[27] $y = \frac{1 + e^{5x}}{e^{3x}} = \frac{1}{e^{3x}} + e^{2x} = e^{-3x} + e^{2x}$

$y' = -3e^{-3x} + 2e^{2x} = \left(\frac{-3}{e^{3x}}\right) + 2e^{2x}$

---

[42] /714 : Find Max. & min.

$\Rightarrow y = \frac{x^2}{e^x}$

A: $y = x^2 \cdot e^{-x}$

$y' = 2x \cdot e^{-x} + x^2 \cdot -e^{-x}$

$y' = 0$

$\Rightarrow \frac{2x}{e^x} = \frac{x^2}{e^x}$

$x^2 - 2x = 0 \rightarrow x(x-2) = 0$

$x = 0$, $x = 2$

أو:

$e^{-x}(2x + x^2) = 0$

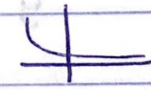
$e^{-x} \neq 0$ $2x + x^2 = 0$

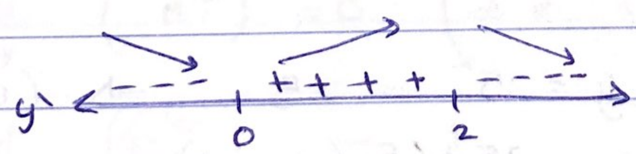
رفض

↓

$e^{-x} > 0$

دائماً





Decreasing : $(-\infty, 0)$, $(2, \infty)$

Increasing : $(0, 2)$

at $x = 0$ Min.

at $x = 2$ max.

Nov 7, 2019
Thursday.

حل تمارين

(48) Sec 11.3 : Implicit differentation الاشتقاق الضمني

* Examples: ① $x^3 + 5y^4 + 10xy = 0$. Find $\frac{dy}{dx}$

$\Rightarrow 3x^2 + 20y^3 \frac{dy}{dx} + 10x \frac{dy}{dx} + 10y = 0$

$20y^3 \frac{dy}{dx} + 10x \frac{dy}{dx} = -3x^2 - 10y$

$\frac{dy}{dx}(20y^3 + 10x) = -3x^2 - 10y$

$\Rightarrow \frac{dy}{dx} = \frac{-3x^2 - 10y}{20y^3 + 10x}$

---

② $x^3 + xy + 4 = 0$ (2, -6)

Find the equation.

$3x^2 + xy' + y = 0$

$3(4) + 2y' + -6 = 0$

$12 + y' - 6 = 0$

$2y' = -6 \rightarrow \boxed{y' = -3}$ = slope.

$\Rightarrow$ Equation:

$y + 6 = -3(x - 2)$

$y + 6 = 6 - 3x$

$\boxed{y = -3x}$ #

page 722

Find $\frac{dy}{dx}$

7 a- $\ln xy = 6$

$\ln x + \ln y = 6$

$\frac{1}{x} + \frac{1}{y} y' = 0$

$\frac{y'}{y} = \frac{-1}{x} \Rightarrow y' = \frac{-y}{x}$

b- $4x^2 + e^{xy} = 6y$

$8x + e^{xy} \cdot (xy' + y) = 6y$

$xy' e^{xy} + ye^{xy} = 6 - 8x$

$xy' e^{xy} = 6 - 8x - ye^{xy}$

$y' = \frac{6 - 8x - ye^{xy}}{xe^{xy}}$ صيك اشي ☺ العلم أبدأ.

Ex-8 / 723: Find the rate of change of demand with respect to price when 19 (hundred) sets are demanded.

$\frac{dx}{dp}$

$P = \frac{10,000}{(x+1)^2}$

$\left(\frac{d}{dx}(P) : \text{الخطأ}\right)$

$\Rightarrow P = 10,000\,(x+1)^{-2}$

$\frac{d}{dp}(P) : 1 = 10,000 \cdot -2\,(x+1)^{-3} \frac{dx}{dp}$

$\frac{dx}{dp} = \frac{-(x+1)^3}{20,000} \rightarrow \left.\frac{dx}{dp}\right|_{x=19} = -0.4$

[20] $(x+y)^2 = 5x^4y^3$, Find $\frac{dy}{dx}$

$\rightarrow 2(x+y).(1+y') = 5(4x^3.y^3 + 3x^4y^2y')$

---

[48] (Page 725): At what points does the curve defined by $x^2 + 4y^2 - 4 = 0$ have.

V. imp

a- Horizontal tangent ? → points من قيمة البسط = صفر

b- Vertical tangent ? → points من قيمة المقام = صفر

$\Rightarrow$ (a) H.T $\Rightarrow y' = 0$

$2x + 8yy' = 0$

$y' = \frac{-2x}{8y} = 0$

$2x = 0 \rightarrow x = 0 \Rightarrow 0 + 4y^2 - 0 - 4 = 0$

$8y \neq 0$ (المقام ≠ صفر) $\quad 4y^2 = 4$

$\boxed{y = \pm 1}$

$\Rightarrow (0,1) / (0,-1)$

Horizontal tangent : ← فش x

① $y - 1 = 0(x - 0)$

$\boxed{y = 1}$

② $\boxed{y = -1}$ #

$y'(0) = 0$

الميل

$\Rightarrow$ (b) V.T $\Rightarrow$ ال points المقام = صفر

$8y = 0 \rightarrow \boxed{y = 0} \Rightarrow x^2 + 0 - 4 = 0$

~~$x(x-4) = 0$~~

~~$x = 0 , 4$~~

~~$(0,0) / (4,0)$~~

Eq: → $\boxed{x = 2}$

$\boxed{x = -2}$ #

$\Rightarrow (2,0) , (-2,0)$

52 Find y'' for $\frac{1}{x} - \frac{1}{y} = 1$

$$xy\left(\frac{1}{x} - \frac{1}{y} = 1\right)$$

$$y - x = xy$$

$$y - xy = x$$

$$y(1-x) = x$$

$$y = \frac{x}{1-x}$$

$$y' = \frac{(1-x)\cdot 1 - (x\cdot 1)}{(1-x)^2} \quad \ldots \text{ etc}$$

بطريقة الدكتور : ← أحسن

$$-\frac{1}{x^2} + \frac{y'}{y^2} = 0$$

$$y' = \frac{y^2}{x^2} = \left(\frac{y}{x}\right)^2$$

$$\Rightarrow y'' = 2\left(\frac{y}{x}\right) \cdot \frac{xy' - y}{x^2}$$

$$y'' = \frac{2xyy' - 2y^2}{x^3}$$

#

Nov 12, 2019
Tuesday

11.5 Applications in Business & Economics : Elasticity of Demand.

D: $p = f(q) = f(x)$.

law of demand $\uparrow p \; q \downarrow$

⊛ Elasticity of demand = $\dfrac{-\dfrac{\text{change in quantity demanded}}{\text{original quantity demanded}}}{\dfrac{\text{change in price}}{\text{original price}}}$ (÷)

$$= -\frac{\Delta q}{q} \div \frac{\Delta P}{P} = \frac{-P}{q} \cdot \frac{\Delta q}{\Delta P}$$

⊛ Point elasticity of demand (η or E_d) ($\eta \equiv$ [illegible])

$$\eta = E_d = \frac{-P}{q} \cdot \frac{dq}{dP} \Bigg|_{(q,P)}$$

⊛ ⇒ If ① $E > 1$; the demand is elastic (If price ↑, the revenue ↓).

② $E < 1$; the demand is inelastic. (If price ↑, the revenue ↑).

③ $E = 1$; the demand is unitary elastic (An increase in price, the revenue does not change).

Example 1/733: $P + 5q = 100$ when:

a- $P = \$40$ b- $P = \$60$ c- $P = \$50$

$$\eta = \frac{-P}{q} \cdot \frac{dq}{dP}$$

$$\Rightarrow \frac{dq}{dP}: \quad 1 + 5\frac{dq}{dP} = 0$$

$$\frac{dq}{dP} = -\frac{1}{5}$$

a- $P = \$40 \rightarrow 40 + 5q = 100$

$5q = 60 \rightarrow \boxed{q = 12}$

$$\Rightarrow \eta = \frac{-40}{12} \cdot \frac{-1}{5} = 0.67$$

$\eta < 1 \equiv$ inelastic. ($P\uparrow$ ∴ Rev.$\uparrow$)

b- $P = \$60 \rightarrow 60 + 5q = 100$

$5q = 40 \rightarrow \boxed{q = 8}$

$$\eta = \frac{-60}{8} \cdot \frac{-1}{5} = 1.5$$

$\eta > 1 \equiv$ elastic ($P\uparrow$ ∴ Rev.$\downarrow$).

c- $P = \$50 \rightarrow 50 + 5q = 100$

$5q = 50 \rightarrow \boxed{q = 10}$

$$\eta = \frac{-50}{10} \cdot \frac{-1}{5} = 1$$

$\eta = 1 \equiv$ unitary elastic ($P\uparrow$ ∴ Rev Doesn't change)

$E = \frac{P}{100-P}$, $E = 1$

$\Rightarrow$ E < 1   E = 1   E > 1

0 ——— 50 ——— 100

$2P = 100$

$\boxed{P = 50}$

unitary elastic

unitary elastic : $P = 50$

elastic : $(50, 100) \Rightarrow$ (فترة)

inelastic : $(0, 50)$

* (الفترات ـــ مفتوحة) .

Ex 2 / 733 + 734 : $P = \frac{1000}{(q+1)^2}$ $\left(= 1000\,(q+1)^{-2} \right)$

Find the elasticity of demand with respect to price when $q = 19$. $\equiv \frac{dq}{dP}$

$\Rightarrow 1 = (-2)(1000) \cdot (q+1)^{-3} \cdot \frac{dq}{dP}$

$\frac{dq}{dP} = \frac{(q+1)^3}{-2000} \Rightarrow \left.\frac{dq}{dP}\right|_{q=19} = \frac{-(20)^3}{2000}$

$\left.\frac{dq}{dP}\right|_{q=19} = -4 \Rightarrow \eta = \frac{-P}{q} \frac{dq}{dP}$

(when $q = 19 \Rightarrow P = 2.5$).

$\eta = \frac{-2.5}{19} \cdot -4$

$\eta = 0.53 < 1 \Rightarrow$ inelastic (price ↑, rev. ↑)

(علاقة طردية بين السعر وال Rev).

* Revenue & Elasticity :

$R = p \cdot q$

$$\frac{dR}{dp} = p \cdot \frac{dq}{dp} + q \cdot 1$$

$$= q \left[\frac{p}{q} \cdot \frac{dq}{dp} + 1 \right]$$

$$\frac{dR}{dp} = q \left[1 - E \right] \quad *$$

q → +

اللي بأثر على إشارتها هو : E

(*) $E > 1 \Rightarrow$ Demand elastic

$$\frac{dR}{dp} = q(1-E) \rightarrow \text{رح يطلع سالب}$$

$\Rightarrow \frac{dR}{dp} < 0 \Rightarrow$ Negative relation between price & Rev.

(*) $E < 1 \Rightarrow$ Demand inelastic

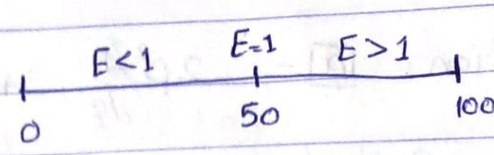
$\frac{dR}{dp} > 0 \quad (q(1-E)) \rightarrow$ رح يطلع موجب

$\Rightarrow$ Positive relation between revenue & price.

(*) $E = 1 \Rightarrow$ Unitary price.

$\frac{dR}{dp} = 0 \Rightarrow$ Max. revenue.

Ex: $p + 5q = 100$



⊛ at $p = 60 \Rightarrow E > 1$ (elastic)

$\therefore \frac{dR}{dp} < 0 \Rightarrow$ negative relation between Revenue & price.

⊛ at $p = \$60$, to increase revenue, should price decreased or increased?

$\Rightarrow$ Decreased.

⊛ at $p = \$20 \longrightarrow E < 1$ (inelastic) $\Rightarrow$ positive ⊕ Relation.

$\therefore \frac{dR}{dp} > 0$.

⊛ Max. Revenue at $E = 1 \Rightarrow p = 50 \therefore q = 10$

$\frac{dR}{dp} = 0 \rightarrow q = \frac{100 - P}{5} = 10$

max. revenue $= \$(50)(10) = \$500$

Excersise 6 / page 739:

Nov 14.2019 Thursday

Suppose that the demand for a product is given by: $2p^2q = 10,000 + 9000p^2$.

x y

a- Find the elasticity when $p = \$50$ and $q = 4502$.

b- Tell what type of elasticity this is.

c- How would revenue be affected by a price increase?

⟷

Solution: [a] - $2p^2 \frac{dP}{dq} + 4pq = 18{,}000\, p$

عشان E بتكون بهاي الصورة .

(4502 , 50)

$2(50^2)\frac{dP}{dq} + 4(50)(4502) = 18{,}000\,(50)$

$\Rightarrow \boxed{\frac{dq}{dp} = \frac{-2}{25}} \Rightarrow E = \frac{-p}{q} \cdot \frac{dq}{dp}$

$E = \frac{-50}{4502} \cdot \frac{-2}{25} < 1 \Rightarrow$ inelastic

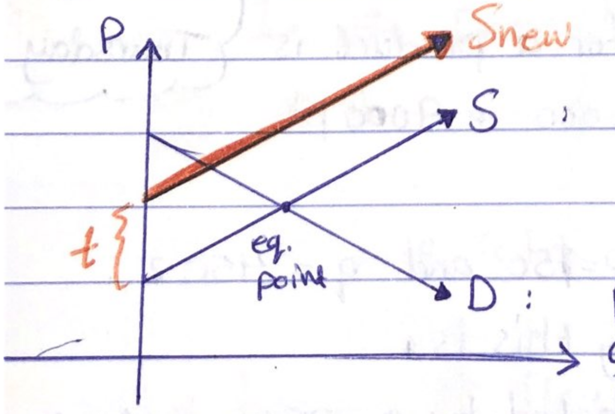
[b] - Inelastic

[c] - since $E < 1 \Rightarrow \frac{dR}{dp} > 0$

$\Rightarrow$ price increases $\Rightarrow$ Revenue increases.

---

* Maximum Tax Revenue → أكبر Rev. للـ Tax



S , $p = g(q)$

D : $P = f(q)$

New eq. point: $S_{new} = D$

$$g(q) + t = f(q)$$

$t = f(q) - g(q)$ (= Demand − Supply).

Let $T \equiv$ tax revenue, then $T = t \cdot q$

(T → Tax rev.; t → Tax ضريبة)

$$T = q\,(f(q) - g(q))$$

Example 4 / page 737:

The demand and supply functions for a product are $p = 900 - 20q - \frac{1}{3}q^2$ and $p = 200 + 10q$, respectively, where p is in dollars and q is the number of units (كنت موافقة بس كنت ما لحكي إلي ما إلوداعي ☺). Find the tax per unit that will maximize the tax revenue T. (≡ Find the max. tax revenue).

$\Rightarrow T = t \cdot q$

$t =$ Demand − Supply

$= 900 - 20q - \frac{1}{3}q^2 - 200 - 10q$

$= -\frac{1}{3}q^2 - 30q + 700$

$T = t \cdot q = -\frac{1}{3}q^3 - 30q^2 + 700q$

مع الملخص عشان نوجد ال max. → $T' = -q^2 - 60q + 700$

$T'' = -2q - 60$ →

$T' = 0 \Rightarrow -q^2 - 60q + 700 = 0$

$(q+70)(q-10) = 0$

$q = -70$ or $\boxed{q = 10}$

ترفض

$T''(10) < 0 \Rightarrow q = 10$ maximizes tax revenue

∴ (max.)

$t = \frac{-1}{3} \cdot 100 - 300 + 700 = 366.67$

Max. tax revenue $\Rightarrow T(10)$

$\Rightarrow T = t.q$

$= (366.67)(10) = 3,666.7$.

Exercise 23/ Page 740 :

If the demand and supply functions for product are $p = 2100 - 10q - 0.5q^2$ and $p = 300 + 5q + 0.5q^2$, respectively, find the tax revenue (t) that will maximiz the tax revenue (T).

$\Rightarrow t =$ Demand − Supply

$= 2100 - 10q - 0.5q^2 - 300 - 5q - 0.5q^2$

$= -q^2 - 15q + 1800$

$t = -q^2 - 15q + 1800$

$T = t.q = -q^3 - 15q^2 + 1800q$

$T' = -3q^2 - 30q + 1800$

$T'' = -6q - 30$

$T' = 0 \Rightarrow q^2 + 10q - 600 = 0$

$(q+30)(q-20) = 0$

$q = -30$ or $\boxed{q = 20}$

نرفض

$\Rightarrow T''(20)\ 20 \Rightarrow$ max

$\therefore q = 20$ is max.

$t = -400 - 300 + 1800 = \$1100$ #

Max. tax revenue $(T) = t.q = (20)(1100) = \boxed{\$22000}$

Nov19.19
Tuesday

Chapter 12
Indefinite Integrals

التكامل الغير محدود

(*) The function $F(x)$ is called an antiderivative of $f(x)$ if:

الاقتران المكامل.

$F'(x) = f(x)$

$(F(x) + c)' = f(x)$

The set of all antiderivative of $f(x)$ $(F(x) + c)$ is called the indefinite integral of $f(x)$ with respect to x

$\Rightarrow \int f(x)\,dx = F(x) + c$

↓ constant of integrals.

Examples: 1- $\int 3x^2\, dx = \frac{3x^3}{3} + C = x^3 + C$

↑ ——— لو اشتقيتها بعطيني

2- $\int x^4 dx = \frac{x^5}{5} + C$.

(*) Rules :

① $\int x^n dx = \frac{x^{n+1}}{n+1} + C$.

② $\int K\, dx = KX + C$ (عدد)

③ $\int (f+g)(x)\, dx = \int f(x)\, dx + \int g(x)\, dx$

(توزيع التكامل)

④ $\int K f(x)\, dx = K \int f(x)\, dx$. بنطلع الثابت برا

Examples:

1- $\int x^{100} dx = \frac{x^{101}}{101} + C$.

2- $\int 5x^3 dx = 5\int x^3 dx = 5\frac{x^4}{4} + C$

3- $\int \sqrt{x}\, dx = \int x^{1/2} dx$

[redacted]

$= \frac{2}{3} X^{3/2} + C$.

في التكامل ممنوع
أشوف ضرب/ قسمة/ جذر
← منحولو

من الكتاب

28

$$\int \left(3x^8 + \frac{4}{x^8} - \frac{5}{\sqrt[5]{x}}\right) dx$$

$$= \frac{x^9}{3} + \frac{4}{7x^7} - \frac{25}{4} x^{4/5} + C.$$

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$$\int \frac{x-3}{\sqrt{x}}\, dx = \int \left(\frac{x}{\sqrt{x}} - \frac{3}{\sqrt{x}}\right) dx$$

$$\left(\text{or, } \int (x-3)\, x^{-1/2}\, dx \right).$$

$$= \int x^{1/2}\, dx - \int 3x^{-1/2}\, dx$$

$$= \frac{2}{3} x^{3/2} - 6\sqrt{x} + C.$$

36

$$\int g(x)\, dx = 11x^{10} - 4x^3 + C$$

(اجا بامتحان الفصل الماضي)

⟸ ما بشيل التكاملات إلا بالدالة (المشتقة)

⟸ الحل: باشتقاق الطرفين:

$$g(x) = 110x^9 - 12x^2$$

⊛ Examples:

1- $$\int (x^2+1)^2\, dx = \int (x^4 + 2x^2 + 1)\, dx$$

$$= \frac{x^5}{5} + \frac{2}{3}x^3 + x + C.$$

⊛ $$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$$

$$\int (u(x))^n . du = \frac{(u(x))^{n+1}}{n+1} + C$$

هاي الطريقة (الأولى)

⟹ power rule

حفظ

$y = f(x)^n \Rightarrow y` = n\,(f(x))^{n-1} \cdot f'(x)$

$\int (f(x))^n \cdot f`(x)\,dx$ → Integration by Substitution

let $u = g(x) \Rightarrow \frac{du}{dx} = g`(x)$

$du = g`(x)\,dx$

differental of $u(x)$ — differental of $f(x)$

طريقة معقدة ملهاش داعي ☹ ومعقدة كمان ☹

$\Rightarrow \int f`(g(x))\,dx$

$= \int f`(u)\,du = f(u) + c \quad . \quad \frac{f(x)^{n+1}}{n+1} + c$

بالعربي : ∫ (س² + ١)¹⁰ ٢س . ءس

أحدهما مشتقة الآخر :. نفرض

نفرض ص = س² + ١ ⇐ ءص / ءس = ٢س ← ءس = ءص / ٢س

= ∫ ص¹⁰ . ~~٢س~~ . ءص / ~~٢س~~ = ص¹¹ / ١١ + جـ = (س² + ١)¹¹ / ١١ + جـ

(*) الطريقة الثانية : (المثال الأول)

let $u = x^2 + 1$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$\Rightarrow = \int u^{10}\,du = \frac{u^{11}}{11} + c = \frac{(x^2+1)^{11}}{11} + c$

2- $\int x^2 \sqrt{x^3 + 10}\ dx$

إلّي برّا مشتقة إلّي جوّا ⇐ نعوض إلي جوا .

let : $u = x^3 + 10$

$\frac{du}{dx} = 3x^2 \Rightarrow dx = \frac{du}{3x^2}$

$\Rightarrow \int x^2 \sqrt{u} \cdot \frac{du}{3x^2} = \frac{1}{3} \int u^{1/2} \cdot du$

$= \frac{1}{3} \cdot \frac{2}{3} u^{3/2} + C = \frac{2x^3 + 20}{9} + C .$

---

7 page 759 : ⓐ $\int (2x^2 - 4x)^2 (x - 1)\ dx$

إلّي برّا الهاملة بمشتقة إلي جوا ⇐ نعوض (إلّي جوّا)

let $u = 2x^2 - 4x$

$\frac{du}{dx} = 4x - 4 = 4(x - 1)$

$dx = \frac{du}{4(x-1)}$

$\Rightarrow \int u^2 \cdot (x-1) \cdot \frac{du}{4(x-1)} = \frac{1}{4} \int u^2 \cdot du$

$= \frac{1}{12} \cdot (2x^2 - 4x)^3 + C .$

ⓑ حل + سؤال

٤٤ صفحة

٧٥٩

Nov 21, 19
Thursday

Ex 6 : $\int (x^2+4)^2 \, d.x$ ... etc مهم

Ex [31]/762 : $\int \frac{x^2-4x}{\sqrt{x^3-6x^2+2}} \cdot dx$

$= \int (x^2-4x) \cdot (x^3-6x^2+2)^{-1/2} \cdot dx$

let : $x^3-6x^2+2 = u$

$\frac{du}{dx} = 3x^2-12x = 3(x^2-4x)$

$\Rightarrow dx = \frac{du}{3(x^2-4x)}$

$= \int (x^2-4x) \cdot u^{-1/2} \cdot \frac{du}{3(x^2-4x)}$

$= \frac{1}{3}\int u^{-1/2} \cdot du = \frac{2}{3}(x^3-6x^2+2) + C.$

[44] $MR = R' = 6000 - \frac{40000}{(10+x)^2}$

$R = \int R' \, d.x = \int \left(6000 - \frac{40000}{(10+x)^2}\right) \cdot dx$

$= 6000x + \frac{40000}{(10+x)} + C$

But : $R(0) = 0$ نعطي ($\Rightarrow$ C عشان نوجد)

$\Rightarrow (6000)(0) + \frac{40000}{10} + C = 0 \Rightarrow C = -4000$

$\therefore R(x) = 6000x + \frac{40000}{(10+x)} - 4000$ //

* Sec 12.3 : Integrals involving exponential and logarithmic functions.

* $\frac{d}{dx} \ln x = \frac{1}{x} \Rightarrow \int \frac{1}{x} dx = \ln|x| + c$

* $\int \frac{du}{u} = \ln|u| + c \Rightarrow$ (لأن البسط مشتقة المقام)

* $\frac{d}{dx} (\log_a x) = \frac{1}{x . \ln a}$ (a عدد)

* $\frac{d}{dx} (e^x) = e^x \Rightarrow \int e^x . dx = \frac{e^x}{\ln e} + c$ ($\ln e = 1$)

$\int e^u . du = e^u + c$.

* $\frac{d}{dx} a^x = a^x . \ln a$

$\Rightarrow \int a^x . dx = \frac{a^x}{\ln a} + c$

---

* Examples : evaluate :

① $\int \frac{5}{(5x+4)} . dx = \ln|5x+4| + c$

لأن البسط مشتقة المقام .

② $\frac{1}{2} \int \frac{1 \cdot (2)}{2x+10} . dx = \frac{1}{2} \ln|2x+10| + c$

← عشان تصير البسط مشتقة المقام

③ $\frac{1}{3} \int \frac{3x^2}{x^3+20} . dx = \frac{1}{3} \ln|x^3+20| + c$

④ $\int \frac{x}{\sqrt{x^2-10}} \cdot dx = \frac{1}{2}\int 2x \cdot (x^2-10)^{-1/2} \cdot dx$

ع التعويض أحسن من طريقتو ☺

$\Rightarrow (x^2-10)^{1/2} + C.$

---

5): [8] /page 770: $\int \left(\frac{x^4-2x^3+4x^2-7x-1}{x^2-2x}\right) \cdot dx$

ملاحظة مهمة : إذا درجة البسط تساوي درجة المقام أو أكبر منها

⇐ قسمة طويلة .

ناتج القسمة: x^2+4

$$\begin{array}{r|l} & x^2+4 \\ x^2-2x & x^4-2x^3+4x^2-7x-1 \\ & -(x^4-2x^3) \\ & 4x^2-7x \\ & -(4x^2-8x) \\ & x-1 \end{array}$$

باقي: $x-1$

⇐ الناتج + الباقي / المقسوم عليه . dx

$= \int \left((x^2+4) + \frac{x-1}{x^2-2x}\right) \cdot dx$

$= \frac{x^3}{3} + 4x + \frac{1}{2}\int \frac{2x-2}{x^2-2x} \cdot dx$

$= \frac{x^3}{3} + 4x + \frac{1}{2} \ln|(x^2-2x)| + C.$

⑥ $\frac{1}{5}\int 5e^{5x}\,.dx = \frac{1}{5}e^{5x} + C$

⇒ Rule: $\int e^{kx}\,.dx = \frac{1}{k}e^{kx} + c.$

⑦ $\int e^{-1/3\,x}\,.dx = -3\,e^{-1/3\,x} + c.$

⑧ $\frac{1}{3}\int 3x^2 e^{x^3+1}\,.dx = \frac{1}{3}e^{x^3+1} + C$

طريقة الأستاذ 😑

طريقتنا :

let $u = x^3+1$

$\frac{du}{dx} = 3x^2 \Rightarrow dx = \frac{du}{3x^2}$

$\Rightarrow \int x^2 . e^u . \frac{du}{3x^2} = \frac{1}{3}e^{x^3+1} + c.$

⑨ $\int 10^x\,dx = \frac{10^x}{\ln 10} + C.$

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⑩ (12/ page 771): $\int \frac{4}{e^{1-2x}}\,.dx = 4\int e^{-(1-2x)}\,.dx$

$= \frac{4}{2}\int 2e^{(2x-1)}\,.dx = 2e^{(2x-1)} + C.$

15 $\int\left(e^{4x} - \frac{3}{e^{x/2}}\right).dx = \frac{1}{4}e^{4x} - 3\int e^{-x/2}\,dx$

$= \frac{1}{4}e^{4x} + 6\,e^{-x/2} + c.$

(42) $\int 5\sqrt{x}\; e^{\sqrt{x}} \,.\, dx$

القوة مش من الدرجة الأولى ← نفرضها.

let $u = \sqrt{x}$ (مشتقة ما بداخل الجذر / ٢ × الجذر)

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$\Rightarrow dx = 2\sqrt{x}\, du$

$dx = 2u\, du$

$\Rightarrow \int 5u\, e^{u} \,.\, (2u)\, du$

$= 10\int u^2 e^{u} \,.\, du$ تكامل بالأجزاء (ولا واحد مشتقة الثاني) ... الخ

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[43] / 772 : $\overline{MR} = R' = 6e^{0.01x}$, find $R(100)$.

Nov 26, 2019 Tuesday

$\Rightarrow R(x) = \int R' . dx = 6\int e^{0.01x} . dx$

$= \frac{6e^{0.01x}}{0.01} + C = 600e^{0.01x} + C$

• $R(0) = 0 \Rightarrow 600 + C = 0$

$\therefore C = -600$

$\Rightarrow R(x) = 600e^{0.01x} - 600$

$R(100) = 600e - 600$

$= 600(e-1)$.

* Sec 12.4 : Applications in Business and Economics.

⊛ Example 2: Rate of change of cost:

$\overline{MC} = \grave{C}(x) = 3(2x+25)^{1/2}$

& Fixed costs = \$11125. Find $c(300)$.

$\Rightarrow c(x) = \int \grave{c}(x)\,.dx = \frac{3}{2}\int 2(2x+25)^{1/2}\,.dx$

$= \frac{3}{2}\frac{(2x+25)^{3/2}}{3/2} + k = (2x+25)^{3/2} + k$

$c(0) = (25)^{3/2} + k = 11125$

$k = 11{,}000$

$\Rightarrow c(x) = (2x+25)^{3/2} + 11000$

$\therefore c(300) = (625)^{3/2} + 11000$ --- etc.

أو بالفرض (التعويض).

7/708.

$\overline{MC} = 3x + 20$, $\overline{MR} = 44 - 5x$

$C(80) = \$11{,}400$

ⓐ Optimal level of production

↳ $\overline{MR} = \overline{MC} \Rightarrow (P = 0,\ \overline{MP} = 0)$

$44 - 5x = 3x + 20$

$8x = 24 \Rightarrow \boxed{x = 3}$