(b-) $P = R - C$

$R = \int (44 - 5x)\,dx = 44x - \frac{5}{2}x^2 + c$

but $R(0) = 0$

$\Rightarrow c = 0$

$\therefore \boxed{R(x) = 44x - \frac{5}{2}x^2}$

$C(x) = \int (3x + 20)\,dx = \frac{3}{2}x^2 + 2x + k$

but $C(180) = 11{,}400$ chéo

$\Rightarrow 11{,}400 = \underbrace{\frac{3}{2}(80)^2 + 20(180)}_{11{,}200} + k$

ليش قسمنا ع 11,200 ؟

$\Rightarrow \boxed{C = 200}$

$\therefore \boxed{C(x) = \frac{3}{2}x^2 + 20x + 200}$

$\Rightarrow P = R - C$

$= 44x - \frac{5}{2}x^2 - \frac{3}{2}x^2 - 20x - 200$

$\boxed{P(x) = -4x^2 + 24x - 200}$ #

(c-) $P(3) = (-4)(9) + (24)(3) - 200 < 0 \Rightarrow$ loss

* National consumption and savings :

let : $y \equiv$ Disposable national income

$C \equiv$ National consumption

$\Rightarrow C = f(y)$, If $S \equiv$ Saving

$y = S + C$

$C = f(y) \Rightarrow \frac{dc}{dy} = f'(y) =$ Marginal propensity to consume (MPC)

$S = y - c$

$\frac{ds}{dy} = 1 - \frac{dc}{dy}$

↳ Marginal propensity to save (MPS)

$\boxed{MPS = 1 - MPC}$.

---

* Example 5 : $C = \$6$ billion when $y = 0$

$MPC = \frac{dc}{dy} = 0.3 + \frac{0.4}{\sqrt{y}} = 0.3 + 0.4 y^{-1/2}$

ss Find $c(y)$.

$\Rightarrow c(y) = \int MPC \, dy = \int \frac{dc}{dy} \, dy$

$= \int (0.3 + 0.4 y^{-1/2}) \, dy = 0.3y + 0.8 y^{1/2} + k$

but : $C(0) = 6$ (billion) $\Rightarrow k = 6$

$\therefore \boxed{c(y) = 0.3y + 0.8 y^{1/2} + 6}$ #

$\boxed{25}$ / 781 : $\frac{ds}{dy} = MPS = 0.2 - \frac{1}{\sqrt{3y+7}}$

$C(0) = \$6$ Billion , find $c(y)$.

$\Rightarrow \quad MPC = \frac{dc}{dy} = 1 - MPS = 1 - \frac{ds}{dy}$

$\frac{dc}{dy} = 1 - \left(0.2 - (3y+7)^{-1/2}\right)$

$= 0.8 + (3y+7)^{-1/2}$

$C = \int \left(0.8 + (3y+7)^{-1/2}\right).dy$

$C = 0.8x + \frac{2}{3}(3y+7)^{1/2} + K$

but: $C(0) = 6$

$\Rightarrow \quad 0 + \frac{2}{3}(7)^{1/2} + k = 6$

$k = 4.2$

$\Rightarrow \quad \boxed{c(y) = 0.8x + \frac{2}{3}(3y+7)^{1/2} + 4.2}$

#

Nov 28, 2019
Thursday

Chapter 13
The definite integral

التكامل المحدود

* Sec $\boxed{13.2}$: The definite integral : The fundemantal theorem of calculus.

* If $\int \underset{\downarrow}{\underline{f(x)}}\,dx = \underset{\downarrow}{\underline{F(x)}} + C \Rightarrow F'(x) = f(x)$

Continuous   Antidirivative

$$\Rightarrow \int_a^b f(x)\,dx = F(x) \Big]_a^b = F(b) - (f(a))$$

ع مافي +C بالتكامل المحدود .

* Examples :

1- $\int_1^3 x^2\,dx = \frac{x^3}{3}\Big]_1^3 = \frac{27}{3} - \frac{1}{3} = \frac{26}{3}$

2- $\int_2^{10} \frac{1}{x}\cdot dx = \ln|x|\Big]_2^{10} = \ln 10 - \ln 2 = \ln\frac{10}{2}$
$= \ln 5$

* Properties of definite integrals :

1- $\int_a^b (f+g)\,dx = \int_a^b f\cdot dx + \int_a^b g\cdot dx$

2- $\int_a^b k\,f\cdot dx = k\int_a^b f\cdot dx$

3- $\int_a^a f(x).dx = 0$

4. $\int_a^b f(x).dx = -\int_b^a f(x).dx$

5- $\int_a^b f(x).dx = \int_a^c f(x).dx + \int_c^b f(x).dx$.

6- If $f(x) \geq 0$, then $\int_a^b f(x).dx \geq 0$

Example : $\boxed{2}$ $\int_1^3 (3x^2 + 6x).dx$

$$= (x^3 + 3x^2)\Big]_1^3 = 27 + 27 - 1 - 3$$
$$= 50$$

$\boxed{3}$ $\int_3^5 (\sqrt{x^2-9} + 2)x\,dx = \frac{1}{2}\int_3^5 2x(x^2-9)^{1/2}.dx + \int_3^5 2x\,dx$

$$= \frac{1}{3}(x^2-9)^{3/2}\Big]_3^5 + x^2\Big]_3^5$$

$$= \frac{1}{3}(64-0) + 25-9 = \frac{112}{3}$$

بالتعويض أيضاً : let $u = x^2 - 9$

$$\frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$$

$$= \int_3^5 \cancel{x}\,\sqrt{u}\,.\frac{du}{2\cancel{x}} = \frac{1}{2}\int_3^5 u^{1/2}.du$$

...etc

30 $\frac{3}{16}\int_0^1 \frac{16x^3}{4x^4+9}\,dx = \frac{3}{16}\ln|4x^4+9|\Big]_0^1$

(لأن البسط مشتقة المقام).

$= \frac{3}{16}\left[\ln|4+9| - \ln|9|\right]$

$= \frac{3}{16}\left[\ln\frac{13}{9}\right] = \boxed{\frac{3}{16}\ln\frac{13}{9}}$

---

34 $\int_1^4 \frac{4\sqrt{x}+5}{\sqrt{x}}\,dx = \int_1^4 4 + 5x^{-1/2}\,dx$

$= \int_1^4 4\,dx + 5\int_1^4 x^{-1/2}\,dx$

$= 4(4-1) + 5\left[2\sqrt{x}\right]_1^4$

$= 12 + 5\left((2).(2) - (2).(1)\right)$

$= 12 + 5(4-2) = 12 + 10 = \boxed{14}$

---

* Examples: If $\int_1^3 f(x)\,dx = 4$, $\int_3^5 f(x)\,dx = -2$

$\int_1^5 g(x)\,dx = 6$. Find:

1- $\int_1^5 f(x)\,dx = \int_1^3 f(x)\,dx + \int_3^5 f(x)\,dx$

$= 4 - -2 = \boxed{6}$

2- $\int_{1}^{1} f(x).dx = 0$

3- $\int_{5}^{3} f(x).dx = -\int_{3}^{5} f(x).dx = -(-2) = 2$

4- $\int_{1}^{5} (3f(x) - 4g(x) + 2).dx$

$= 3\int_{1}^{5} f(x).dx - 4\int_{1}^{5} g(x)\,\cancel{dx} + \int_{1}^{5} 2.dx$

$= (3)(6) - (4)(6) + 2(5-1)$

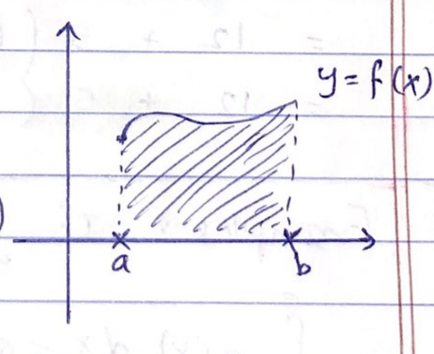
$= 18 - 24 + 8 = 2$

* Rule : $\int_{a}^{b} k.dx = k(b-a)$

حيث K = عدد .

---

f(x) continuous , $f(x) \geq 0$,
$x \in [a,b]$, then
the area between (enclosed)
f(x) and the x-axis
$a \leq x \leq b$ is :



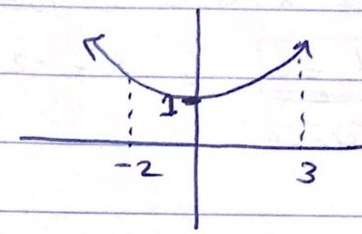
$A = \int_{a}^{b} f(x)\,dx$.

---

* Example : $f(x) = x^2+1 \Rightarrow (y = x^2+1)$.

Find the area under $f(x)$:

1- from : $x = -2$ to $x = 3$

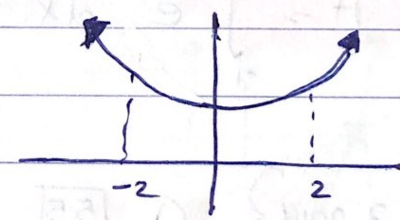


$$\Rightarrow A = \int_{-2}^{3} (x^2+1) \cdot dx$$

$$A = \left.\frac{x^3}{3}\right]_{-2}^{3} + 1(3 - -2)$$

$$= \frac{27}{3} - \frac{-8}{3} + 5 = \frac{35}{3} + \frac{15}{3} = \boxed{\frac{50}{3}}$$

2- $-2 \leq x \leq 2$



$$\Rightarrow A = \int_{-2}^{2} (x^2+1)\, dx$$

$$= \left.\frac{x^3}{3}\right]_{-2}^{2} + 1(2 - -2)$$

$$= \frac{8}{3} - \frac{-8}{3} + 4 = \frac{16}{3} + \frac{12}{3} = \boxed{\frac{28}{3}}$$

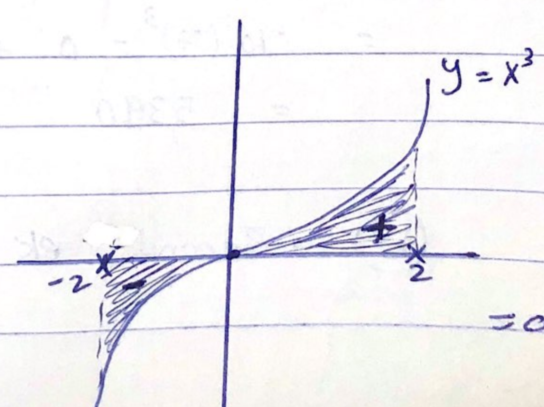
طريقة أخرى: بما أنه متماثل حول محور ص ←

$$A = 2\int_{0}^{2} f(x) \cdot dx$$ $\boxed{-2 \leq x \leq 2}$ مجال

---

* Example : $f(x) = x^3$

$-2 \leq x \leq x$



$$A = \int_{-2}^{2} x^3 = 0$$

$$A = \left| \int_{-2}^{0} f(x).\,dx \right| + \left| \int_{0}^{2} f(x)\,.dx \right|$$

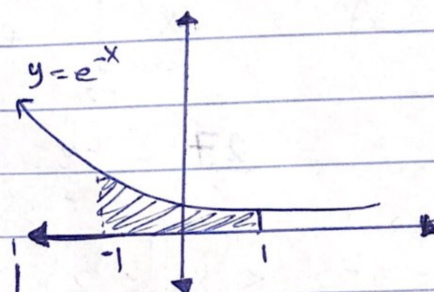
منقسم التكامل ⇐ إذا كانت الرسمة تحت ال x-asis

$\boxed{x = 0}$, $x = -2$, $x = 2$ وصف أفق : عنا

عند نقطة الأصل

٣ نقاط ⇐ لا لازم مساحتين .

Q $\boxed{44}$ $y = e^{-x}$



$-1 \leq x \leq 1$

$$A = \int_{-1}^{1} e^{-x}\,dx = -e^{-x} \Big|_{-1} \cdots \text{etc.}$$

Dec 3, 2019
Tuesday

Q $\boxed{55}$ / 817. $S'(t) = -30t^2 + 360t$

$0 \leq t \leq 30$

ⓐ → $\int_0^7 S'(t)\,dt = \int_0^7 (-30t^2 + 360t)\,dt$

First week

$= -10t^3 \Big]_0^7 + 180t^2 \Big]_0^7$

$= -10.(7)^3 - 0 + 180(7)^2 - 0$

$= 5390$

ⓑ → Second week $= \int_7^{14} S'(t)\,dt = (-10t^3 + 180t^2)\Big]_7^{14}$

... etc

* Sec 13.3 : Area between 2 curves.

Suppose that $f(x)$, $g(x)$ are continuous on $[a, b]$ and $0 \leq f(x) \leq g(x)$, then the area between $f(x)$ and $g(x)$ $(a < x < b)$ is :

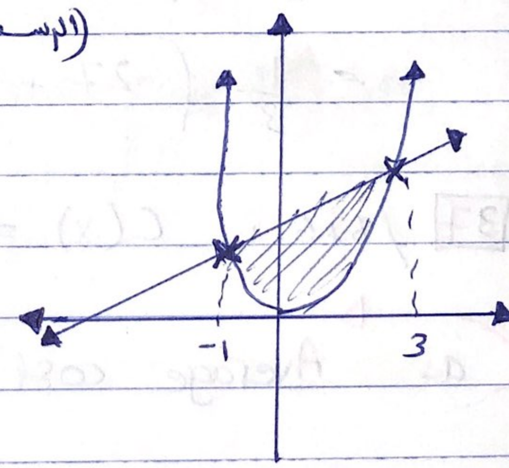
$$A = \left| \int_a^b (g(x) - f(x)).dx \right| \quad \text{(القيمة المطلقة)}$$

---

* Example 2 : $y = x^2$, $y = 2x + 3$

لإيجاد نقاط التقاطع : متساوي الاقترانين .

(الرسمة مش مطلوبة طبعاً) .



$$\Rightarrow x^2 = 2x + 3$$

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

$$x = 3 \quad , \quad x = -1$$

$$\therefore A = \left| \int_{-1}^{3} ((2x+3) - x^2).dx \right|$$

$$= \left| \cancel{\int} \left(x^2 + 3x - \frac{x^3}{3}\right) \Big]_{-1}^{3} \right|$$

$$= \left| 9 + 9 - 9 - \left(1 - 3 + \frac{1}{3}\right) \right|$$

$$= \left| \frac{15}{3} + \frac{1}{3} \right| = \left| \frac{16}{3} \right| = \boxed{\frac{16}{3}}$$

Def: let $f(x)$ be continuous on $[a, b]$, then the average value of $f(x)$, $a \leq x \leq b$ is :

$$AV(f) = \frac{1}{b-a} \int_a^b f(x) \cdot dx.$$

* Example: $f(x) = 3x^2 + 2x + 10$, $x \in [0, 3]$.

Find the average value of $f(x)$.

$$\Rightarrow AV(f(x)) = \frac{1}{3-0} \int_0^3 (3x^2 + 2x + 10)\, dx$$

$$= \frac{1}{3} \left[(x^3 + x^2 + 10x) \Big|_0^3 \right]$$

$$= \frac{1}{3} (27 + 9 + 30 - 0) = \frac{66}{3} = 22$$

$\boxed{37}$ / 826 : $C(x) = \frac{x^2 + 400x + 2000}{x}$.

a- Average cost of producing 1000 unit.

$$\bar{C}(x) = \frac{C(x)}{x} \Big|_{x=1000} = x + 400 + \frac{2000}{x} \Big|_{x=1000}$$

$$= 1400 + 2 = \$1402 \text{ per unit}$$

b- Average value of $C(x)$ over the interval from 0 to 1000

$$Av(C(x)) = \frac{1}{1000-0} \int_0^{1000} x^2 + 400x + 2000$$

$$= \frac{1}{1000}\left[\left(\frac{x^3}{3} + 200x^2 + 2000x\right)\Big|_0^{1000}\right]$$

$$= \frac{1}{\cancel{1000}}\left(\frac{(1000)^{\cancel{3}\,2}}{3} + 200\,(1000)^{\cancel{2}} + 2000\,\cancel{(1000)}\right)$$

$$= \frac{1000000}{3} + 200000 + 2000$$

... etc.

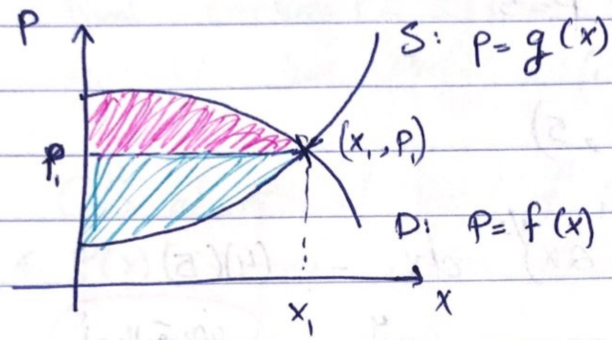
$\boxed{40}$ D: $p = 500 + \frac{100}{q+1}$. Average price as demand change from 49 to 99.

$$AV\,(p(q)) = \frac{1}{99-49}\int_{49}^{99}\left(500 + \frac{100}{q+1}\right).dx$$

...

Dec 10, 2019
Thursday.

※ See $\boxed{13.4}$ Applications in Business and Economics.
(Consumer's surplus, producer's surpluc)



⇒ Consumer's surplus.

⊛ $p > p_1$ ⇒ في ناس عندهم إمكانية يشتروا
الي consumer's surplus (C.S)

$\Rightarrow C.S = \int_0^{x_1} \underbrace{f(x)}_{\text{demand}}\, dx - \underbrace{x_1 p_1}_{\text{مساحة المستطيل}}$

or: $C.S = \int_0^{x_1} (\underbrace{f(x) - p_1}_{\text{معادلة خط مستقيم (معروفة).}})\, dx$

⊛ $p < p_1$ ⇒ $\boxed{\text{Producer's surplus}}$ (P.S)

$$P.S = x_1 p_1 - \int_0^{x_1} (\underbrace{g(x)}_{\text{Supply}})\, dx$$

Ex 5 & 8

D: $p = \sqrt{49 - 6x}$
S: $p = x + 1$

a. Find the equilibrium point ⇒ D = S

$$\left(\sqrt{49 - 6x}\right)^2 = (x+1)^2$$
$$49 - 6x = x^2 + 2x + 1 \longrightarrow$$

$\Rightarrow x^2 + 8x - 48 = 0$

$(x+12)(x-4) = 0 \quad \Rightarrow \boxed{x = 4}$

$\Rightarrow$ at $x = 4 \quad \Rightarrow p = 5$

$\Rightarrow$ Eq. point : $(4, 5)$

$$C.S = \frac{1}{-6}\int_0^4 -6(49-6x)^{1/2}\, dx - (4)(5)$$

أو بالتعويض

$$= \left(\frac{-1}{6} \cdot \frac{2}{3}(49-6x)^{3/2}\right)\Big|_0^4 - 20$$

$$= \left(\frac{-1}{9}(49-6x)^{3/2}\right)\Big|_0^4 - 20$$

$$= \frac{-1}{9}\left[125 - 343\right] - 20 = 4.2$$

(لازم يطلع موجب والا الحل بكون غلط ☺)

Producer surplus : $x_1 p_1 - \int_0^{x_1} S\, dx$

$$= 20 - \int_0^4 (x+1)\, dx$$

$$= 20 - \left(\left(\frac{x^2}{2} + x\right)\Big|_0^4\right)$$

$$= 20 - 12 = \boxed{8} \quad \#$$

(cost)

$\boxed{25}$ Page 836 : $C = 1000 + 120x + 6x^2$

D: $p = 360 - 3x - 2x^2$

Find consumer's surplus at the point as the munopoly has profit. (تأكدي من صيغة السؤال)

C.S at optimal level of production

⊛ $P(x)$ is max. when $\overline{MC} = \overline{MR}$

$R = px = 360x - 3x^2 - 2x^3$

$R' = 360 - 6x - 6x^2$

$C' = 120 + 12x$

$\Rightarrow R' = C'$

$360 - 6x - \cancel{16}6x^2 = 120 + 12x$

$6x^2 + 18x - 240 = 0$

$x^2 + 3x - 40 = 0$

$(x+8)(x-5) = 0$

$\boxed{x = 5}$

$x = 5 \Rightarrow P = 360 - 15 - 50 = 295$

$\Rightarrow (5, 295)$

⊛ Consumer's surplus at $(5, 295)$ is :

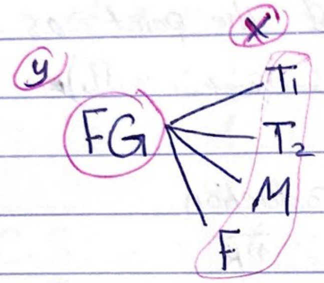
$$\int_0^5 (360 - 3x - 2x^2)\,.dx - (5)(295)$$

$$= \left(360x - \frac{3x^2}{2} - \frac{3x^3}{2}\right)\Bigg]_0^5 - 1475 =$$

مجال ال (fn)
لازم أكبر من صفر
(مش أكبر أو يساوي)

Chapter 14

Functions of Several variables

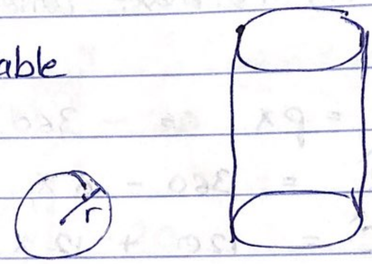


$y = f(x)$

$\underset{\text{Dep.}}{y} = f\underset{\text{ind.}}{(x)}$

functions in one variable

$V(r, t_1) = \pi r^2 h$



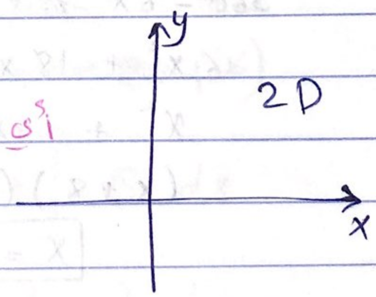
$q \rightarrow p$

$\boxed{q = f(P)}$

(المساحة) $A(r) = \pi r^2$

$y = f(x)$

$\underset{\text{dep.}}{Z} = f\underset{\text{independent variable}}{(x, y)} \Rightarrow$ أي قيمة لـ Z : تعتمد على x,y

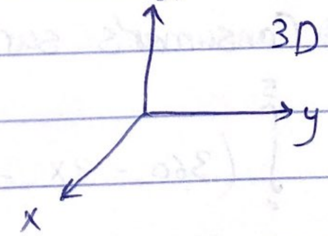


Function in 2 variables (3D ، تري دي)

$W = f(x, y, z)$

⋮

4D



Ex. $f(x, y) = \sqrt{5x - 4y}$

$f(1, 1) = \sqrt{5 - 4} = 1$

Domain : $5x - 4y \geq 0$

$\boxed{y \leq \frac{5}{4} x}$

Jan. 14, 2020
Tuesday

Functions in 2 variables : $\boxed{Z = f(x, y)}$

$Z = f(x, y) = \sqrt{x^2 - 2y}$.

$\Rightarrow$ The domain of $f(x, y)$: $x^2 - 2y \geq 0$

$x^2 \geq 2y$

$f(2,1)$ بتصير

$\Rightarrow f(2,1) = \sqrt{4-2} = \sqrt{2}$

but : $f(1,2) = \cancel{\sqrt{1-4}}$

Ex 4 / 874 : $U = f(x, y) = x^2 y^2$

$x = 10$ & $y = 2$

ⓐ $f(10, 2) = (10)^2 (2)^2 = 400$

ⓑ If $x = 5$, $y = ??$, $U = 400$

$400 = (25)(y)^2 \Rightarrow y^2 = 16 \Rightarrow \boxed{y = 4}$

$y = f(x) \Rightarrow \frac{dy}{dx} = f'(x)$

$\hookrightarrow$ Rate of change of y with respect to x $\equiv$ Marginal

$P = f(q) \Rightarrow P' = \frac{dP}{dq} \equiv$ Rate of change of Price with respect to quantity.

$\equiv$ Marginal.

$y = f(x)$ independend (1 variable)

$Z = f(x, y) \Rightarrow$ 2 independed variables.

$\cancel{\frac{dZ}{dx}}$, $\cancel{\frac{dZ}{dy}}$???

$\Rightarrow$ Partial Derivative الاشتقاق الجزئي

Let $Z = f(x, y)$

لمّا أشتق بالنسبة ل y

بعامل x كإنها ثابت ، والعكس.

* $Z_x = f_x = \frac{\partial Z}{\partial x} = 1^{st}$ p.d of Z with respect to x

عشان نميزها عن الاشتقاق العادي.

$\frac{\partial Z}{\partial x} \equiv \frac{\text{Partial } Z}{\text{Partial } x} = 1^{st}$ P.d ----

نثبت y و بشتق بالنسبة ل x.

← (الاشتقاق من الدرجة الأولى ← بشتق مرة وحدة).

* $Z_{xx} = \cancel{f}_{xx} = \frac{\partial^2 Z}{\partial x^2} = 2^{nd}$ p.d of Z with respect to x

يعني اشتق مرتين

* $Z_{yy} = f_{yy} = \frac{\partial^2 Z}{\partial y^2} = 2^{nd}$ p.d of Z with respect to y.

* Mixed
$$\begin{cases} Z_{xy} = \frac{\partial}{\partial y}(Z_x) \\ Z_{yx} = \frac{\partial}{\partial x}(Z_y) \\ Z_{xyy} \text{ وهكذا} \end{cases}$$

$\Rightarrow \boxed{Z_{xy} = Z_{yx}}$

بالمادة تبعتنا دائماً

$\underline{\underline{Ex}}$ ① $Z = f(x,y) = x^3 + 4y^2 + 10$

ثابت لما نشتق بالنسبة لـ x .

$Z_x = f_x = 3x^2 + 0 + 0 = 3x^2$

مشتقة

$Z_y = f_y = 0 + 8y + 0 = 8y$.

---

② $f(x,y) = x^4 + 5x^3y^4 + y^2 + 10xy$.

$f_x = 4x^3 + 15y^4x^2 + 0 + 10y$.

$f_{xx} \equiv$ بشتق بالنسبة لـ x ، وبرضو بشتق بالنسبة لـ x

$= 12x^2 + 30xy^4 + 0$.

ثابت

$f_{xy} \equiv \frac{\partial}{\partial y}\left(4x^3 + 15x^2y^4 + 10y\right)$

$= \boxed{0 + 60x^2y^3 + 10}$

---

③ $f(x,y) = x^4 + 5x^3y^4 + y^2 + 10xy$

$f_y = 0 + 20x^3y^3 + 2y + 10x$

$f_{yy} = 60x^3y^2 + 2 + 0$

$f_{yx} = \frac{\partial}{\partial x}\left(20x^3y^3 + 2y + 10x\right)$

$= \boxed{60x^2y^3 + 0 + 10}$

$\Rightarrow \boxed{f_{xy} = f_{yx}}$

④ $Z = f(x,y) = e^{x^2+2y}$

$f(1,0) = e^1 = e$

$f_x = e^{x^2+2y} \cdot (2x)$

$f_x(1,0) = e^1 (2)(1) = 2e$

مشتقة ضرب

$f_{xx} = 2x \cdot \underbrace{e^{x^2+2y}(2x)}_{\text{بالنسبة لـ } x} + e^{x^2+2y} \cdot 2$

$= 4x^2 e^{x^2+2y} + 2e^{x^2+2y}$.

---

⑤ $f_y = (e^{x^2+2y})(2)$. تكملة ع المثال السابق

$= 2e^{x^2+2y}$

⑥ $f_{yy} = 4e^{x^2+2y}$

$f_{yy}(0,0) = 4e^0$

$= 4$

---

⑦ $Z = f(x,y) = (x^3 + 4y^2)^{10}$ $\Rightarrow$ chain rule

$\Rightarrow Z_x = f_x = 10(x^3 + 4y^2)^9 \cdot 3x^2$

$= 30x^2 (x^3 + 4y^2)^9$.

$f_{xx} = \ldots$

$Z_y = f_y = 10(x^3 + 4y^2)^9 \cdot 8y$

$= 80y(x^3 + 4y^2)^9$.

⑧ $Z = x \ln y$

$Z_x = (1) \ln y = \ln y$

$Z_{xx} = 0$, $Z_{yy} = \frac{-x}{y^2}$

$Z_y = x\left(\frac{1}{y}\right) = \frac{x}{y}$

(*) Cost = C

Row materials (مواد خام) → x

laber → y

$\Rightarrow C(x, y)$

الكوست بيعتمد على x و y

$\equiv$ Joint cost function.

eg $C(x,y) = x^3 + 10x^2 y + 100$

$C(10, 10) = 1000 + 10000 + 100$

$= 1100$

$C_x(10,10)$??

$C_y(10,10)$?? what does it mean $\Rightarrow$ Marginal

Jan. 16, 2020 Thursday

* Sec $\boxed{14.3}$: Applications of functions of 2 variables:

$C(x,y)$ = Joint cost of producing 2 product X & Y.

$\Rightarrow C_x \equiv$ The marginal cost of x.

$C_y \equiv$ The marginal cost of y.

Ex: If $C(x,y) = 100 + 3x + 10xy + y^2$ →

1) Find the cost of producing 5 units of product x

and 10 units of y.

$\Rightarrow C(5, 10) = 100 + 15 + 10(5)(10) + 10^2 = 715$

2) Find the marginal cost with respect to x when

$$\underbrace{x = 5 \quad , \quad y = 10}_{\text{آخر خطوة}} .$$

$C_x = 3 + 10y$

$C_x \Big|_{y=10} = 3 + 10(10) = \$103$.

~~Interp~~ $\boxed{\text{Interpret}}$: producing 1 extra unit at $x = 5$ and y is constant, the cost will increase by \$103

عشان المارجينال لـ x ⇐ y بتضل ثابتة

يعني عند الوحدات ـــــــــ بصير 6 .

3) Find the marginal cost with respect to Y when

$x = 5$, $y = 10$. $\qquad C(x, y) = 100 + 3x + 10xy + y^2$

$C_y = 10x + 2y$

$C_y \Big|_{\substack{x=5 \\ y=10}} = 50 + 20$

$= \$70$

$\Rightarrow$ Producing 1 extra unit at $y = 10$, and x is constant, the cost will increase by \$70.

نفس فكرة الفوق (2) .

$\boxed{6}$ The total cost of a product is given by:

$$c(x,y) = 30 + 0.5x^2 + 30y - xy$$

where x is the hourly labor rate and y is the cost per pound of raw materials. The current hourly rate $25 and raw materials cost is $6 per pound. How will an increase of:

a- $1 for raw materials affect the total cost?

↳ Derivative (المشتقة)

$\Rightarrow C_y = 30 - x$

$= 30 - 25 = \$5$

مش بتكون ال cost = $5

بل ⇐ يزيد بمقدار $5

$1 ما معروف في محل

Y ≡ raw materials
X ≡ hourly labor

b- $1 in the hourly labor affect the total cost?

$C_x = x - y = 25 - 6 = \$19$

تفسيرهم زي تفسير المثال السابق: إذا زادت x بمقدار واحد والـ Y ثابتة ⇐ السعر ح يزيد بمقدار $19.

---

* Let $Z = f(x,y)$ be a production function for a product depends on input x and y.

Z_x = The marginal producting of x.

Z_y = The " " of y.

14 Suppose that a company's production function for a certain product is : $Z = 60\, x^{2/5}\, y^{3/5}$ where x is the capital expenditures and y is the number of work-hours. Find the marginal productivity of work hours at $x = 32$, $y = 32$.

$$Z_y = 60\, x^{2/5} \left(\frac{3}{5}\, y^{-2/5} \right)$$

$$= \frac{36\, x^{2/5}}{y^{2/5}} = \frac{36\,(32)^{2/5}}{(32)^{2/5}} = 36 .$$