② $f(x) = \sqrt{2x+1} \quad = (2x+1)^{1/2}$

$f'(x) = \frac{1}{\cancel{2}}(2x+1)^{-1/2} \cdot \cancel{2}$

طريقة أخرى : مشتقة ما بداخل الجذر
٢× الجذر

$f''(x) = -(2x+1)^{-3/2}$

Tuesday
Oct 22, 19

9.9 Marginal Revenue, Cost and Profit.

(* Marginal = مشتقة)

p619 ← Ex. 3 : $C(x) : 0.001x^3 - 0.3x^2 + 32x + 2500$

$\Rightarrow C'(x) = 0.003x^2 - 0.6x + 32$

$C'(80) = \$3.2$ per unit ⇒ تكلفة إنتاج وحدة

زيادة بعد الوحدة رقم 80 ، وإذا الإشارة سالبة بتقلل من التكلفة ،

Ex 5/621 : $P(x) = 20\sqrt{x+1} - 2x - 22$

What is the marginal profit at a production level of 15 units?

$\Rightarrow P'(15)$.

$P'(x) = \frac{10}{\sqrt{x+1}} - 2$

$P'(15) = \frac{10}{\sqrt{16}} - 2 = \frac{10}{4} - \frac{8}{4} = \frac{1}{2} = \$0.5$

Competitive ≠ mopoly

حدا بتحكّم بالسعر ≠ ولاحدا بتحكّم بالسعر.

⊛ In $\boxed{9.8}$: Ex 30 / 615: $f(x) = \frac{1}{\sqrt{x^2+7}}$, $f''(3)$.

$\Rightarrow \quad f(x) = (x^2+7)^{-1/2}$

$f'(x) = -\frac{1}{\cancel{2}} (x^2+7)^{-3/2} \cdot \cancel{2}x$

$f'(x) = \quad -x (x^2+7)^{-3/2}$

$f''(x) = \frac{3}{\cancel{2}} x (x^2+7)^{-5/2} \cdot \cancel{2}x$

$f''(x) = \frac{3x^2}{\sqrt{(x^2+7)^5}}$

$f''(3) = \frac{27}{\quad} \; ....$

Ex 36 / 616 : $R(x) = 70x + 0.5x^2 - 0.001x^3$.

How fast is the marginal revenue $\overline{MR}$ when $x = 100$?

[Note:] ⊛ How fast is the Marginal (Revenue) ⇒ مشتقة ثانية

⊛ How fast is the $R(x)$ ⇒ مشتقة أولى

$\Rightarrow R' = 70 + \cancel{x} - 0.00\cancel{3}x^2$

$R'' = 1 - 0.006x$

$R''(100) = 1 - 0.6 = \$0.4$

$R'(101) - R'(100) \approx R''(100)$

$\Downarrow$ exact $\quad \Downarrow$ approximate

9.9 تكملة

Ex 6/622: $p = \$200$ per unit , Cost per unit $= 80 + X$

↳ العلاقة بـ R

Find the marginal profit function.

$$\Rightarrow P(x) = R(x) - C(x)$$
$$= 200x - (80 + x)x$$
$$= 200x - 80x + x^2$$
$$= 120x - x^2$$
$$\boxed{\overline{MP} = P'(x) = 120 - 2X} \quad \#$$

---

* Exercise 4/624:

$R(X) = 25X - 0.05X^2$.

a- Find R(50) and tell what it represents.

$\Rightarrow R(50) = 1250 - 125 = \$1125$

R(50) represents Producing and Selling 50 units

b- Find the $\overline{MR}$ function.

$\overline{MR} = R'(x) = 25 - 0.1X$

c- Find the $\overline{MR}$ function at $x = 50$, and tell what it predicts about the sale of the next unit and the next 2 units.

$R'(50) = \$20 \Rightarrow$ Revenue will increase by about \$20 if a 51^{st} unit is sold.

($20 لإنتاج الوحدة رقم 51 ، بس 52 لازمها حسابات جديدة).

d →

d- Find R(51)-R(50) and what this value represent.

$\Rightarrow R(51) - R(50) \approx R'(50) \approx \$20$

R(51) - R(50) is the actual revenue from the sale of the 51^{st} unit.

Chapter 10: Applications of Derivatives

$\boxed{10.1} \Rightarrow f'(x)$:
- extreme value.
- increasing and decreasing.
- graph x .
- Critical values. (القيم الحرجة)

$\boxed{10.2} \Rightarrow f''(x)$:
- concavity (التقعّر)
- extreme values
- Infliction points (نُقط الانعطاف)

$\boxed{10.1}$ Relative Maxima and Minima (القيم القصوى المحلية)

* Relative = local (محليّة)

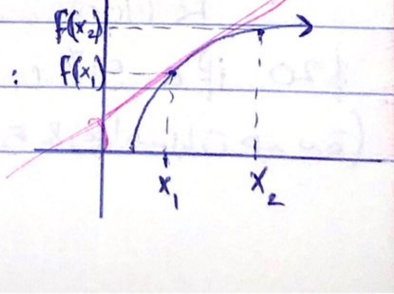
* Absolute = Global (مُطلَقة)

ويكون الميل سالب
(لاحادة)

* Def. : f(x) is increasing if :

$x_1 < x_2$

$f(x_1) < f(x_2)$

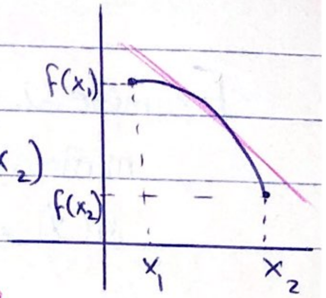


f(x) is decreasing if: $x_1 \quad x_2$

$f(x_1) \quad f(x_2)$

الميل بكون موجب

(✗ منفرجة)



مش $f'(x) \neq$ (انتباه)

* Notes: ① If $f'(x) > 0$ on I ⇒ f(x) is increasing on I.

$f'(x) < 0$ on I ⇒ f(x) is decreasing on I.

② If f(x) has a local max. or a local min. at $x = c$, then $f'(c) = 0$ or $f'(c)$ is undefined.

③ If $f'(c) = 0$ or $f'(c)$ is undefined, then $x = c$ is called a critical value. (نقطة حرجة)

$(c, f(c))$ ⇒ critical point.

④ If $x = c$ is a critical value, then $x = c$ may be Max. or Min.

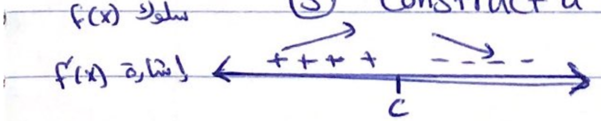
⇒ The First derivative test: اختبار المشتقة الأولى

① find $f'(x)$

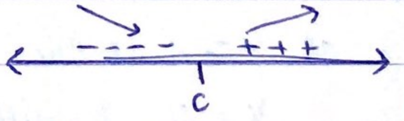
② find the critical value(s) ($f'(x) = 0$ / undefined).

$x = c$ ⇒ critical value.

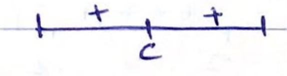
③ Construct a sign diagram for $f'(x)$:



Max.

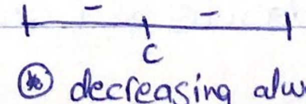


Min.



⊛ increasing always

⊛ neither max. nor min.



⊛ decreasing always

⊛ neither max. nor min.

Example 2 / 643: Find the relative maxima, relative minima, and horizontal points of inflection of $h(x) = \frac{1}{4}x^4 - \frac{2}{3}x^3 + 8x + 4 - 2x^2$.

$\Rightarrow$ ⊛ $h'(x) = x^3 - 2x^2 - 4x + 8$

حلّها بالتجزير: بنشوف عوامل الحد المطلق (8) وبصير أجرّبهم بالاقتران ⇐ بس يطلع الجواب صفر بكون هاد العامل المطلوب ⇐ قسمة طويلة

(⇐ بطلع 2)

$x - 2 \,\big|\, x^3 - 2x^2 - 4x + 8$ $\Rightarrow$ at $x = 2$ $f'(x) = 0$

⊛ $f'(x) = (x-2)(x^2-4)$

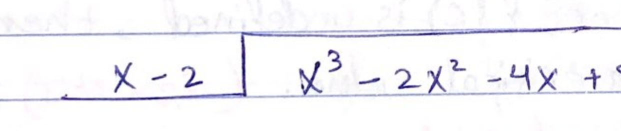
$= (x-2)(x-2)(x+2)$

$= (x-2)^2(x+2)$

→ عملنا هيك عشان $(x-2)^2$ دائماً موجب فَ بضل علينا إشارة $(x+2)$

⊛ $f'(x) = 0 \Rightarrow x - 2 = 0$ or $x + 2 = 0$

$\Rightarrow \boxed{x = \pm 2}$ (⇐ عشان الرسم والإشارات)



$\Rightarrow$ f(x): increasing $(-2, \infty)$

decreasing $(-\infty, -2)$

at $x = -2 \Rightarrow$ Min (Abs.)

$x = 2 \Rightarrow$ neither max. nor min.

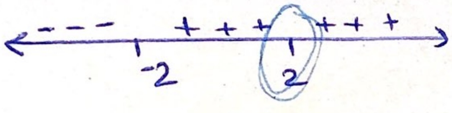
HPI

ملاحظة مهمة جداً (لا يجاد Horizontal points of inflection)

⇐ باستخدام المشتقة الأولى :

النقطة لي قبلها وبعدها نفس الإشارة (على خط أعداد اختبار المشتقة 1)

هي بتكون Horizontal point of inf.



$\Rightarrow (2, f(2))$

أمّا باستخدام المشتقة الثانية (نقطة انعطاف أفقي) :

$$f'(x) = (x-2)(x^2-4)$$

$$\Rightarrow f''(x) = (x-2)(2x) + (x^2-4)(1)$$

$$= 2x^2 - 4x + x^2 - 4$$

$$= 3x^2 - 4x - 4$$

$$0 = 3x^2 - 4x - 4$$

$$0 = (3x + 2)(x + 2)$$

$\boxed{x = -\frac{2}{3}}$ or $\boxed{x = 2}$

$\left(-\frac{2}{3}, f\left(-\frac{2}{3}\right)\right)$ $\quad$ $(2, f(2))$

⇐ هاي اسمها inflection point

أمّا المطلوب : HPI

(بختلفوا عن بعض) .

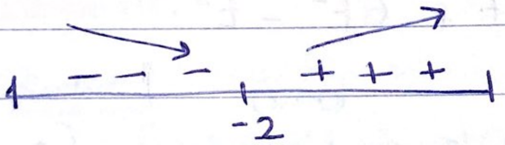
Example 3 / 643: Find the relative maxima and minima (if any) of the graph of $y=(x+2)^{2/3}$.

$\Rightarrow y' = \frac{2}{3}(x+2)^{-1/3} = \frac{2}{3\sqrt[3]{x+2}}$

$y' = 0 \Rightarrow 3\sqrt[3]{x+2} \neq 0 \Rightarrow$ undifined

$\Rightarrow$ at $x = -2 \Rightarrow$ undefined

So: at $x = -2$ there is a critical value



⊛ y: decreasing $(-\infty, -2)$

increasing $(-2, \infty)$

⊛ at $x = -2 \Rightarrow$ Min.

Exercises: [36] $f(x) = x - 3x^{2/3}$

$f'(x) = \frac{x^{1/3} - 2}{x^{1/3}}$ → خطأ

$\Rightarrow f'(x) = 0 \Rightarrow x^{1/3} - 2 = 0$

$(x^{1/3})^3 = (2)^3 \Rightarrow \boxed{x = 8}$

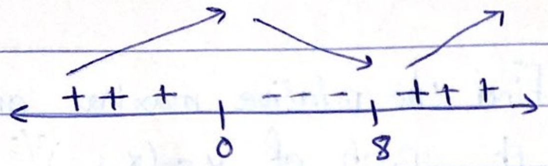
$(f'(x))$ $x^{1/3} = 0$ is undefined $\Rightarrow \boxed{x = 0}$

Critical values.

critical points: $(8, f(8))$

$(0, f(0))$

→



(عشان نعرف الإشارة
منعوض في $f'(x)$)

$\Rightarrow$ $f'(x)$: increasing : $(-\infty, 0)$, $(8, \infty)$
decreasing : $(0, 8)$.

at $x = 0$ $\Rightarrow$ Max.
$x = 8$ $\Rightarrow$ Min.

$\boxed{31}$ $P(t) = 27t + 6t^2 - t^3$ $0 \le t \le 8$

(متصل) فترة مغلقة

Note: If $f(x)$ is continuous on $[a, b]$, then $f(x)$ has an absolute maximum and absolute minimum.
$\Rightarrow$ end points + critical values.

Remember!

القانون العام:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\Rightarrow$ Find 1st derivative.

$P'(t) = 27 + 12t - 3t^2$

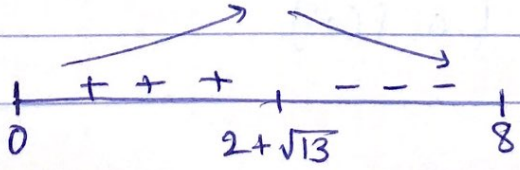
$0 = 27\cancel{8} + 12t - 3t^2$

$0 = 9 + 4t - t^2$

$t = \dfrac{4 \pm \sqrt{16 + 36}}{2}$

$t = \dfrac{4 \pm 2\sqrt{13}}{2}$ $\Rightarrow$ $\boxed{t = 2 \pm \sqrt{13}}$ $\Rightarrow$ Critical values.

نرفض $2 - \sqrt{13}$ $\Rightarrow$ $\notin]0, 8[$



- Inc : $(0, 2+\sqrt{13})$
- Dec : $(2+\sqrt{13}, 8)$
- at $x = 2 + \sqrt{13}$
$\Rightarrow$ Max. abs. (قيمة عظمى مطلقة)

Oct 29.19
Tuesday

10.2 Concavity ; Points of Inflection

التقعّر   نقط الانعطاف

㊟ If $f''(x) > 0$ on an interval, then $f(x)$ is concave up (مقعّر للأعلى) on this interval.

㊟ If $f''(x) < 0 \Rightarrow f(x)$ is concave down.

㊟ The point $(c, f(c))$ is an inflection point (نقطة انعطاف) if the function changes concavity around $x = c$.

㊟ If $(c, f(c))$ is an inflection point, then $f''(c) = 0$ or undefined.

㊟ منحنى: يوجد نقطة انعطاف عند $x = c$ ، لأن :

① ق'' (س) = صفر أو غير موجودة

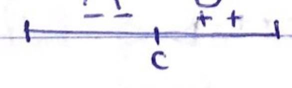
② لأن ق''(س) يغيّر من اتجاه تقعّره حول $x = c$ من الأعلى للأسفل (أو العكس).

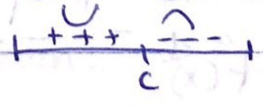
Test : ① Find f', f''

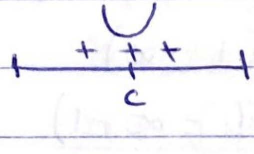
② Find $f'' = 0$ or undefined

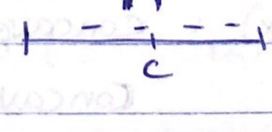
$x = c$

③ construct a sign diagram for f'' :









concave up   concave down.

مقعّر للأعلى   مقعّر للأسفل

Example: $f(x) = x^3 - 3x^2 + 10$

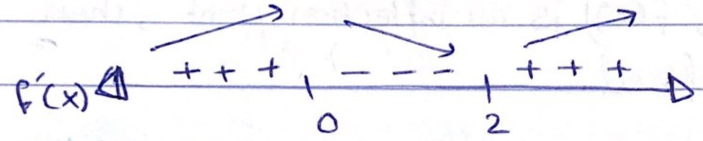
$f' = 3x^2 - 6x$

$f'' = 6x - 6$

$f' = 0 \Rightarrow 3x^2 - 6x = 0 \Rightarrow 3x(x-2) = 0$

$\boxed{x = 0}$ or $\boxed{x = 2}$

Critical values الحل



f(x): Increasing $(-\infty, 0)$, $(2, \infty)$

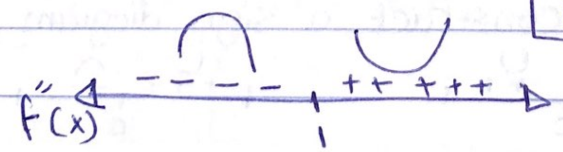
Decreasing $(0, 2)$

at $x = 2 \Rightarrow$ min.

$x = 0 \Rightarrow$ max.

$f'' = 6x - 6 = 0 \Rightarrow 6(x-1) = 0$

$\boxed{x = 1}$



f(x): concave up $(1, \infty)$

Concave down $(-\infty, 1)$.

$\Rightarrow (1, f(1)) = (1, 8)$ is an inflection point.

(f'' مش)

Page 660

↑

* Exercises : ⑱ $y = x^4 - 8x^3 + 16x^2$.

Find the relative min. & max. and the inflection point(s).

$\Rightarrow \quad y = x^4 - 8x^3 + 16x^2$

$y' = 4x^3 - 24x^2 + 32x$

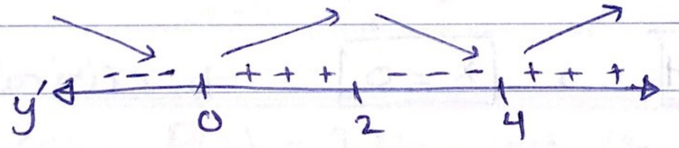
$0 = 4x(x^2 - 6x + 8)$

$4x = 0$ or $x^2 - 6x + 8 = 0$

$\boxed{x = 0}$ $(x-2)(x-4) = 0$

$\boxed{x = 2, 4}$

$\Rightarrow x = 0, 2, 4$



at $x = 0, 4 \Rightarrow$ min.

at $x = 2 \Rightarrow$ max.

To find points of inflection:

$y'' = 12x^2 - 48x + 32 = 0$

[redacted] $4(3x^2 - 12x + 8) = 0$

$x = \frac{-b \pm \sqrt{b^2 + 4ac}}{2a}$

$= \frac{12 \pm \sqrt{144 - 96}}{6}$ ... etc.

بيجي زي هيك

㉒ $y = x^{4/3}(x-7)$ (نفس المطاليب السابقة)

$y' = \frac{4}{3}x^{1/3}(x-7) + x^{4/3}\cdot(1)$

⇓

بتصير أصعب

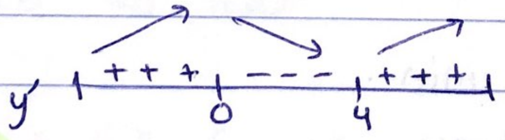
الأسرع ⇒ $y = x^{7/3} - 7x^{4/3}$

$y' = \frac{7}{3}x^{4/3} - \frac{28}{3}x^{1/3}$

$y'' = \frac{28}{9}x^{1/3} - \frac{28}{9}x^{-2/3}$

$\Rightarrow y' = 0 \rightarrow \frac{7}{3}x^{1/3}(x-4) = 0$

$\boxed{x=4}$, $\boxed{x=0}$ ⇒ critical values.



⇒ y : increasing $(-\infty, 0)$, $(4, \infty)$

decreasing $(0, 4)$.

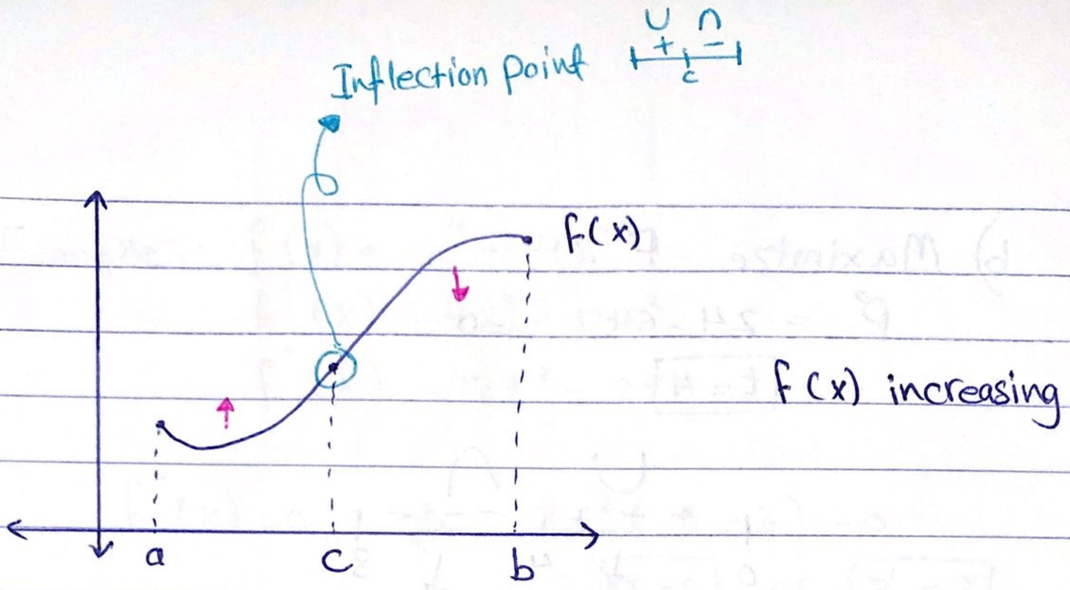
y'' ???

⊛ $y'' = 0$

$x^{1/3} = x^{-2/3} \rightarrow x^{1/3} - x^{-2/3} = 0$

$x^{1/3}(1 -$

...etc.



$R' = 0 \Rightarrow$ Max.

$\downarrow$

max of R'

$\downarrow$

inflection point of R

$\Rightarrow$ $\boxed{\text{The point of diminishing returns}}$ $\Rightarrow$ بالاقتصاد

$\triangleleft$ شرط: من أعلى لأسفل (مش العكس)

$$\overset{\cup}{+\;+\;+}\;\underset{c}{|}\;\overset{\cap}{-\;-\;-}$$

35/662: $P(t) = 27t + 12t^2 - t^3$ $\quad 0 \leq t \leq 8$

a) Find the # of units before the function is maximized.

$P'(t) = 0$

$p' = 27 + 24t - 3t^2 = 0$

$-3(t^2 - 8t - 9) = 0$

$(t+1)(t-9) = 0$

$t = -1$, $t = 9$

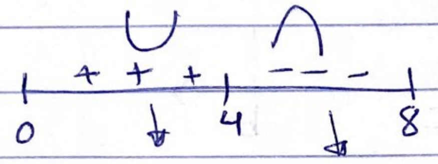
$$\underset{-1}{|}\;\times\;\underset{0}{|}\;+\,+\,+\,+\,+\,+\,+\,+\;+\,+\,+\;\underset{8}{|}\;\;\underset{9}{|}\quad P'(x)$$

Increasing.

b) Maximize P'.

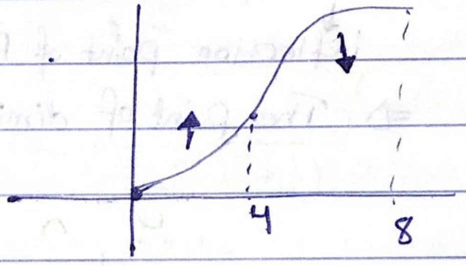
$P'' = 24 - 6t \quad = 0$

$\boxed{t = 4}$



concave up   concave down :

$t = 4$ maximizes P'.



(من أعلى للأسفل)

point of diminishing

$\boxed{t = 4} \longrightarrow$ Max. rate of change

$\longrightarrow$ Point of diminishing returns . v.imp

(=inflection point)

⊛ Second Derivative Test
(for maxima and minima) :

① Find f', f''

② Find critical values for f , $x = c$

③ ⊛ $f''(c) > 0 \Rightarrow$ min.

⊛ $f''(c) < 0 \Rightarrow$ max.

⊛ $f''(c) = 0 \Rightarrow$ test fails $\longrightarrow$ نرجع لمختبر المشتقة الأولى

* Example: $f(x) = x^4 - 4x^3$

$$f'(x) = 4x^3 - 12x^2$$

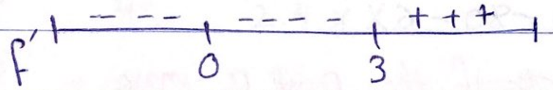
$$f''(x) = 12x^2 - 24x$$

$f'(x) = 0 \Rightarrow 4x^2(x-3) = 0$

$\boxed{x=0}$ or $\boxed{x=3}$

$f''(0) = 0$ → test fails → 1st derivative test

$f''(3) = (12)(9) - (24)(3) > 0 \Rightarrow$ min.



$f' = 0$ ⇒ Horizontal point of inflection **v.imp**

$f'' = 0$. (نقطة انعطاف أفقي)

لو قبلها وبعدها نفس الإشارة ، + المشتقة الثانية والأولى = صفر

انعطاف . أفقي

Oct 31.19
Thursday

10.3 Optimization in Business and Economics.

find max. and min. (abs) ⇒ . المطلوب منها

$f(x)$ continuous $[a, b]$

local , absolute لازم يكونوا :

⇒ - critical values

- end points.

⊛ Example 1/665 : $R(x) = 8000x - 40x^2 - x^3$

x is the number of units sold per day. If only 50 units can be sold per day, find the number of units that must be sold to maximize revenue. Find the maximum revenue.

$\Rightarrow \quad 0 \leq x \leq 50$ مُعطى

← منّا

Abs. max. , Abs. min. لازم يكون له

$\Rightarrow R' = 8000 - 80x - 3x^2$

$R''(x) = -80 - 6x$

⇒ To check if the point is max.

$R'(x) = 0$

$0 = -(3x^2 + 80x - 8000)$

$(3x + 200)(x - 40) = 0$

$\boxed{x = -\frac{200}{3}}$ or $\boxed{x = 40}$ → لازم نفحص

بمختبر المشتقة الاولى أو الثانية

مش نحكي إنها Max. ع طول

← - علامتش

Regrected

$R''(40) < 0 \Rightarrow x = 40$ maximizes the revenue, and the maximum revenue is $\$R(40) = \$192000$ when 40 units are produced and sold.

(يعني عدد الوحدات = ٤٠ عشان نوصل لل Max. revenue).

# of units ≡ level of production ⇒ (نفس, معنى)

⊛ Example 5/669 : $P = 168 - 0.2X$

$\bar{C} = 120 + X$

$(\bar{C})$: average $= \left(\frac{C}{x}\right)$

$P(x) = R(x) - C(x)$

$R = pX = 168X - 0.2X^2$

$C = X\bar{C} = 120X + X^2$

a- $P = R - C$

$= 168X - 0.2X^2 - 120X - X^2$

$= 48X - 1.2X^2$

$P' = 48 - 2.4X$

$P'' = -2.4 \Rightarrow P' = 0$

$\Rightarrow X = \frac{48}{2.4} = 20$

$P''(20) < 0 \quad (-2.4) \Rightarrow X = 20$ is max.

# of units that maximize the profit is call the (optimal) level of production.

$P' = 0$ or $\overline{MR} = \overline{MC}$

b- Selling price at optimal.

$p\Big|_{X=20} = 168(20) - 0.2(20)^2 = \$164$

c- Find the max. profit.

$\Rightarrow P(20) = (48)(20) - (1.2)(20)^2 = \$480$.

Ex 6 : p = selling price = \$200 $\Rightarrow R(x) = 200x$

$\bar{C} = 80 + x \Rightarrow C(x) = 80x + x^2$

$$P = R - C = 200x - 80x - x^2$$

$$P = 120x - x^2$$

$$P' = 120 - 2x$$

$$P'' = -2$$

$P' = 0 \Rightarrow x = 60$

$P''(60) < 0 \Rightarrow x = 60$ is max.

(ماطلب ال Max. profit عشان هاي تثبت ال fixed costs).

Ex 19: $C(x) = 100(0.02x + 4)^3$

x = # of 100 units produced

Find the min. average cost.

① Average cost $(\bar{C}) = \frac{C}{x} = \frac{100(0.02x+4)^3}{x}$

$$(\bar{C})' = 100\left[\frac{x(3)(0.02x+4)^2(0.02) - (0.02x+4)^3(1)}{x^2}\right]$$

صعب أوجد المشتقة الثانية so ← بسط الأولى

$$= \frac{(100)\boxed{(0.02x+4)^2}}{\boxed{x^2}}\left[0.06 - 0.02x - 4\right]$$

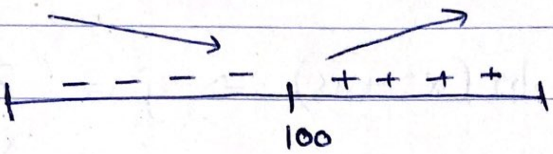
x^2 ← دائماً موجبة

x = صفر ← غير معرف ← ما بستفيد منها

هاي ما رح أستفيد من إشارتها ← دائماً موجبة

هاي اللي بتهمني

$\Rightarrow 0.04x - 4 = 0 \quad \Rightarrow \boxed{x = 100}$



$x = 100$ is min.

$\Rightarrow$ The minimum average cost is $\bar{C}(100) \left(= \frac{C(100)}{100} \right)$

$$= \frac{\cancel{100}\left((0.02)(100)+4\right)^3}{\cancel{100}} = 6^3 = \boxed{\$216} \;\#$$

عند ال units إلي رح تعطيني min. ave. cost هو

$\Rightarrow$ 100 × 100 = 10,000 units.

الوحدة بـ ١٠٠

Nov 5
Tuesday

Chapter 11: Derivative continued

* Sec: 11.1, 11.2 $\Rightarrow$ Derivative of exponential and logarithmic functions

$y = a^x$, $y = e^x$ } مبدئيا نمشي على
$y = \log_a x$, $y = \ln x$ } هدول القوانين

* **Rules**:

① $\frac{d}{dx} \ln x = \frac{1}{x}$ [(زي اللوغاريتمات) $\frac{\text{مشتقة ما بداخل ln}}{\text{ما بداخل ln}}$]

$$\frac{d}{dx} \ln\left[u(x)\right] = \frac{u'(x)}{u(x)}$$

Ex $\rightarrow$

Examples: $\boxed{a-}$ $y = \ln 5x \Rightarrow y' = \frac{5}{5x} = \frac{1}{x}$

$\boxed{b-}$ $y = \ln(x^2+10) \Rightarrow y' = \frac{2x}{x^2+10}$

$\boxed{c-}$ $y = \ln\sqrt{x} = \ln x^{1/2} = \frac{1}{2}\ln x$

$y' = \frac{1}{2}\cdot\frac{1}{x} = \frac{1}{2x}$

$\boxed{d-}$ $y = \sqrt{\ln x} = (\ln x)^{1/2}$

$y' = \frac{1}{2}(\ln x)^{-1/2}\cdot\frac{1}{x} = \frac{\ln x}{2x\sqrt{\ln x}}$

$\boxed{e-}$ $y = x^2 \ln x$

$y' = x^2\cdot\frac{1}{x} + 2x\ln x$

$= x + 2x\ln x.$

Ex 12/708: Find $\frac{ds}{dq}$ if $s = \ln\left(\frac{q^2}{4}+1\right)$

$\Rightarrow \frac{ds}{dq} = \frac{1}{2}q \Big/ \frac{q^2}{4}+1.$

$\boxed{22}$ Find $\frac{dy}{dx}$ if $y = \ln\left(\frac{3x+2}{x^2-5}\right)^{1/4}$

$y = \frac{1}{4}\ln\left(\frac{3x+2}{x^2-5}\right) = \frac{1}{4}\left[\ln 3x+2 - \ln x^2-5\right]$

$\frac{dy}{dx} = \frac{1}{4}\left[\frac{3}{3x+2} - \frac{2x}{x^2-5}\right].$

في الامتحان سؤال شبيه فيه .

(46) Suppose that the supply of x units of a product at price p dollars per unit is given by :

$$P = 10 + 50 \ln(3x+1) \cdot$$

a- Find the rate of change of supply price with respect to the number of units supplied.

$$\Rightarrow \frac{dP}{dx} = 50 \cdot \frac{3}{3x+1} = \frac{150}{3x+1}$$

b- Find the rate of change of supply price when the number of units is 33 .

$$\Rightarrow \left.\frac{dP}{dx}\right|_{x=33} = \frac{150}{3(33)+1} = 1.5 \text{ \$ per unit}$$

$\left(\frac{dP}{dq}\right.$ نفس$\left.\right)$

الـ إذا $\underset{\smile}{\text{الطلب}}\overset{(D)}{}$ زاد وحدة واحدة ، ال Supply بزيد (P) بمقدار \$1.5

c- Approximate the price increase associated with the number of units supplied changing from 33 to 34 .

$$\Rightarrow P(34) - P(33) \approx 1.5$$

Rule ② : $\log_b a = \frac{\ln a}{\ln b}$

$$* \quad \frac{d}{dx}\left(\log_a x\right) = \frac{d}{dx}\left(\frac{\ln x}{\underset{\text{عدد}}{\boxed{\ln a}}}\right) = \frac{1}{\ln a} \cdot \frac{1}{x}$$

$$* \quad \frac{d}{dx}\left(\log_a u(x)\right) = \frac{u'(x)}{u(x)\ln a}$$

$\longrightarrow$

* Examples :

① $\frac{d}{dx}\left(\log_5 x\right) = \frac{1}{\ln 5 . x}$

② $\frac{d}{dx}\left(\log x\right) = \frac{1}{x \ln 10}$.

③ $\frac{d}{dx}\left(\log_2 (x^2+2)\right) = \frac{2x}{(\ln 2)(x^2+2)}$

Rule ③ : ✱ $\frac{d}{dx}(e^x) = e^x$ ⟹ (نفسها) [ص = (ص)′]

✱ $\frac{d}{dx}(e^{u(x)}) = e^{u(x)} . u'(x)$.

Examples : ① $\frac{d}{dx}(e^{10x}) = e^{10x} . 10$

② $\frac{d}{dx}(e^{100}) = 0$ (⟹ e^{100} = عدد)

③ $\frac{d}{dx}\left(\underset{=x}{e^{\ln x}}\right) = 1$

④ $\frac{d}{dx}\left(\sqrt{e^x}\right) = \frac{d}{dx}(e^x)^{1/2} = \frac{d}{dx} e^{x/2}$

$= e^{x/2} . \frac{1}{2}$

Rule ④ : $\frac{d}{dx} a^x = a^x . \ln a$ ⇒ V. important.

⊛ $\frac{d}{dx}\left(a^{u(x)}\right) = a^{u(x)} . \ln a . u'(x)$.

* Examples: ① $\frac{d}{dx} 5^x = 5^x . \ln 5 . 1$

② $\frac{d}{dx}\left(10^{2x+3}\right) = 10^{2x+3} . 2 . \ln 10$

③ $\frac{d}{dx}\left(e^x\right) = e^x . \ln e = e^x$.

④ Find the equation of the tangent to $y = 5^{x+2}$ at $x = 0$

$y' = 5^{x+2} . \ln 5$

$y'\big|_{x=0} = 25 \ln 5$ = slope.

at $x=0$ → $y = 5^2 = 25$ ⇒ $(0, 25)$

⇒ $y - 25 = 25 \ln 5 \, (x - 0)$

$y = 25x \ln 5 + 25 = \boxed{25(x \ln 5 + 1)}$ #

[25] $y = e^{-3x} \ln(2x)$

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$y' = -3e^{-3x} . \ln(2x) + e^{-3x} . \frac{1}{x}$.