Ch9: Derivatives (المشتقات)

- Sec 9.1 : limits :

$f(x)$

$\lim_{x\to a} f(x) =$

$f(a)$ = The value of $f(x)$ at $x = a$.

---

* $\lim_{x\to 1} f(x)$ →
- $\boxed{C}$
- $\boxed{\frac{C}{0}}$: DNE (does not exist)
- $\boxed{\frac{0}{0}}$ ⇒ factory

تحليل

ثم ⇓

بطلع جواب

Ex: 22/553 :

$\lim_{x\to -4} \frac{x^2-16}{x+4}$ ① بالتعويض : $x = -4$

النهاية = صفر / صفر

⇐ نلجأ إلى التحليل

$\Rightarrow \lim_{x\to -4} \frac{(x-4)\cancel{(x+4)}}{\cancel{(x+4)}} = -4-4 = \boxed{-8}$.

\#

28/553 : $\lim_{x\to 10} \frac{x^2-8x-20}{x^2-11x+10}$

① بالتعويض ينتج $\frac{0}{0}$ ⇐ ∈ تحليل :

$= \lim_{x\to 10} \frac{(x+2)\cancel{(x-10)}}{\cancel{(x-10)}(x-1)} = \lim_{x\to 10} \frac{x+2}{x-1} = \frac{12}{9} = \boxed{\frac{4}{3}}$

\#

$$f(x) = \begin{cases} x^2 - 4 & ; \quad x < -1 \\ 1-4x & , \quad x \geq -1 \end{cases}$$

$\lim_{x \to -1} f(x)$??

A: $\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} (1-4x) = 5$

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} x^2 - 4 = -3$

$\Rightarrow \lim_{x \to -1^+} f(x) \neq \lim_{x \to -1^-} f(x)$

$\therefore \lim_{x \to -1} f(x)$ DNE .

$\boxed{14}$ (لنفس القسم)

$$f(x) = \begin{cases} 4 - x^2 & , \quad x \leq -2 \\ x^2 + 2x & , \quad x > -2 \end{cases}$$

a) find $\lim_{x \to -2} f(x)$

$\lim_{x \to -2^-} 4 - x^2 = 0$, $\lim_{x \to -2^+} x^2 + 2x = 0$

$\therefore \lim_{x \to -2} f(x) = 0$.

b) find $f(-2) = 4 - (-2)^2 = 0$.

موجودة عند المساواة .

Alawi

32 find $\lim_{x \to 2} f(x)$; $f(x) = \begin{cases} \dfrac{x^3-4}{x-3}, & x \leq 2 \\ \dfrac{3-x^2}{x}, & x > 2 \end{cases}$

نقطة تفرع ⇐ يمين ويسار .

A: $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \dfrac{3-x^2}{x} = \boxed{-\dfrac{1}{2}}$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \dfrac{x^3-4}{x-3} = \dfrac{4}{-1} = \boxed{-4}$

$\Rightarrow \lim_{x \to 2^+} f(x) \neq \lim_{x \to 2^-} f(x)$

$\therefore \lim_{x \to 2} f(x)$ DNE .

في الاقتران → $f(2)$
لك عندو مساواة
$\left(\dfrac{x^3-4}{x-3} \right)$

---

52 If $\lim_{x \to 5} [f(x) - g(x)] = 8$, and $\lim_{x \to 5} g(x) = 2$,

find : a) $\lim_{x \to 5} f(x)$

A: $\lim_{x \to 5} [f(x) - g(x)] = \lim_{x \to 5} f(x) - \lim_{x \to 5} g(x) = 8$

$\lim_{x \to 5} f(x) = 8 + 2 = \boxed{10}$ #

b) $\lim_{x \to 5} \left[(g(x))^2 - f(x) \right]$

$= \lim_{x \to 5} (g(x))^2 - \lim_{x \to 5} f(x)$

$= 4 - 10 = \boxed{-6}$ #

$\boxed{\text{Sec 9.2}}$ Continuous functions, limits at infinity :

الاقترانات المتصلة .

* $f(x)$ is continuous at $x=a$ if

$$\lim_{x\to a} f(x) = f(a)$$

النهاية = ~~[illegible]~~ الصورة .

① Polynomial: (كثير حدود)

$$f(x) = x^2 + 3x - \frac{4}{5}x^3 + 1$$

$\Rightarrow$ continuous .

اقترانات (كثير حدود) ⇐ دائماً متصلة .

* $f(x) = \sqrt{x} - 1$ not polynomial .

② Rational functions (نسبي)

$$f(x) = \frac{x^2 - 3x + 1}{x^2 - 1}$$

نجد أصفار المقام

* لأن الاقتران النسبي متصل عدا عند أصفار المقام .

$x^2 - 1 = 0 \Rightarrow \boxed{x = \pm 1}$

$\Rightarrow$ continuous except at $x = -1, 1$.

sec 9.4 ← متصل غير عن قابل للاشتقاق

③ $f(x) = \begin{cases} x^2 - 3x, & x < 1 \\ -2x, & x \geq 1 \end{cases}$ (متعدد القاعدة)

القيمة المطلقة ↓ دائماً متصل

If $f(x)$ continuous at $x = 1$?

إذا كانت النهاية = الصورة ⇐ ∴ متصل.

A: $\cancel{\lim_{x \to 1^+} f(x)}$ $f(1) = -2$

$\lim_{x \to 1^+} f(x) = -2$, $\lim_{x \to 1^-} f(x) = -2$

$\therefore \lim_{x \to 1} f(x) = -2$ (Exist)

$f(1) \stackrel{??}{=} \lim_{x \to 1} f(x)$ ✓ yes.

$\therefore$ $f(x)$ is continuous at $x = 1$

Oct 23rd. 19
Wednesday

Sec 9.2 : Continuous functions and limits at infinity.

$\boxed{8}$ $f(x) = \begin{cases} x^2+1, & x \leq 1 \\ 2x^2-1, & x > 1 \end{cases}$

Is $f(x)$ cont. at $x=1$?

A: $\lim_{x\to 1^+} f(x) = 1$ $\quad$ $\lim_{x\to 1^-} f(x) = 2$ $\quad$ $f(1) = 2$

$\Rightarrow$ لازم هدول ال ٣ متساويين
عشان نحكي إنه ق(س) متصل عند س=١

$\Rightarrow \therefore f(x)$ isn't cont. at $x=1$

$\boxed{20}$ $f(x) = \begin{cases} x^2+4, & x \neq 4 \\ 8, & x = 4 \end{cases}$

Is $f(x)$ continuous at $x=4$?

A: $\lim_{x\to 4} f(x)$, $f(4)$

فش يمين ويسار عشانا في الاقتران الثاني

في $\neq$ عشان بدنا اللي حواليها.

$= 20$ $\quad$ 8

$\neq$ فش أكبر أو أصغر

$\therefore f(x)$ is discont. at $x=4$

* $\lim\limits_{x\to\infty} \frac{f(x)}{g(x)}$

① If the degree of $f(x)$ is less than the degree of $g(x)$, then the $\lim = 0$

② If the degree of $f(x)$ = the degree of $g(x)$, then $\text{Lim} = \frac{\text{معامل أكبر قوة في البسط}}{\text{معامل أكبر قوة في المقام}}$

③ If the degree of $f(x) >$ The degree of $g(x)$, then the $\lim = \pm\infty$.

Ex $\boxed{30}$ / 546 :

تمارين على ما سبق

$$\text{Lim} \frac{4x^2+5x}{x^2-4x} = \frac{4}{1} = 4$$

- Ex: $\lim\limits_{x\to\infty} \frac{1-2x}{4x-5} = \frac{-2}{4} = -\frac{1}{2}$.

$\boxed{25}$ $\lim\limits_{x\to\infty} \frac{3}{x-11} = 0$

$\boxed{32}$ $\lim\limits_{x\to-\infty} \frac{5x^3-8}{4x^2+5x} = -\infty$

Ex: $\lim\limits_{x\to\infty} \frac{1-4x^5}{x-3} = -\infty$ $\longrightarrow$ $\frac{-4x^5}{x}$

$= -4x^4$

$= -4(-\infty)^4$

$= -\infty$

* Sec 9.3 : Rates of changes and derivative

$f(x) \longrightarrow [a, b]$.

$\Rightarrow$ average rate of change (متوسط التغير)

$$= \frac{f(b) - f(a)}{b - a}$$

rate of change = المشتقة = معدل التغير

average " " " = متوسط التغير

imp

$\boxed{39}$ Suppose total cost (in \$) is given by

$C(X) = 0.0001X^3 + 0.005X^2 + 28X + 3000$

find the average rate of change when production changes from:

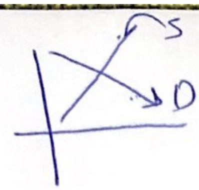
a) 100 to 300

A: average rate of change $= \dfrac{C(300) - C(100)}{300 - 100}$

$$= \frac{14450 - 5950}{200}$$

$= 43$

$\boxed{42}$ $\dfrac{R(20) - R(10)}{20 - 10}$



$q_D > q_S$

$\Downarrow$

Shortage

★ See 9.4 : Derivative formulas.

Rate of change = derivative = marginal Cost / Profit / Revenue

= slope of the tanget

★ Page 588 :

$\boxed{13}$ Find the derivatives,

$$g(x) = 2x^{12} - 5x^6 + 9x^4 + x - 5$$

A: $g'(x) = 24x^{11} - 30x^5 + 36x^3 + 1$

الشرح دايمًا بيجي

$\boxed{30}$ write the equation of the tangent line

$\downarrow$ $f'(x)$

$$f(x) = \frac{x^3}{3} - \frac{3}{x^3} \quad \text{at } x = -1$$

A: $f(x) = \frac{1}{3}x^3 - 3x^{-3}$

$f'(x) = x^2 + 9x^{-4} \qquad = x^2 + \frac{9}{x^4}$

$\Rightarrow f'(1) = 1 + \frac{9}{1} = \boxed{10}$ ##

تكملة (هيئة المعادلة) : f'

$y - y_1 = m(x - x_1) \qquad \Rightarrow y_1 = f(-1) = +\frac{8}{3}$

$y - \frac{8}{3} = 10(x - -1)$

$y = 10x + 10 + \frac{8}{3} \Rightarrow \boxed{y = 10x + \frac{38}{3}}$ ##

Oct 30, 19
Wednesday

مهم ص أ ج ٣

بيجي مثله

Ex: 34 Find the coordinates of points where $f(x)$ has Horizontal tangent
$\rightarrow$ ① slope = 0
①+② $f'(x) = 0$

$$f(x) = 3x^5 - 5x^3 + 2$$

$$f'(x) = 15x^4 - 15x^2$$

$$0 = 15x^2(x^2 - 1)$$

$15x^2 = 0$ or $x^2 - 1 = 0$

$\boxed{x = 0}$ $\boxed{x = \pm 1}$

$$\Rightarrow \begin{aligned} (0, f(0)) &= (0, 2) \\ (1, f(1)) &= (1, 0) \\ (-1, f(-1)) &= (-1, 4) \end{aligned}$$ #

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48 The total revenue $R(x) = 300x - 0.02x^2$

a) What is the $\overline{MR}$ when 40 units are sold
↳ slope = derivative (R')

A: $R'(x) = 300 - 0.04x$

$$R'(40) = 300 - (0.04)(40)$$
$$= 298.4$$

b) Interpret your result.

العائد المتوقع

$\Rightarrow$ The expected (or approximated [التقريبي]) revenue from the sale of 41 st unit.

c) Find $R(41) - R(40)$ and interpret.

(تقريباً زي ج بعض (بس في فرق صغير)

$R(41) = 300(41) - 0.02(41)^2$
$= 12{,}266.38$

هاي أدق من المشتقة

$R(40) = 300(40) - 0.02(40)^2$
$= 11968$

$\Rightarrow R(41) - R(40) = 298.38$

$\Rightarrow$ The exact revenue from the sale of 41st unit

$(R(41) - R(40)) \cong R'(40)$

estimated / expected / approximated : لمّا يطلب
$\Rightarrow$ ع المشتقة

أمّا : exact $\Rightarrow$ ع المارجينال العادي

(زي فرع c)

v.imp

51 The demand for q units of a product is $q = \frac{1000}{\sqrt{P}} - 1$. Find and explain the instantaneous rate of change $\Rightarrow$ التغير اللحظي of demand with respect to price when $p = \$25$.

يعني متحول ع المشتقة

q' هنا يعني بدنا

with respect to price: بالنسبة لل P

بدنا: q'

$\longrightarrow$

$$q' = 1000\left(-\frac{1}{2}\right)p^{-3/2}$$ لازم تطلع سالب

$$= \frac{-500}{p^{3/2}} = \frac{-500}{25^{3/2}} = -4$$

$\Rightarrow$ An increase in the price by \$1 (\$26), the quantity demanded decreases by 4 units.

- Sec 9.5 : The product rule and quotient Role :

① $h(x) = f(x) \cdot g(x)$

$$h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

2) $h(x) = \dfrac{f(x)}{g(x)}$

$$h'(x) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{g^2(x)}$$

Ex: 16/596: $y = 200x - \dfrac{100x}{3x+1}$

find $\dfrac{dy}{dx}$ (y')

A: $$\frac{dy}{dx} = 200 - \frac{(3x+1)100 - (100x)(3)}{(3x+1)^2}$$

$$= 200 - \frac{\cancel{300x} + 100 + \cancel{300x}}{(3x+1)^2} = 200 - \frac{100}{(3x+1)^2}$$

Sec 9.6 : The chain rule and the power rule

(قاعدة السلسلة)

$(f \circ g)(x) = f(g(x))$

$\rightarrow$ let $h(x) = (f \circ g)(x)$

find $h'(x)$.

$h'(x) = f'(g(x))\, g'(x)$

- Ex: let $g(-1) = 2$, $f(-1) = 4$

$g'(-1) = -3$ $\quad f'(-1) = 5$

$f'(2) = 2$

a) let $h(x) = (f \circ g)(x)$ find $h'(-1)$

A: $h'(x) = f'(g(x))\, g'(x)$

$h'(-1) = f'(g(-1)) \cdot g'(-1)$

$= f'(2) \cdot -3$

$= 2 \cdot -3 = \boxed{-6}$ #

b) Let $h(x) = f(x) \cdot g(x)$ find $h'(-1)$

$h'(-1) = f(-1)\, g'(-1) + f'(-1)\, g(-1)$

$= 4 \cdot -3 + 5 \cdot 2$

$= -12 + 10$

$= \boxed{-2}$ #

Nov 6. 2019
Wednesday

Sec 9.6 : The chain and power rule.

الأس

(قاعدة السلسلة)

$$(f \circ g)(x) = f(g(x))$$

$$\frac{d}{dx}(f \circ g)(x) = f'(g(x)) . g'(x)$$

Ex : $f(x) = (x^2 - 4x + 1)^5$

$f'(x) = 5(x^2 - 4x + 1) . (2x - 4)$

* Page 603 :

(10) $p(q) = 4(3q^2 - 1)^4 - 13q$

$p'(q) = 16(3q^2 - 1)^3 . 6q - 13$

$= 96q(3q^2 - 1)^3 - 13$.

(18) $y = \frac{1}{(3x^3 + 4x + 1)^{3/2}} = (3x^3 + 4x + 1)^{-3/2}$

$y' = \frac{-3}{2}(3x^3 + 4x + 1)^{-5/2} . (9x^2 + 4)$

$y' = \frac{-3(9x^2 + 4)}{2(3x^3 + 4x + 1)^{5/2}}$

(42) $y = \frac{90}{\sqrt{P+5}} = 90(P+5)^{-1/2}$

What is the rate of change of sales volume when the price is \$20 ?

A: $y' = \frac{-1}{2} \cancel{\times} 90(P+5)^{-3/2} \cdot (1)$

$= -45(P+5)^{-3/2}$

$y'(20) = -45(25)^{-3/2} = \frac{-45}{125} = -0.31$

Sec 9.7 : Using derivative formular :

(7) $y = (x^2-2)(x+4)$

$y' = (x^2-2)(1) + (x+4)(2x)$

$= (x^2-2) + 2x^2 + 8x$

$= 3x^2 + 8x - 2$.

★ (30) $y = 3x\sqrt[3]{4x^4+3}$

$y' = \cancel{3}x \cdot \frac{1}{\cancel{3}}(4x^4+3)^{-2/3} \cdot 16x^3 + 3\sqrt[3]{4x^4+3}$

$y' = \frac{16x^3}{\sqrt[3]{(4x^4+3)^2}} + 3\sqrt[3]{4x^4+3}$

* Rate of change of the marginal revenue:

$= R''(X)$

مشتقة مرتين.

Page 615 :

⑥ find the 2^{nd} derivative : $y = 3x^2 - \sqrt[3]{x^2}$

$\Rightarrow y = 3x^2 - x^{2/3}$

$y' = 6x - \frac{2}{3} x^{-1/3}$

$y'' = 6 + \frac{2}{9} x^{-4/3} = 6 + \frac{2}{9\sqrt[3]{x^4}}$

$y'' = 6 + \frac{2}{9x\sqrt[3]{x}}$ فلسفة زيادة :)

(27) $f''(3)$; $f(x) = x^3 - \frac{27}{x}$ → طريقة أخرى: سالب البسط المقام²

$\Rightarrow f'(x) = 3x^2 + \frac{27}{x^2}$

$f''(x) = 6x - 54x^{-3} = 6x - \frac{54}{x^3}$

$f''(3) = 18 - \frac{54}{27} = 18 - 2 = \boxed{16}$

#

(35) $R(x) = 100x - 0.01x^2$

~~Find the rate of change~~

Find the instaneuous rate of change of the marginal revenue.

التغير اللحظي ⇒ مشتقة

كمان مشتقة ⇒ المشتقة الثانية

$\Rightarrow R'(x) = 100 - 0.02x$

$R''(x) = -0.02$

Sec 9.9 : Applications :

تذكير

P : profit

p : selling price

Marginal cost $(\overline{MC}) = C'(x)$

Marginal Revenue $(\overline{MR}) = R'(x)$

Marginal Profit $(\overline{MP}) = P'(x)$

مشتقة

تفسير (بيجي بماذا يعني ⇒ ...)

Approximated / estimated / expected (⇒ إذن مشتقة) revenue from the sale of the 10^{th} Unit.

$\Rightarrow R'(9)$

- The exact revenue from the sale of 10^{th} unit. ⇒ مش مشتقة

$R(10) - R(9)$.

$\Rightarrow R(10) - R(9) \cong R'(9)$.

$\Rightarrow$ In general ; $R(x+1) - R(x) \cong R'(x)$

eg

* find the marginal revenue at $X = 3$ and interpret.

$\Rightarrow R'(3)$ the expected revenue from the sale of 4^{th} Unit.

(4) $R(X) = 25X - 0.05X^2$

a) $R(50)$ and tell what it represents.

$\Rightarrow R(50) = 25(50) - 0.05(50)^2 = 1125$

the total revenue from the sale of 50 units.

Nov 13.19 Wednesday

Sec $\boxed{10.1}$ Relativa maxima and minima

(18) Find the critical values, critical points, find internal where $f(x)$ is increasing, decreasing, find the relative maxima, minima, horizontal point of inflection.

(1) $f'(x)$, (2) $f'(x) = 0$ and undefined

(3) sign diagram. (4) max. & min., inc. & dec.

$$f(x) = \frac{x^4}{4} - \frac{x^3}{3} - 2$$

$f'(x) = x^3 - x^2$

$0 = x^3 - x^2 \longrightarrow x^2(x-1) = 0$

$\boxed{x = 0}$ or $\boxed{x = 1}$

critical values

$*f'(x)$ is undefined : No values .

$f'(x)$: +++ | 0 | --- | 1 | +++

$f(x)$: decreasing $(-\infty, \phi)$

increasing $(1, \infty)$.

at $x = 1 \Rightarrow$ min. (relative / local)

$\Rightarrow$ Critical points: $(0, f(0)) = (0, -2)$

$(1, f(1)) = \left(1, \frac{-25}{12}\right)$

HPI : $(0, f(0))$ (النقطة الي قبلها وبعدها نفس الإشارة بخط f')

(no max. point)

at $x = 1 \Rightarrow$ min $\Rightarrow (1, f(1))$

$= \left(1, \frac{-25}{12}\right)$

(لازم أوجد النقطة) .

$f(x)$ has a relative min. at : في صح 0 :

مش النقطة $(x = 1)$ (مثال السؤال السابق)

(بس الرقم هاد) .

But: The relative min. of $f(x)$ is: $f(1)$ imp

See $\boxed{10.2}$: Concavity ; Points of inflection

1. $f''(x)$.
2. $f''(x) = 0$, $f''(x)$ undefined.
3. sign diagram.
4. Concave up , down , inflection point
   " diminishing point of returns "

نقطة تناقُص العوائد (بالاقتصاد).

Ex : 1. $f(x) = x^3 - 3x^2 + 1$, concave up , down

page 660

$\Rightarrow f'(x) = 3x^2 - 6x$

$f''(x) = 6x - 6 = 0$

$6(x-1) = 0 \Rightarrow \boxed{x = 1}$

Inflection value

$\Rightarrow$ inflection point : $(1, f(1)) = (1, -1)$

$\Rightarrow f''(x)$ undefined : No value . (مش مقام)

$\cap$ $\quad$ $\cup$

$f''(x)$: $+++$ $\;|_{1}\;$ $+++$

$f(x)$: concave down $(-\infty, 1)$
concave up $(1, \infty)$

منعوّض ب $f''(x)$ بالاا
مش ب $f(x)$

(inflection point : $(1, -1)$)

نقطة الانعطاف

Ex: Let $f''(x) = x(x-2)$ find point(s) of inflection.

$\Rightarrow f''(x) = (x-2)x = 0$

$\boxed{x=0}$ or $\boxed{x=2}$

$$\begin{array}{cccccc} & \cup & & \cap & & \cup \\ f''(x) & +++ & 0 & ---- & 2 & +++ \end{array}$$

(concave مش لطالب)

$\Rightarrow$ Inflection points: $(0, f(0)) = (0,$

$(2, f(2)) = (0,$

لو طالب (Inflection point at: $x = 0, 2$)

بس هيك .

Exercise 35: سؤال الليكتشر :

$p(t) = 27t + 12t^2 - t^3$.

b) Find the point of diminishing returns.

$\Rightarrow P'(t) = 27 + 24t - 3t^2$

$P''(t) = 24 - 6t$

$0 = 24 - 6t \Rightarrow \boxed{t = 4}$

$$\begin{array}{ccc} \cup & & \cap \\ + & 4 & -- \end{array}$$

$P(t)$ undefined: no values.

$\Rightarrow$ The point of diminishing: $(4, P(4))$

$= (4, \ )$

$\Rightarrow$ = inflection point

$$\begin{array}{ccc} ++ & | & -- \end{array}$$

بس إنو

من أعلى لأسفل (دائماً) .

See $\boxed{10.3}$: Optimization in Economics :

(18) : If the total cost $C(X) = 250 + 6X + 0.1X^2$ producing how many units will minimize the average cost, find the min. average cost ?

المشتقة!

$$\bar{C} = \frac{C(x)}{X} = \frac{250}{X} + 6 + 0.1X$$

$$\Rightarrow \text{min} : (\bar{C})' = \frac{-250}{X^2} + 0.1 = 0$$

$$\Rightarrow \frac{250}{X^2} = 0.1$$

$$0.1X^2 = 250 \Rightarrow \boxed{X = 50}, -50$$

نرفض لأن x = عدد وحدات

$(\bar{C})'$ undifined at $\boxed{x = 0}$

($0 = X^2$ ⇍ المقام = صفر)

$f'(x)$: 0 --- 50 +++

فش عدد وحدات سالب

⇍ فش داعي

$\Rightarrow$ minimum average cost at $X = 50$ ← له

\# of unit

But: minimum average cost $= \bar{C}(50)$

للدوائر

$= 16$.

Nov 20, 19
Wednesday

$\boxed{\text{Sec}}\boxed{11.1}$ Derivatives of logarithmic functions

- Properties of logarithms:

1) $\log_a (x \cdot y) = \log_a x + \log_a y$

2) $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

3) $\log_a x^n = n \log_a x$

4) $\log_a a^x = x \log_a a = x$

5) $\ln 1 = 0$

6) $\ln e = 1$

7) $\log x = \log_{10} x$

* $\ln = \log_e$

* If $y = \log_a f(x)$

$\Rightarrow y' = \dfrac{f'(x)}{f(x) \ln a}$

مشتقة مابداخل اللوغاريتم
مابداخل اللوغاريتم X ln الأساس

* If $y = \ln f(x)$.

$\rightarrow y' = \dfrac{f'(x)}{f(x)(\ln e)} = \dfrac{f'(x)}{f(x)}$

$\ln e = 1$

Ex 11 / 708: Find $\frac{dP}{dq}$ if $P = \ln(q^2+1)$

$\frac{dP}{dq} = \frac{2q}{q^2+1}$.

ln
لا نوزع على الجمع والطرح

$\boxed{15}$ $y = \frac{1}{3} \ln(x^2 - 1)$

$$y' = \frac{1}{3} \cdot \frac{2x}{x^2-1} = \frac{2x}{3x^2-3}$$

$\boxed{19}$ $p = \ln\left(\frac{q^2-1}{q}\right)$, Find $\frac{dP}{dq}$.

$$p = \ln(q^2-1) - \ln q$$

* القسمة تتحول لطرح .

$$\frac{dP}{dq} = \frac{2q}{q^2-1} - \frac{1}{q}.$$

$\boxed{34}$ $y = \log_6 (x^4 - 4x^3 + 1)$

$$y' = \frac{4x^3 - 12x^2}{(\ln 6)(x^4 - 4x^3 + 1)}$$

* find y' if $y = \log x$.

$$y' = \frac{1}{(x)(\ln 10)}$$

* But: find y' : $y = \ln x$

$$y' = \frac{1}{x \underbrace{(\ln e)}_{1}}$$