrow y' = -1(\ln x)^{-2} . \frac{1}{x} = \frac{-1}{x(\ln x)^2}$$

Find the relative maxima and minima :

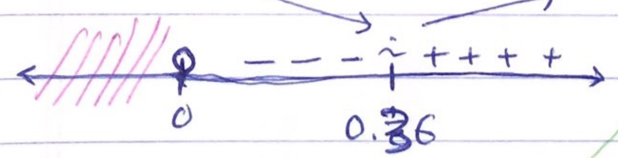
39 $y = x . \ln x$

$$\rightarrow y' = (x) . \left(\frac{1}{x}\right) + (\ln x)(1) = 1 + \ln x$$

⊛ $y' = 0 \rightarrow \ln x = -1$

$\frac{1}{e} = x \quad (e^{-1} = x).$

$\boxed{x = 0.36}$



⊛ y' is undifined at $x = 0$.

(أي جوّا ال ln ≠ صفر)

ماننسى يا غارا

∴ min at $x = e^{-1}$

$$\Rightarrow (e^{-1}, e^{-1}\ln e^{-1}) = (e^{-1}, -e^{-1})$$

Sec 11.2 : Derivatives of exponential function :

مهم جداً

① $y = a^{f(x)}$ $\qquad$ (a = constant)

$\rightarrow y' = a^{f(x)} \cdot f'(x) \cdot \ln a$

---

8 / 714 : $y = e^{x^2-1}$

$y' = e^{x^2-1} \cdot 2x \cdot \ln e$

$= \boxed{2xe^{x^2-1}}$

---

12 $\quad y = e^{\sqrt{x^2-9}}$

$y' = e^{\sqrt{x^2-9}} \cdot \dfrac{\cancel{2}x}{\cancel{2}\sqrt{x^2-9}} \cdot \ln e$

$\Rightarrow y' = e^{\sqrt{x^2-9}} \cdot \dfrac{x}{\sqrt{x^2-9}}$

---

14 $\quad y = e^3 + e^{\ln x} = e^3 + x$

$y' = 0 + 1$

(لأن e^3 عدد)

$\boxed{y' = 1}$

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$\boxed{34}$ $y = 5^{2x-1}$ $\Rightarrow$ ~~$y' = (2x-1)(5^{2x-2})$~~ X

$y' = (5)^{2x-1} \cdot (2) \cdot \ln 5$

$\boxed{36}$ What is the slope of the line tangent to $y = \dfrac{e^{-x}}{1+e^{-x}}\Big|_{x=0}$

$1 * \ln e$

$\Rightarrow y' = \dfrac{(1+e^{-x})\cdot(-e^{x}) - (e^{-x}\cdot -e^{-x})}{(1+e^{-x})^2}$

$y'\Big|_{x=0} = \dfrac{(2)\cdot(-1) - (1)(-1)}{(1+1)^2} = \dfrac{-2+1}{4} = -\dfrac{1}{4}$

b) write the equation of the tangent line at $x=0$?

$y - y_1 = m(x - x_1)$

$y - \frac{1}{2} = \frac{1}{4}(x - 0)$ $\qquad y(0) = \frac{1}{2}$

$\boxed{y = \frac{1}{4}x - \frac{1}{2}}$ #

- Sec 11.3 : Implicit differentiation : الاشتقاق الضمني

(⇐ بكونوا x, y ع نفس الطرف بالمعادلة)

$\boxed{6}$ $x^2 + 5xy + 4 = 0$ at $(1, -1)$

Find $\dfrac{dy}{dx}$ (y')

Find

$2x + 5xy' + 5y = 0$ $\longrightarrow$

Sec 11.5

بيجي عليه أكل إشي 20 علامة

$2x + 5x\boxed{y'} + 5y = 0 \qquad (1, -1)$

$2 + 5y' - 5 = 0 \Rightarrow 5y' = 3$

$\boxed{y' = \frac{3}{5}}$ #

[13] If $xy^2 - y^3 = 1$, find $\frac{dy}{dx}$

$y^2 + 2y\frac{dy}{dx}(x) - 3y^2 \cdot \frac{dy}{dx} = 0$

تنسيش تشتقي ال 1

$\cancel{y^2 + \frac{dy}{dx}(2y^3 - 3y) = 1}$

$\cancel{-y\frac{dy}{dx} = 1 - y^2}$

بتقل إلي ما معهم y' عند ال 1 :

$2xy\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0 - y^2$

$\frac{dy}{dx}(2xy - 3y^2) = -y^2$

$\boxed{\frac{dy}{dx} = \frac{-y^2}{2xy - 3y^2}}$ #

[37] If $x \cdot e^y = 6$ find $\frac{dy}{dx}$?

$x \cdot e^y \cdot \frac{dy}{dx} + e^y = 0 \longrightarrow$

$$Xe^{y}\frac{dy}{dx} = -e^{y} \Rightarrow \frac{dy}{dx} = \frac{-e^{y}}{Xe^{y}} = \boxed{\frac{-1}{X}}$$

48 At what points does the curve defined by $x^2 + 4y^2 - 4 = 0$ have:

a) Horizantal tangent. $\Rightarrow$ slope $= 0$ $\Rightarrow \frac{dy}{dx} = 0$.

بالاشتقاق إضمني :

$$2x + 8y \cdot \frac{dy}{dx} = 0$$

$$8y \cdot \frac{dy}{dx} = -2x \Rightarrow \frac{dy}{dx} = \frac{-x}{4y}$$

$$0 = \frac{-x}{4y} \Rightarrow \boxed{x = 0}$$

$$0 + 4y^2 - 4 = 0$$

$$y^2 = 1 \Rightarrow y = \pm 1$$

$\Rightarrow$ Points: $(0,1)$, $(0,-1)$

دائماً

slope = undefined

b) Vertical tangent? $\Rightarrow$ مماس عمودي $\Rightarrow$: slope = undefined

(لأن عند $y=0$ ← المقام ≠ صفر) الميل غير معرّف

$$\frac{dy}{dx} = \frac{-\cancel{x}}{4y} \Rightarrow 4y = 0 \Rightarrow y = 0$$

$$x^2 + 0 - 4 = 0$$

$$x^2 = 4 \Rightarrow \boxed{x = \pm 2}$$

$\Rightarrow$ points $(2,0)$, $(-2,0)$.

مهم جداً

Sec 11.5 : Applications in Economics :

The elasticity of demand

$$\boxed{\eta \text{ or } E_d = \frac{-P}{q} \cdot \frac{dq}{dP}}$$; P : price

q : quantity

$\frac{dq}{dP} = q'$

$\Rightarrow \eta$
- ① $\eta > 1$: The demand is elastic
- ② $\eta < 1$: The demand is inelastic
- ③ $\eta = 1$: The demand is unitary elastic.

ال demand بتناسب بعلاقة عكسية .

$\Rightarrow$ ①: $(\eta > 1) \Rightarrow$ If price increases, the revenue decreases. ↑↓

②: $(\eta < 1) \Rightarrow$ If price increases, the revenue increases. ↑↑

③: $(\eta = 1) \Rightarrow$ An increase in price, the revenue does not change.

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* Sec 11.5 : Applications in Economics :

$$\eta \text{ "}E_d\text{" } = \frac{-p}{q} \frac{dq}{dp}$$ "E_d" دايماً منشتق ضمني بال

$\boxed{6}$ The demand $2p^2 q = 10000 + 9000\,p^2$

a- Find the elasticity when $p = 50$ and $q = 4502$

(إيتا) $\leftarrow \eta = \frac{-p}{q} \frac{dq}{dp}$

$$2p^2 q = 10000 + 9000\,p^2$$

$$4pq + 2p^2 \frac{dq}{dp} = 18000\,p$$

$p = 50$, $q = 4502$

$$\rightarrow 4(50)(4502) + 2(50^2) \frac{dq}{dp} = 18000(50)$$

$$\Rightarrow \frac{dq}{dp} = -0.08$$

$$\Rightarrow \eta = \frac{-50}{4502} \cdot -0.08 = 0.00088$$

$\eta < 1$ (The demand is inelastic).

b- Tell what type of elasticity $\Rightarrow$ inelastic.

c- How what price increase affect revenue.

$\Rightarrow$ Revenue will increase. علاقة طردية

مهمة لازم نكتبها (علاقة)

$\frac{dR}{dp} > 0$ طردية

$\boxed{\frac{dR}{dp} < 0}$: بال elastic

عكسية

$\boxed{11}$ $p = 120\sqrt[3]{125-q} = 120\,(125-q)^{1/3}$

$$\eta = \frac{-p}{q}\,\frac{dq}{dp}$$

a) $\Rightarrow$ $1 = 120 \cdot \frac{1}{3}(125-q)^{-2/3} \cdot \frac{-dq}{dp}$

$$1 = \frac{-40}{(125-q)^{2/3}} \cdot \frac{dq}{dp}$$

$$\boxed{\frac{dq}{dp} = \frac{-\sqrt[3]{(125-q)^2}}{40}} \ \#$$

مش معطيني أعوّض قيم p و q

$$\eta = \frac{-p}{q} \cdot \frac{-(125-q)^{2/3}}{40}$$

$$\eta = \boxed{\frac{p\,(125-q)^{2/3}}{40\,q}} = \frac{120\sqrt[3]{125-q}\cdot(125-q)^{2/3}}{40\,q}$$

هيك حل

بسأل إذا حكى:

Find "E_d" as a function of q

يعني الجواب لازم يكون بدلالة q

$$\Rightarrow \eta = \frac{3\,(125-q)^{1/3}\cdot(125-q)^{2/3}}{q}$$

$$\boxed{\eta = \frac{3\,(125-q)}{q}} \ \#$$

(*) b) Find where the revenue is maximized ?

ال Rev. بكون max. لما يكون ال Unitary elastic

$(\eta = 1)$, $\frac{dR}{dP} = 0$

$\eta = 1$

$\frac{3(125-q)}{q} = 1$

$q = 3(125) - 3q$

$4q = 375 \Rightarrow q = 93.75$ units

$P = 120\sqrt[3]{125 - 93.75} = \boxed{377.97}$

---

* Ex : If $E_d = \frac{P}{900-P}$; $0 < P < 900$,

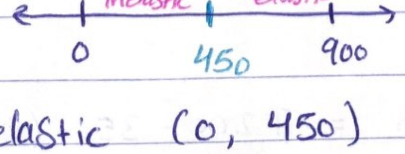
a- Find the price at which the demand is unitary elastic , elastic , inelastic .

أول إشي منبلش في ال Unitary :

$E_d = \frac{P}{900-P} = 1$

$\Rightarrow 900 - P = P$

$2P = 900 \Rightarrow P = 450$



inelastic $(0, 450)$ → فترات مفتوحة

elastic $(450, 900)$ → " "

$\boxed{26}$ If the monthly demand $p = 7230 - 5q^2$ and S: $p = 30 + 30q^2$. what tax per item will maximize the total revenue.

مشكلة حل :

tax rev. per item → $D = S + t$ S_{new}

1) $t = D - S$ 	إذا ما كان بده اكتبوا الاقتران ← بس هاد القانون 	$t = D - S$

$t = 7230 - 5q^2 - 30 - 30q^2$

$$\boxed{\therefore t = 7200 - 35q^2}$$

2) Write the total tax revenue.

$T = t \cdot q = (7200 - 35q^2)\, q$

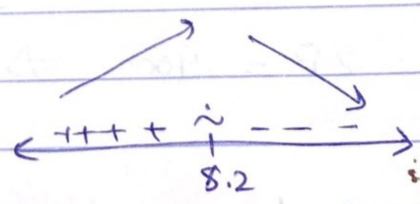
$T = 7200q - 35q^3$

3) Max. : $T' = 7200 - 105q^2 = 0$

$7200 = 105q^2$

$q^2 = 68.5$

$$\boxed{q = 8.2}$$



max at ~~q~~ $q = 8.2$

max. tax rev. per item $= 7200 - 35 - (8.2)^2$

$= 4800.45$

max. tax rev. : لو حكا

← معوّض بـ T

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Chapter 12

* See $\boxed{12.1}$: Indefinite integrals : التكامل الغير محدود

$$f(x) \xrightarrow{d/dx} f'(x)$$
$$\int$$

Rules

1- $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$

2- $\int K . dx = Kx + c$.

3- $\int K \, u(x) . dx = k \int u(x) . dx$.

C ≡ Constant of integral.
(ثابت التكامل)

* Example $\boxed{9}$: $\int 3^3 + x^{13} \, dx$

$$= 27x + \frac{x^{14}}{14} + c .$$

$\boxed{18}$ $\int \left(17 + \sqrt{x^3}\right) . dx = 17x + \int x^{3/2} \, dx$

$$= 17x + \frac{2}{5} x^{5/2} + c .$$

$\boxed{20}$ $\int 3 \sqrt[3]{x^2} . dx = 3 \int x^{2/3} \, dx$

$$= \frac{3.3}{5} x^{5/3} = \frac{9}{5} x^{5/3} .$$

$\boxed{28}$ $\int \left(3x^8 + \frac{4}{x^8} - \frac{5}{\sqrt[5]{x}}\right) \cdot dx$

$= \frac{x^9}{3} + \int 4x^{-8}\,dx - 5\int x^{1/5}\,dx$

$= \frac{x^3}{3} + \frac{-4}{7x^7} - \frac{25}{6} x^{6/5} + C.$

$\boxed{31}$ $\int \frac{x+1}{x^3} \cdot dx = \int x^{-2}\,dx + \int x^{-3}\,dx$

$= \frac{-1}{x} + \frac{-2}{x^2} + C$

* See $\boxed{12.2}$: The power rule :

$\int 2(2x+1)^2 \cdot dx$

① مشتقها = 2

② 2 موجودة برا ← آه، ما بضيف

$x \leftarrow$: بضيف من عندي

③ بكامل عادي : $\frac{(2x+1)^3}{3}$

$= \frac{(2x+1)^3}{3} + C.$

مناخد القوس إلي عليه قوة ومنشوف إذا مشتقتو برا.

دائماً أول خطوة

$\boxed{8}$ $\int \underbrace{(4x^2-3x)}_{\downarrow\ 8x-3}{}^4 (8x-3)\,dx.$

$= \frac{1}{5}(4x^2-3x)^5 + C$

$\boxed{16}$ $\frac{1}{2}\int 2(2x^2 - x)\,(\underbrace{x^4 + x^2}_{4x^3 - 2x\ =\ 2(2x^2 - x)})^6$

مش موجودة : بضيفها.

موجودة

$= \frac{1}{2}\,\frac{(x^4 + x^2)^7}{7}$.

$\boxed{32}$ $\int \frac{x^2+1}{\sqrt{x^3+3x+10}}\,.dx = \frac{1}{3}\int 3(x^2+1)(\underbrace{x^3+3x+10}_{3x^2+3})^{-1/2}$

$= 3(x^2+1)$

موجودة

$= \frac{2}{3}\,(x^3+3x+10)^{1/2}$

لو بالمقام ما في جذر (يعني القوة 1)

← ما بصير نطلع ع البسط.

بتصير لوغاريتم ← لأن البسط مشتقة المقام ...

$\boxed{34}$ $\int g(x)\,.dx = (5x^2+2)^6 + C$

Find $g(x)$

ما بشيل التكاملات إلا الدالاح ☺ ← : باشتقاق الطرفين

$\Rightarrow g(x) = 6\,(5x^2+2)^5 \cdot 10x$

$g(x) = 60x\,(5x^2+2)$

* Sec $\boxed{12.3}$:

1- $\int \frac{1}{x}\,dx = \ln|x| + C$

$\boxed{17}$ $\displaystyle\int \frac{3x^2}{x^3+4}\,.dx = \ln|x^3+4| + C$

لأن البسط مشتقة المقام .

$\boxed{27}$ $\displaystyle\frac{1}{3}\int \frac{3\cancel{Z^2}+1}{Z^3+3Z+17}\,.dZ.$

قوة المقام = ١

!! ما بصير أطلّع البسط .

$3Z^2+3 = 3\,(Z^2+1)$

$= \frac{1}{3}\ln|Z^3+3Z+17| + C$

Ex. $\displaystyle\int e^x\,dx = e^x \cdot 1 \cdot \ln e + C$

$= e^x + C.$

$\boxed{10}$ $\displaystyle\frac{1}{4}\int 4x\,e^{2x^2}\,.dx = \frac{1}{4}\,e^{2x^2} + K$

$4x$ موجودة

زي ماهي

$\boxed{14}$ $\displaystyle\int \frac{x^3}{e^{4x^4}}\,.dx$

بس نشوف e ← منشوف مشتقة القوة إلها و منشوّر عليها برّا.

$\displaystyle= \frac{1}{-16}\int -16x^3\,e^{-4e^4}\,.dx$

$-16\,e^3$

موجودة

$= \frac{-1}{16}\,e^{-4x^4} + C.$

$= \frac{1}{16\,e^{4x^4}} + C.$

* Sec $\boxed{12.4}$: Application ....

* Optimal level of production $\Rightarrow$: max. profit.

$\Rightarrow \overline{MC} = \overline{MR}$

* $\overline{MR} = R'(x) \Rightarrow$ Find $R(x) \Rightarrow \int R'(x)\,dx$

هاي المعلومة بيجي عليها كثير أسئلة

$\therefore$ to find C : $R(0) = 0$.

$C \neq$ صفر

* $\overline{MC} = C'(x) \Rightarrow C(x) = \int C'(x)\,dx$

$\therefore$ to find C : $C(x) =$ constant (مش شرط صفر)

$\Rightarrow$ any additional information is available ↓

$\Rightarrow$ fixed cost $\neq C$ إلي هي ال fixed cost

بتجي بالدوائر .

But: C depends on fixed cost. ✓

$\boxed{8}$ $\overline{MC} = 6X + 60$, $\overline{MR} = 180 - 2X$

its total cost of producing 10 items is \$1000

a- Find the optimal level of production.

$\Rightarrow \overline{MC} = \overline{MR}$

$6X + 60 = 180 - 2X$

$8X = 120$

$X = 15$ units.

→

b- Find the profit function :

$$P(x) = R(x) - C(x).$$

$$R(x) = \int (180 - 2x).\,dx \quad = 180x - x^2 + c$$

$$R(0) = 0 \Rightarrow \boxed{c = 0}$$

$$\therefore \boxed{R(x) = 180x - x^2}$$

كل واحد منهما لحاله أحسن
مش أطرح $\overline{MC}$ من $\overline{MR}$
وأكامل عليك بيصير في
مشكلة بإيجاد ثابت التكامل.

$$C(x) = \int (6x + 60)\,dx$$

$$= 3x^2 + 60x + c$$

$$C(10) = 1000 \Rightarrow 300 + 600 + c = 1000$$

$$c = 1000 - 900$$

$$\boxed{c = 100}$$

$$\Rightarrow \boxed{C(x) = 3x^2 + 60x + 100}$$

$$\therefore P(x) = 180x - x^2 - 3x^2 - 60x - 100$$

$$\boxed{P(x) = -4x^2 + 120x - 100}$$

#

Saturday
Dec 14. 2019

* National consumption function $C(y)$ $\boxed{MPC}$
⇒ لما نشتق ⇒ marginal propensity to consume
$C'(y)$ or $\frac{dc}{dy}$

* National saving function $S(y)$ {y = income
⇒ $S'(y)$ or $\frac{ds}{dy}$ ⇒ marginal propensity to save $\boxed{MPS}$

* ① $C(y) + S(y) = y$
* ② $C'(y) + S'(y) = 1$.
⇒ $\frac{dc}{dy} + \frac{ds}{dy} = 1$

17 If consumption is \$8 billion when income is \$0, and if the marginal propensity to consume $\frac{dc}{dy} = 0.3 + \frac{0.2}{\sqrt{y}}\, dy$. Find the national consumption function.

$$C(y) = \int \left(0.3 + 0.2\, y^{-1/2}\right) . dy$$

$$= 0.3\, y + 0.4 \sqrt{y} + K$$

but $C(0) = 8$
⇒ $K = 8$

$$\therefore \boxed{C(y) = 0.3y + 0.4\sqrt{y} + 8} \quad \#$$

$C(0) = 3$

$\boxed{26}$ If consumption is \$3 billion when deisosable income is \$0, and if the marginal propensity to save is $\frac{ds}{dy} = 0.2 + e^{-1.5y}$. Find the consumption function.

$$\Rightarrow \frac{ds}{dy} + \frac{dc}{dy} = 1 \quad \Rightarrow \frac{dc}{dy} = 1 - \frac{ds}{dy}$$

$$c'(y) = 0.8 - e^{-1.5y}$$

$$\Rightarrow c(y) = \int (0.8 - e^{-1.5y})\,dy = 0.8y + \frac{e^{-1.5y}}{1.5} + k$$

but: $C(0) = \frac{e}{1.5} + k = 3$

$$\Rightarrow k = 2.33$$

$$\Rightarrow \boxed{C(y) = 0.8y + \frac{e^{-1.5y}}{1.5} + 2.33} \quad \#$$

$\boxed{Q\ 43}$ If $\overline{MR} = 6e^{0.01x}$; find the revenue function

$$R(x) = \int 6e^{0.01x}\,dx$$

$$= 600\,e^{0.01x} + k$$

But $R(0) = 0$

$\Rightarrow k = -600$

$$\Rightarrow \boxed{R(x) = 600\,e^{0.01x} - 600} \quad \#$$

* Sec $\boxed{13.2}$: The definite integrals : التكامل المحدود

$\boxed{10}$ $$\int_{-1}^{4} (6x-9)\,dx = 3x^2 \Big|_{-1}^{4} - 9(4-1)$$

$$\left[\text{or} : = (3x^2 - 9x)\Big|_{-1}^{4} \right]$$

$$= \{(48 - 36) - (3+9)\}$$

$$= 12 - 12 = 0.$$

$$= 48 - 3 - 45$$

$$= 0$$

$\boxed{30}$ $$\int_0^1 \frac{3x^3}{4x^4+9}\,dx = \frac{3}{16}\int_0^1 \frac{16x^3}{4x^4+9}\,.dx$$

$$= \frac{3}{16} \ln|4x^2+9| \Big|_0^1$$ (لأنّ البسط مشتقة المقام)

$$= \frac{3}{16} \ln 13 - \ln 9 = \frac{3}{16} \ln \frac{13}{9}$$

---

* Examples : If $\int_1^3 f(x)\,dx = -4$

and $\int_1^2 f(x)\,dx = 2$. Find $\int_2^3 f(x)\,dx$.

$$\Rightarrow \int_2^3 f(x)\,.dx = \int_2^1 f(x)\,.dx + \int_1^3 f(x)\,.dx$$

$$= -2 + -4 = \boxed{-6}$$

ص = صفر

* Areas :

لما يعطيني مساحة بين اقتران ومحور س ، مساحة معينة (حل ع طول) .

42 Find the area between $y = x^2 + 3x + 2$ and the x-axis from $x = -1$ to $x = 3$

$y = 0$: (مش حتر)

$$\text{Area} = \int_{-1}^{3} (x^2 + 3x + 2)\, dx$$

$$= \left(\frac{x^3}{3} + \frac{3x^2}{2} + 2x\right)\Big|_{-1}^{3} = \boxed{}$$

44 Find the area between $y = e^{-x}$ from $x = -1$ to $x = 1$

$$A = \int_{-1}^{1} e^{-x} \, dx = -e^{-x} \Big|_{-1}^{1} = \frac{-1}{e} - -e$$

$$= e - e^{-1}$$

مهم

***** The average value of a function : $f(x)$, $[a, b]$

مش rate (هاد وعي) .

$$\Rightarrow \frac{1}{b-a} \int_a^b f(x) \, dx$$

38 $f(x) = \frac{1}{2}x^3 + 1$ over $[-2, 0]$. Find the average value of $f(x)$.

$$\Rightarrow \frac{1}{0 - -2} \int_{-2}^{0} \left(\frac{1}{2}x^3 + 1\right) dx = \frac{1}{2}\left[\left(\frac{1}{8}x^4 \Big|_{-2}^{0}\right) + 2\right]$$

$$1 + 1 = \boxed{\cancel{2}}$$

بطلع صفر

$\boxed{37}$ $C(x) = x^2 + 400x + 2000$, Find the average value of the cost over $[0, 1000]$.

$$\Rightarrow \frac{1}{1000-0} \int_0^{1000} (x^2 + 400x + 2000)\, dx$$

$$= \frac{1}{1000} \left[\left(\frac{x^3}{3} + 200x^2 + 2000x \right) \Big|_0^{1000} \right]$$

$$= \frac{1}{1000} \left[\frac{1000^3}{3} + 200(1000)^2 + 2000(1000) - 0 \right]$$

.... etc.

$\boxed{13}$ $f(x) = x^2 + 1$, $y\ f(x) = -x^2$; $x = 0$ to $x = 2$

لازم نوجد نقاط التقاطع لما يكون في اقترانين ⇐ منساويهم في بعض

$$\Rightarrow x^2 + 1 = -x^2 \Rightarrow 2x^2 + 1 = 0$$

$$x^2 \neq -\frac{1}{2}$$

أصلاً من الرسمة ⇐ ما بتقاطعوا .

$$\Rightarrow \int_0^2 (x^2 + 1 + x^2)\, dx \qquad \boxed{\left(\int_0^2 \left((x^2+1) - (-x^2) \right) dx \right)}$$

$$= \int_0^2 (2x^2 + 1)\, dx$$

$$= \frac{2}{3} x^2 \Big|_0^2 + \underbrace{2}_{1(2-0)}$$

$$= \frac{2}{3}(4 - 0) + 2 = \frac{8}{3} + \frac{6}{3} = \boxed{\frac{14}{3}}$$

* Sec $\boxed{13.4}$:

① Consumes Surplus : D: $\boxed{}$

S: $\boxed{}$

eq. point $\Rightarrow (x_1, p_1)$

$$\Rightarrow CS = \int_0^{x_1} D \, dx - x_1 p_1$$

② producer surplus :

$$PS = x_1 p_1 - \int_0^{x_1} S \, dx$$

* Ex : If the demand and supply functions are $p = 49 - x^2$ and $p = 4x + 4$:

a) Find the consumer surplus.

مربع ← eq. point

$D = S$

$x^2 + 4x - 45 = 0$

$\boxed{x = +5}$, $\boxed{x = -9}$

تُرفض

$x_1 = 5$

$\therefore p_1 = 24 \Rightarrow$ eq. point $(5, 24)$.

منعوضها في أي اقتران

$$\Rightarrow CS = \int_0^{5} (49 - x^2) \, dx - \underbrace{120}_{x_1 p_1} = \dots \text{ etc}$$