جامعة بيرزيت
BIRZEIT UNIVERSITY

Summary

MATH 2351

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قال تعالى: ﴿ مَنْ عَمِلَ صَالِحًا مِنْ ذَكَرٍ أَوْ أُنْثَى وَهُوَ مُؤْمِنٌ فَلَنُحْيِيَنَّهُ حَيَاةً طَيِّبَةً وَلَنَجْزِيَنَّهُمْ أَجْرَهُمْ بِأَحْسَنِ مَا كَانُوا يَعْمَلُونَ ﴾

Section 1.6

(Application of Function in Business and Economic)

* 1.6 / Applications of functions in Business and economic.

التطبيق الاقترانات في الأعمال التجارية والاقتصاد.

الإيرادات

* Revenue $\xrightarrow{\text{رمزها}}$ $R(X)$.
* التكلفة Cost $\xrightarrow{\text{رمزها}}$ $C(X)$.
* الأرباح profit $\xrightarrow{\text{رمزها}}$ $p(X)$.

* Revenue :

↳ is result from The sale of item ← هي ناتج من البيع....

عبارة عن صيغة الاقتران. ← : $f(X) = aX + b$

① aX → الـ slope و ② b → الـ y-intercept

* example :

find the slope and y-intercept, if $f(X) = 10X + 50$.

solution :

① the slope = $\boxed{10}$ ② y-intercept = $\boxed{50}$

*

Revenue $= p \cdot X$

→ p → (price per unit) سعر كل وحدة

→ X → (number of units) عدد الوحدات

* Cost :

* the equation المعادلة

: Cost = variable costs + fixed costs

: التكلفة = التكاليف متغيرة + التكاليف ثابتة.

تعريف → Variable costs : are those تلك directly مباشرة related to the numbers of units produced.

← هي تلك المرتبطة مباشرة بأعداد وحدات الإنتاج.

يرمزله $\boxed{VC}$.

تعريف → Fixed costs : remain تبقى constants ثابتة regardless of بغض النظر the numbers of units produced.

← تبقى ثابتة بغض النظر عن أعداد وحدات الإنتاج.

يرمز له $\boxed{FC}$.

* variable cost = cost per unit • number of units.

* التكلفة المتغيرة = تكلفة كل وحدة • عدد الوحدات.

بتمثل عدد الوحدات

$\boxed{a x + b}$

تمثل "تكلفة كل وحدة". ← fixed cost.

* marginal cost (MC) * تعريف : التكلفة الحدية :

→ The marginal cost is the slope "the cost Line", production of each additional Unit.

← التكلفة الحدية هي الميل "خط التكلفة"، وإنتاج كل وحدة إضافية.

* example : (1) :

if the cost of producing, 50 unit is 1000, and the cost of producing 100 unit is 1100, assume Line cost model, find the cost : :

* إذا التكلفة الإنتاجية 50 وحدة هي 1000 و التكلفة الإنتاجية 100 وحدة هي 1100 ، جد التكلفة :

: السؤال طالب : معادلة التكلفة / الإقتران

* assume line cost model ⇐

* find the cost ⇐

solution :

لما تكون السؤال معطين ~~أرقام نقاط~~ نقطتين :

$\Rightarrow C(\overset{x_1}{50}) = \overset{y_1}{1000} \Rightarrow (\overset{x_1}{50}, \overset{y_1}{1000})$

$\Rightarrow C(\overset{x_2}{100}) = \overset{y_2}{1100} \Rightarrow (\overset{x_2}{100}, \overset{y_2}{1100})$

ف لإيجاد الإقتران منحل على قاعدة الميل بعد إيجاد الميل :

$\therefore$ slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{1100 - 1000}{100 - 50} = \dfrac{100}{50} = 2 \rightarrow$ slope

$\therefore \quad y - y_1 = m(x - x_1)$ ⇐ منا خذ أي نقطة

$y - 1000 = 2(x - 50)$ فلنكن الأولى (50, 1000)

$y - \cancel{1000} = 2x - 100$

$+ \cancel{1000} \qquad + 1000$

$y = 2x + 900$

$\therefore \boxed{C(x) = 2x + 900}$.

* example (2):

If the cost of producing 60 unit is 2000, and the cost of producing 110 unit is 2500, assume Line cost model, find the cost: ..

solution:

$C(60) = 2000 \Rightarrow (60, 2000)$ (x_1, y_1)

$C(110) = 2500 \Rightarrow (110, 2500)$ (x_2, y_2)

$\therefore$ slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2500 - 2000}{110 - 60} = \dfrac{500}{50} = \boxed{10}$ → the slope.

$\therefore$ $y - y_1 = m(x - x_1)$

$y - 2000 = 10(x - 60)$

$y - 2000 = 10x - 600$
$+2000 \qquad +2000$

$y = 10x + 1400$

$\therefore$ $\boxed{C(x) = 10x + 1400}$

* marginal revenue : (MR) *تعريف : الإيرادات الحدية :

→ The marginal revenue is the slope "revenue Line", productions and selling of each additional unit.

← الإيرادات الحدية هي الميل "الإيراد الخطي"، والناتج كل وحدة إضافية.

* profit: الربح

→ net procees (الربح) or what remains (لتبقى) from revenue after costs substreat.

← صافي الربح أو ما تبقى من الإيرادات بعد التكاليف المزعية

* profit = revenue − cost

$\boxed{P(X) = R(X) - C(X)}$

* الربح = الإيرال ـ التكلفة

* $p(X) \longrightarrow$ profit > 0 : "profit" ← إذا الربح كان أكبر من صفر فيكون في عنا ربح

* $p(X) \longrightarrow$ profit < 0 : "Loss" ← إذا الربح كان أقل من صفر بكون في عنا خسارة

* $p(X) \longrightarrow$ profit $= 0$: "Break even" ← إذا الربح تساوى صفر ف هاد يعني انه

الايرادات تساوت مع التكاليف فيوجد خسارة

* marginal profit (Mp) :

→ The marginal profit is the slope "profitLine", production of each additional unit.

* example :

A manufacturer sells a product for \$ 10 per unit. The manufacturer's variable costs are \$ 2.50 per unit and the cost of 100 units is \$ 1450. How many units must the manufacturer produce each month to break even?

solution :

: break even = revenue − cost

: revenue $\Rightarrow p \cdot X$

$\boxed{10X}$

: cost $\Rightarrow$ variable cost $= \boxed{2.50}$

$\Rightarrow C(100) = 1450$

$\therefore C - 1450 = 2.50(X - 100)$

$C - 1450 = 2.50X - 250$

$+1450 \qquad\qquad +1450$

$\boxed{C = 2.50X + 1200}$

break even $= 10X = 2.50X + 1200$

$10X - 2.50X = 1200$

$= 7.5X = 1200 \Rightarrow \boxed{X = 160}$

* example:

A company break-even if it sale are 36000, if yearly fixed cost are $ 12000, and each sale for $ 30.

* شركة الـ breakeven تاعها، إذا البيع هو 36000 $، إذا التكلفة الثابتة سنويًا هي 12000، و الـ sale يساوي 30

(a)- find revenue, cost, profit function?

المعطيات:

break even = 36000, fixed cost = 12000, Revenue ⇒ price = 30 per unit

:: solution:

* revenue function:

$\Rightarrow R(x) = p \cdot x$

$= \boxed{30x}$

* cost function:

$\Rightarrow$ cost = variable cost + fixed cost

ax + fixed cost (FC).

a بالناقيمة ← $\boxed{ax + 12000}$

:: break-even = revenue

$$\frac{36000}{30} = \frac{30x}{30}$$

$1200 = x$

:: $ax + 12000$

$36000 = a\,1200 + 12000$

$-12000 \qquad - 12000$

$$\frac{24000}{1200} = \frac{a\,1200}{1200}$$

$20 = a$.

:: $\boxed{c(x) = 20x + 12000}$

* profit function

$p(x) = R(x) - c(x)$

$= 30x - [20x + 12000]$

$= \cancel{10x}$

$\boxed{p(x) = 10x - 12000}$

(b). if the fixed cost increased to 14000, and variable cost decreased to 16 $ per unit, find the new B.E :

* اطانا التكلفة الثابتة زادت الى 14000 و التكلفة المتغيرة نقصت الى 16 كل وحدة جد ال breakeven الجديد

solution :

new. break-even

* revenue function

$\Rightarrow R(X) = 30X$

* cost $\Rightarrow$ variable cost + fixed cost

$$ax + b$$

$$\Rightarrow \boxed{C(X) = 16X + 14000}$$

∴ new. break even

= revenue = cost

$$R(X) = C(X)$$

$$30X = 16X + 14000$$

$$30X - 16X = 14000$$

$$\frac{14X}{14} = \frac{14000}{14}$$

نعوضها في أي معادلة عشان اطلع (Y) فالتكل الإيفنو، $\boxed{X = 1000}$.

$$\therefore R(X) = 30X \Rightarrow X = 1000$$

$$= 30(1000)$$

$$= \boxed{30{,}000}$$

$$\therefore \text{new B.E} \Rightarrow \overset{X,\ Y}{(1000, 30{,}000)}$$

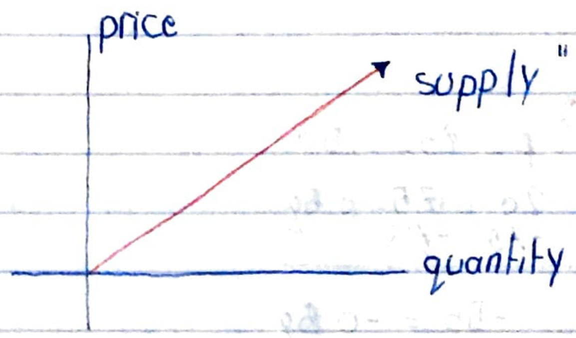
* supply, Demand and equilibrium.

* supply. العرض

→ if Low the supply: ①- increase the price, ②- increase the quantity.

←إذا انخفض العرض: ①- زاد السعر، ②- زادت الكمية.

→ Graph. *رسمتها



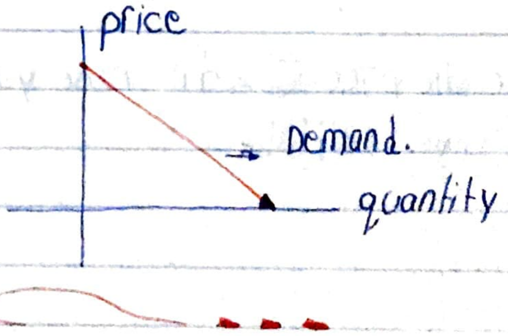
* Demand. الطلب

→ if high the supply: discrease the quantity.

ارتفع

←إذا ارتفع العرض: قلت الكمية.

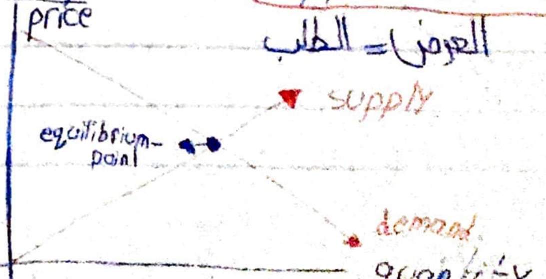
→ Graph رسمتها



* equilibrium - point:

: equilibrium - point ⇒ supply = demand.

العرض = الطلب



* classification "surpluss, shortag"

* التصنيف "زيادة، نقص".

①- supply > demand: surpluss

②- demand > supply: shortage

* example:
Consider of the following supply and demand function:

$S: p = 15 + 0.1q$
$D: p = 75 - 0.5q$

Determine wether there is a shortage or sorpluss at price of $20.

solution:

∴ price ⇒ $\boxed{20}$

$S: p = 15 + 0.1q$
$20 = \cancel{15} + 0.1q$
$-15 \quad -\cancel{15}$

$\frac{5}{0.1} = \frac{\cancel{0.1}q}{\cancel{0.1}}$

$S \Leftarrow \{50 = q\}$

$D: p = 75 - 0.5q$
$20 = \cancel{75} - 0.5q$
$-75 \quad -\cancel{75}$

$\frac{-55}{-0.5} = \frac{\cancel{-0.5}q}{\cancel{-0.5}}$

$D \Leftarrow \{110 = q\}$

∴ Demand > supply ⇒ shortage.

q ↔ q
110 > 50

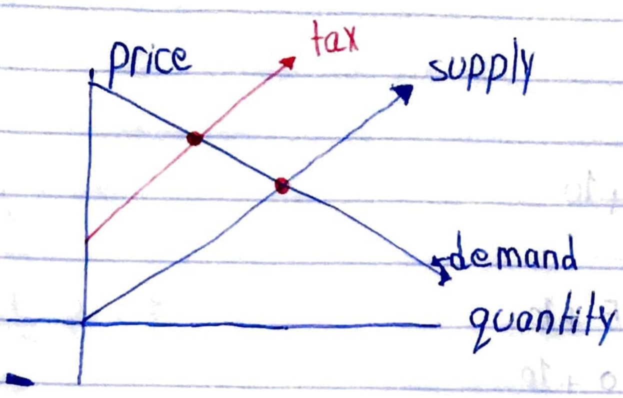
* Tax　　　　　　　　*الضريبة:
→ if increase tax: ①. increase the price. ②- discrease quantity.
اذا زادت الضريبة: ①- زاد السعر. ②- قلت الكمية.

→ * supply ⇒ p = f(a)

* Demand ⇒ p = f(b)

∴ $\{ P_{new} = f(a) + tax \}$

* Graph "tax".



* example:

The demand for a certain commtity is $5p + 2X = 200$, and supply is $p = \frac{4}{5}X + 10$.

@- find the equilibrium price and quantity.

solution:

- Demand $\Rightarrow 5p + 2X = 200$
- supply $\Rightarrow \boxed{p = \frac{4}{5}X + 10}$

$D: 5p + 2X = 200$

$-2X \quad -2X$

$\Rightarrow \frac{5p}{5} = \frac{200 - 2X}{5}$

$P = \frac{-2X}{5} + \frac{200}{5} \Rightarrow p = \frac{-2X}{5} + 40$

∴ equilibrum-point $\Rightarrow$ supply = demand

$\frac{4}{5}X + 10 = \frac{-2X}{5} + 40$

$-10 \quad -10$

$\frac{4}{5}X = \frac{-2X}{5} + 30$

$0 = -\frac{4X}{5} - \frac{2X}{5} + 30$

$0 = \frac{-6X}{5} + 30$

$-30 \quad -30$

$-30 \div \frac{-6}{5}$

$\Rightarrow -30 \cdot \frac{5}{-6}$

$\Rightarrow \frac{-150}{-6} \Rightarrow \boxed{25}$

$\frac{-30}{\frac{-6}{5}} = \frac{\frac{-6}{5}X}{\frac{-6}{5}} \Rightarrow X = 25$

$\boxed{X = 25}$

← نعوّض بنفس العادلات
المعادلات

$\therefore \frac{4}{5}X + 10$

$\Rightarrow \frac{4}{5} \cdot 25 + 10$

$\Rightarrow 20 + 10$

$P = \boxed{30}$

$\therefore$ equilibrium-point $\Rightarrow \overset{X,\ P}{(25, 30)}$.

(b). find the equilibrium price and quantity after a tax of \$6 per unit imposed.

solution:

$\therefore$ tax $\Rightarrow P_{new} = \underbrace{f(a)}_{supply} + tax$

$= \frac{4}{5}X + 10 + 6$

$= \boxed{\frac{4}{5}X + 16}$

$\therefore$ equilibrium = supply = demand

$\frac{4}{5}X + \cancel{16} = \frac{-2X}{5} + 40$

$-16 \qquad -16$

$\frac{4}{5}X = \frac{-2X}{5} + 24$

$0 = -\frac{4}{5}X - \frac{2X}{5} + 24$

$0 = \frac{-6}{5}X + 24$

$-24 \qquad -24$

$-24 = \frac{-6}{5}X$

$\frac{-24}{\frac{-6}{5}} \qquad \frac{\frac{-6}{5}X}{\frac{-6}{5}}$

$-24 \div \frac{-6}{5}$

$\Rightarrow \boxed{-24 \cdot \frac{5}{-6}}$

$\Rightarrow \frac{-120}{-6} = \boxed{20}$

$\frac{-24}{\frac{-6}{5}} = X \Rightarrow \boxed{20}$

$\boxed{10}$

$\boxed{X = 20}$

نعوضها في احدى المعادلات.

$\frac{4}{5}X + 16$

$\frac{4}{5} \cdot 20 + 16$

$\Rightarrow 16 + 16$

$\therefore \boxed{p = 32}$.

$\therefore$ equilibrium-point "tax" $= \overset{X}{(20}, \overset{p}{32)}$ ✓

* example (2):
if supply and demand function given by:

$D: 5p + 2X = 200$

$S: 5p = 4X + 50$

find the equilibrium price and quantity after Tax of 8 per unit impose:

solution:

$\frac{5p}{5} = \frac{4X + 50}{5}$

$\therefore p = \frac{4X}{5} + 10$

supply

Tax $\Rightarrow p_{new} = f(a) + tax$

$= \frac{4X}{5} + 10 + 8$

$= \boxed{\frac{4X}{5} + 18}$

$5p + 2X = 200$

$-2X \quad -2X$

$\frac{5p}{5} = \frac{-2X + 200}{5}$

$p = \frac{-2X}{5} + 40$

$\therefore$ equilibrium-point

$=$ supply $=$ demand

$\frac{4X}{5} + 18 = \frac{-2X}{5} + 40$

$-18 \quad -18$

$\frac{4X}{5} = \frac{-2X}{5} + 22$

$-22 = \frac{-6X}{5}$

$-22 \cdot \frac{5}{-6} = \boxed{18.3}$

$\Rightarrow \therefore \boxed{X = 18.3}$

$\boxed{X = 18.3}$

بتعويضها في احدى المعادلات

$\frac{4}{5}X + 18$

$\therefore \frac{4}{5}(18.3) + 18$

$P = \boxed{32.6}$.

$\therefore$ equilibrium-point = $(18.3, 32.6)$ "tax"

X, P

* example :

$\therefore R(X) \Rightarrow p = 50$ "price per unit"

* suppose a firm manufactures MP3 players and sells them for $50 each. The costs incurred in the production and sale of the MP3 players are $200,000 plus $10 for each player produced and sold.

fixed cost

$\hookrightarrow a$

Write the profit function for the production and sale of X players.

solution :

profit function = revenue − cost

$\therefore p(X) = R(X) - c(X)$.

① revenue function :

$\Rightarrow R(X) = p \cdot X$

$= \boxed{50X}$.

② cost function :

$\Rightarrow$ * cost = variable cost + fixed cost

$(a \cdot X + \text{fixed cost})$

$\Rightarrow (10X + 200{,}000)$.

$\therefore p(X) = R(X) - c(X)$

$= 50X - [10X + 200{,}000]$

$= 50X - 10X - 200{,}000$

$\therefore \boxed{p(X) = 40X - 200{,}000}$.

* example:

suppose that the cost (in dollars) for a product is $C = 21.75x + 4890$. What is the marginal cost for this product, and what does it mean?

solution:

∴ marginal cost ⇒ "the slope" = $\boxed{21.75}$.

∴ it mean ⇒ marginal cost is the slope "cost line", production of each additional unit will be "21.75", more, at any level of production.

cost

∴ ziad alulu.

∴ قال تعالى: "مَنْ عَمِلَ صَالِحًا فَلِنَفْسِهِ وَمَنْ أَسَاءَ فَعَلَيْهَا وَمَا رَبُّكَ بِظَلَّامٍ لِلْعَبِيدِ"

...

* حل أسئلة الأوت لاين: (1, 6, 13, 15, 23)...

* exercises:

* E1: suppose a calculator manufacturer has the total cost function $C(X) = 34X + 6800$, and the total revenue function $R(X) = 68X$.

(a)- What is the equation of the profit function for the calculator?

solution:

∴ profit function = revenue − cost

$$\boxed{p(X) = R(X) - C(X)}$$

$$= 68X - [34X + 6800]$$

$$= 68X - 34X - 6800$$

$$\boxed{p(X) = 34X - 6800}$$

(b)- what is the profit on 3000 units?

solution:

$$\boxed{X = 3000}$$

∴ profit function ⟶ $p(X) = 34X - 6800$

$$= p(3000) = 34(3000) - 6800$$

$$= 102{,}000 - 6800$$

$$= \boxed{95{,}200}.$$

* E6: A Linear cost function is $C(X) = 27.55X + 5180$.

(a)- what are the slope and the C-intercept?

solution:

* the slope ⟶ $\boxed{27.55}$, * C-intercept ⟶ $\boxed{5180}$.

(b)- what is the marginal cost, and what does it mean?

solution: marginal cost ⟶ $\boxed{27.55}$, * it mean:

⟶ The marginal cost is the slope "cost line", production of each additional unit. at any level of production.

ⓒ- How are your answer to parts (a) and (b) ralated?

solution:

$\Rightarrow$ the slope equal the marginal cost, C-intercept = fixed costs.

ⓓ- what is the cost of producing one more item if 50 are currently being produced? What is it if 100 are currently being produced?

solution: $C(X) = 27.55X + 5180$

* producing one more

$X = 50 \Rightarrow C(50) = 27.55(50) + 5180 = 1377.5 + 5180 = \boxed{6,557.5}$

$\boxed{X = 51} \Rightarrow 27.55(51) + 5180 = \boxed{6,585.05}$

$C(X) = 27.55X + 5180$

$C(100) = 27.55(100) + 5180 = \boxed{7,935}$

* E13: Extreme protection, Inc. manufactures helmets for sking and snow boarding are $6600 per month. Materials and Labor for each helmets of this model are $35, and the company sells this helmet to dealers for $60 each.

ⓐ- For this helmet, write the function for monthly total costs.

solution:

function $\Rightarrow$ total cost

∴ cost = variable cost + fixed cost

(aX + fixed cost)

$C(X) = 35X + 6600$

* المعطيات:

* fixed cost = 6600

* varable cost = 35

* Revenue $\Rightarrow$ p "price per unit" = 60

ⓑ- Write the function for total revenue:

solution:

function $\Rightarrow$ total revenue

$R(X) = p \cdot X$

$= \boxed{60X}$

ⓒ- write the function for profit:

solution:

function $\Rightarrow$ profit

∴ profit = revenue - cost

$p(X) = R(X) - C(X)$

$= 60X - [35X + 6600]$

$= 60X - 35X - 6600$

$p(X) = (25X - 6600)$

ⓓ- Find $C(200)$, $R(200)$, and $p(200)$ and interpret each answer.

solution:

$C(200) \Rightarrow C(X) = 35X + 6600$

$C(200) = 35(200) + 6600$

$= 7000 + 6600$

$= \boxed{\$13600}$.

∴ interpret "التفسير" ⟹ \$13600 the cost of producing 200 iterms.

$R(200) \Rightarrow 60X$

$= 60 \cdot 200$

$= \boxed{12{,}000}$.

∴ interpret "التفسير" ⟹ The total revenue of producing and selling 200 items.

$p(200) \Rightarrow p(X) = 25X - 6600$

$p(200) = 25(200) - 6600$

$= 5000 - 6600$

$= \boxed{-1{,}600}$.

∴ interpret "التفسير" ⟹ we will lose \$1600 from producing and selling 200 units.

ⓕ- Find the marginal profit and write a sentence that explains its meaning.

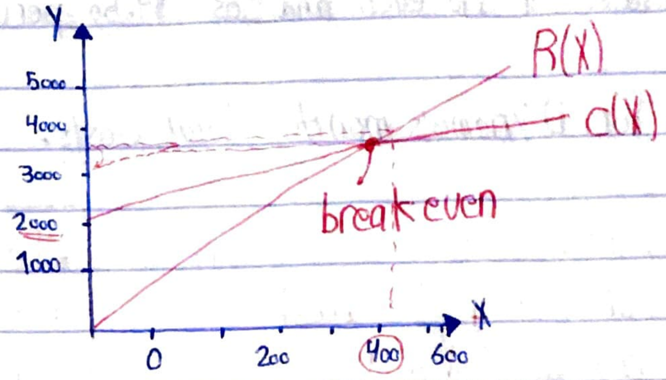
solution:

$p(X) = 25X - 6600$

⟹ the marginal profit = 25

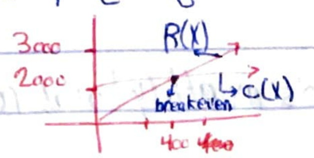
⟹ meaning: the marginal profit is the slope, production of each additional unit will profit "25", at any Level production.

E (15). The figure shows graphs of the total cost function and the total revenue function for a commodity.



(a)- Label each function correctly. سمية كل وظيفة بالشكل الصحيح

solution: →



(b)- Determine the fixed costs.

solution: fixed costs ⟹ 2000

(c)- Locate the break-even point and determine the number of units sold to break even.

solution:

break even (400, 3000).

(d)- Estimate the marginal cost and marginal revenue.

solution: * the marginal cost ⟹ $(\overset{x_1}{0}, \overset{y_1}{2000})$.

* the marginal revenue ⟹ $(\overset{x_2}{400}, \overset{y_2}{3000})$.

the slope $= \frac{3000-2000}{400-0} = \frac{1000}{400} = \boxed{2.5}$

قال تعالى: "وما تقدموا لأنفسكم من خير تجدوه عند الله، إن الله بما تعملون بصير".

E 23: Electronic equipment manufacturer Dynamo Electric, Inc. makes several types of surge protectors. Their base model surge protector has monthly fixed costs of $1045. This particular model wholesales for $10 each and costs $4.50 per unit to manufacture.

ⓐ- write the function for Dynamo's monthly total costs.

solution:

function ⟹ total cost

cost = variable cost + fixed cost

$= (aX + \text{fixedcost})$

$C(X) = 4.50X + 1045$

*الملاحظات:

* fixed cost = $\boxed{1045}$.
* variable cost = $\boxed{4.50}$.
* revenue ⟹ p "price per unit"

$p = \boxed{10}$.

ⓑ- write the function for Dynamo's monthly total revenue.

solution:

Function ⟹ total revenue

$R(X) = p \cdot X$

$= \boxed{10X}$

ⓒ- write the function for Dynamo's monthly profit.

solution:

function profit:

∴ profit = revenue − cost

$p(X) = R(X) - C(X)$.

$= \cancel{10X - 4a}$

$p(X) = 10X - [4.50X + 1045]$.

$= \cancel{10X - 4.50X - 1045}$

$= \cancel{5.5X - 1045}$

$= \cancel{1{,}039.5X}$

ⓓ- Find the number of this type of surge protector that Dynamo must produce and sell each month to break even.

solution: break even = (revenue = cost)

$R(X) = C(X)$

$10X = 4.50X + 1045$

$10X - 4.50X = 1045$

$= \dfrac{5.5X}{5.5} = \dfrac{1045}{5.5} \Longrightarrow \boxed{X = 190}$ ..ziad alulu.

Section 2.3

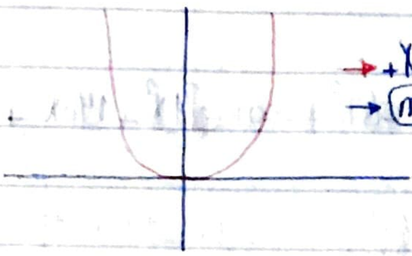
(Business Application Using Quadratics)

2.3 / Business application using quadratics. *الاقتران التربيعي

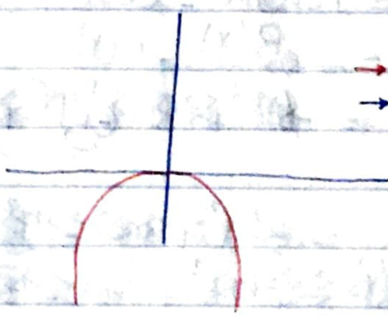
* Quadratic function : *الاقتران التربيعي

صيغته → $ax^2 + bx - c$, a, b, and c $\neq 0$
number.

* Graph - Quadratic function :



→ $+x^2$
→ minimum



→ $-x^2$
→ maximum

* $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

* vertex :

$$x = \left(\frac{-b}{2a}\right), \quad f(x) \Rightarrow f\left(\frac{-b}{2a}\right)$$

* example :

a company has a fixed cost of 150 for its product and variable cost given by 140 + 4x dollar per unit. where x is the total number of unit. The demand function for the product is given by $p = 300 - 6x$.

@- write cost, revenue and profit function.

*المعطيات :

* fixed cost = 150
* variable cost = $140 + 4x$
* demand function = $p = 300 - 6x$

solution :

Cost = variable cost + fixed cost

$$ax + b$$

$$(140 + 4x)\,x + 150$$

$$140x + 4x^2 + 150$$

$$\therefore \boxed{C(x) = 4x^2 + 140x + 150}.$$

∴ Revenue $= p \cdot X$

$p = 300 - 6X$

$= (300 - 6X)\, X$

$= 300X - 6X^2$

$= \boxed{-6X^2 + 300X}$.

∴ profit function:

profit = revenue - cost

$p(X) = R(X) - c(X)$

$= \cancel{-6X^2 + 300X - [4X^2 + 140X + 150]}$ $= -6X^2 + 300X - [4X^2 + 140X + 150]$

$= \cancel{-6X^2 + 300X - 4X^2 - 140X - 150}$ $= -6X^2 + 300X - 4X^2 - 140X - 150$

$\cancel{p(X) = -10X^2 + 300X - 150}$ $\boxed{p(X) = -10X^2 + 160X - 150}$.

$\cancel{2X^2 + 3X^2 = 5X^2}$

Ⓑ- find the break even point(s).

solution:

∴ break even $= \boxed{\text{revenue} = \text{cost}}$

$= R(X) = c(X)$

$= -6X^2 + 300X = 4X^2 + 140X + 150$

$-6X^2 + 300X - 4X^2 - 140X = 150$

$= \boxed{-10X^2 + 160X = 150}$

$= \frac{-10X^2}{-10} + \frac{160X}{-10} - \frac{150}{-10} = \frac{0}{-10}$

$\boxed{X^2 + -16X + 15 = 0}$

$(X - 1)\ (X - 15) = 0$

∴ $X - 1 = 0$ or $X - 15 = 0$

$\boxed{X = 1}$ or $\boxed{X = 15}$

∴ التعويض في أي معادلة

$R(X) = -6X^2 + 300X \Rightarrow \boxed{X = 1}$

$R(1) = -6(1)^2 + 300(1)$

$= -6 + 300$

$= \boxed{294}$

$R(X) = -6X^2 + 300X \Rightarrow \boxed{X = 15}$

$R(15) = -6(15)^2 + 300(15)$

$= -1350 + 4500$

$= \boxed{3150}$.

ⓒ- find the maximum profit :

solution:

$$p(x) = -10x^2 + 160x - 150$$

$\boxed{a = -10}$, $\boxed{b = 160}$, $\boxed{c = -150}$

∴ vertex $\Rightarrow$ $x = \frac{-b}{2a}$, $f(x) = \frac{-b}{2a}$

$$x = \frac{-(160)}{2(-10)}$$

$$= \frac{+160}{+20}$$

تعويضها في الاقتران . ← $= \boxed{8}$

$$p(x) = -10x^2 + 160x - 750 \Rightarrow \boxed{x = 8}$$

$$p(8) = -10(8)^2 + 160(8) - 150$$

$$= -640 + (1280 - 150)$$

$$= -640 + 1130$$

$$= \boxed{490} \text{ maximum.}$$

ⓓ- find the maximum revenue :

solution:

$$R(x) = -6x^2 + 300x$$

$\boxed{a = -6}$, $\boxed{b = 300}$, $\boxed{c = 0}$ ← فش قيمة

vertex $\Rightarrow$ $x = \frac{-b}{2a}$, $f(x) = \frac{-b}{2a}$

$$x = \frac{-(300)}{2(-6)}$$

تعويضها في الاقتران ← $= 25$

$$R(x) = -6x^2 + 300x$$

$$R(25) = -6(25)^2 + 300(25).$$

$$= -3750 + 7500$$

$$= \boxed{3750} \text{ maximum.}$$

ⓔ- what is the price per unit that producted maximum profit:

solution:

maximum profit = $\boxed{8 = X}$

Demand: $p = -6X + 300$

عن السؤال

$\therefore p = -6X + 300$

$p = -6(8) + 300$

$= -48 + 300$

$= \boxed{252}$.

ⓕ- what is the price per unit that producted maximum revenue:

solution:

maximum revenue = $\boxed{25 = X}$.

$p = -6X + 300$

$\therefore p = -6X + 300$

$= -6(25) + 300$

$= -150 + 300$

$= \boxed{150}$.

ⓖ- what is the price per unit that producted maximum cost:

solution:

maximum cost:

$C(X) = 4X^2 + 140X + 150$

$\boxed{a = 4}$, $\boxed{b = 140}$

$\therefore$ vertex $\Rightarrow X = \frac{-b}{2a}$, $f(X) = \frac{-b}{2a}$

$= \frac{-(140)}{2(4)}$

$= \frac{-(140)}{8}$

$= \boxed{-17.5}$

$p = -6X + 300$

$p = -6X + 300$

$= -6(-17.5) + 300$

$= 105 + 300$

$= \boxed{405}$.

ⓓ. How many units should be sold to garunt no loss?

solution:

no Loss $\Rightarrow$ break even = revenue = cost

$$B(X) = C(X)$$
$$= -6X^2 + 300X = 4X^2 + 140X + 150$$

$$= -6X^2 + 300X - 4X^2 - 140X - 150$$
$$= \frac{-10X^2}{-10} + \frac{160X}{-10} - \frac{150}{-10} = \frac{0}{-10}$$

$$X^2 + -16X + 15 = 0$$
$$(X-1)(X-15)$$
$$X-1=0 \text{ or } X-15=0$$
$$\boxed{X=1} \text{ or } \boxed{X=15}.$$

$$X = \{1, 5\}$$

ⓖ. if the supply function given by $p - 2X^2 = 220$. find the equilibrium point

solution:

∴ equilibrium point = supply = demand

$$= 2X^2 + 220 = -6X + 300$$
$$= 2X^2 + 220 + 6X - 300$$

$p - 2X^2 = 220$.

supply ← $\boxed{P = 2X^2 - 220}$

demand — $P = -6X + 300$

$$= \cancel{2X^2 - 80} = \frac{2X^2}{2} + \frac{6X}{2} - \frac{80}{2} = \frac{0}{2}$$

$$\boxed{X^2 + 3X - 40 = 0}$$
$$(X+8)(X-5) = 0$$
$$X+8=0 \text{ or } X-5=0$$
$$\boxed{X=-8} \text{ or } \boxed{X=5}$$

X ✓

$$\therefore\ p = -6X + 300$$
$$= -6(5) + 300$$
$$= -30 + 300$$
$$\boxed{P = 270}.$$

∴ eqcilibriom-point = $\boxed{(5, 270)}$.

* examples:

The supply and demand for a product is given by: $2p = q + 50$ and $pq - 20q = 100$, respectively find the equilibrium point.

solution:

supply: $2p = q + 50$

demand: $pq - 20q = 100$

* equilibrium - point = $\boxed{\text{supply} = \text{demand}}$

بدلنا الـ q عشان المعادلة اللي بعدها

$$2p = q + 50$$
$$-50 \quad -50$$
$$2p - 50 = q$$

$$pq - 20q = 100 \qquad q = 2p - 50$$

$$p(2p - 50) - 20(2p - 50) = 100$$

$p \cdot 2p = 2p^2$

$$= [2p^2 - 50p] - [40p - 1000] = 100$$

$$= 2p^2 - 50p - 40p + 1000 = 100$$

عشان نتخلص من الـ 2 أقسم الـ 2 على جميع المعادلة.

$$= \frac{\cancel{2}p^2}{\cancel{2}} - \frac{90p}{2} + \frac{1000}{2} = \frac{100}{2}$$

$$= p^2 - 45 + 500 = 50$$
$$-50 \quad -50$$

بدلنا قيم الـ price

$$\boxed{p^2 - 45 + 450 = 0}$$

$\therefore$ $\boxed{a = 1}$, $\boxed{b = -45}$, $\boxed{c = 450}$

"فَصَبْرٌ جَمِيلٌ وَاللَّهُ الْمُسْتَعَانُ عَلَىٰ مَا تَصِفُونَ".

$\therefore\ p = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(-45) \pm \sqrt{(-45)^2 - 4(1)(450)}}{2(1)}$

$= \dfrac{45 \pm \sqrt{+2025 - 1800}}{2}$

$= \dfrac{45 \pm \sqrt{225}}{2}$

$\dfrac{45+15}{2}$ or $\dfrac{45-15}{2}$

$= \boxed{30 \text{ or } 15}$

$\therefore\ 2p = q + 50 \Rightarrow \boxed{p = 30}$

$2(30) = q + 50$

$60 = q + \cancel{50}$

$-50 \quad\quad -\cancel{50}$

$\boxed{10 = q}$. ✓

$\therefore\ 2p = q + 50 \Rightarrow \boxed{p = 15}$

$2(15) = q + 50$

$30 = q + \cancel{50}$

$-50 \quad\quad -\cancel{50}$

$\boxed{-20 = q}$. X

بناخذ القيمة الموجبة

* example:

For the total cost function $C(X) = 3600 + 100X + 2X^2$ and the total revenue function $R(X) = 500X - 2X^2$, find the number of units that maximizes profit and find the maximum profit:

solution: profit = revenue − cost

$[500X - 2X^2] - [3600 + 100X + 2X^2]$

$500X - 2X^2 - 3600 - 100X - 2X^2$

$\boxed{p(X) = -4X^2 + 400X - 3600}$

* maximizes profit:

$p(X) = -4X^2 + 400X - 3600$

$\boxed{a = -4}$, $\boxed{b = 400}$, $\boxed{c = -3600}$

$\therefore$ vertex $= X = \dfrac{-b}{2a}$, $f(X) = \dfrac{-b}{2a}$

number of unit

$\therefore\ X = \dfrac{-(400)}{2(-4)} = \dfrac{+400}{+8} = 50$

$X = 50$, $p(X) = -4X^2 + 400X - 3600$

$p(50) = -4(50)^2 + 400(50) - 3600$

$= -10{,}000 + 20{,}000 - 3600$

$= -10{,}000 + 16400$

$= \boxed{6{,}400}$

∴ Ziadalulu.

* حل أسئلة الأرقام كالتالي : (1, 5, 13, 17, [redacted], 32, 34) ...

* exercises ...

E: ① : The total costs for a company are given by $c(x) = 2000 + 40x + x^2$ and the total revenues are given by $R(x) = 130x$ find the break-even point:

solution:

$$\text{break even} = R(x) = c(x)$$

$$130x = 2000 + 40x + x^2$$

$$2000 - 90x + x^2 = 0$$

$$\Rightarrow x^2 - 90x + 2000 = 0$$

$$(x - 40)(x - 50) = 0$$

$$(x - 40) = 0 \quad \text{or} \quad (x - 50) = 0$$

$$\boxed{x = 40} \qquad \boxed{x = 50}$$

E: ⑤ : Given that $p(x) = 11.5x - 0.1x^2 - 150$ and that production is restricted to fewer that 75 units, find the break-even points:

solution:

$$\therefore \text{break even} = R(x) = c(x)$$

$$p(x) = 0$$

$$\frac{11.5x}{-0.1} - \frac{0.1x^2}{-0.1} - \frac{150}{-0.1} = \frac{0}{-0.1}$$

$$x^2 - 115x + 1500 = 0$$

$$(x - 100)(x - 15) = 0$$

$$x - 100 = 0 \quad \text{or} \quad x - 15 = 0$$

$$\boxed{x = 100} \quad \text{or} \quad \boxed{x = 15}$$

$$\boxed{x = 15} \ldots ✓$$

E: 13 :

(a) - Graph the profit function $p(X) = 80X - 0.1X^2 - 7000$

solution:

X - intercept :

$$p(X) = 0 \Rightarrow \frac{80X}{-0.1} - \frac{0.1X^2}{-0.1} - \frac{7000}{-0.1}$$

$$-800X + X^2 + 70000 = 0$$

$$\Rightarrow X^2 - 800X + 70000 = 0$$

$$(X - 100) = 0 \text{ or } (X - 700) = 0$$

$$\boxed{X = 110}, \boxed{X = 700}$$

$$\therefore (100, 0), (700, 0)$$

vertex:

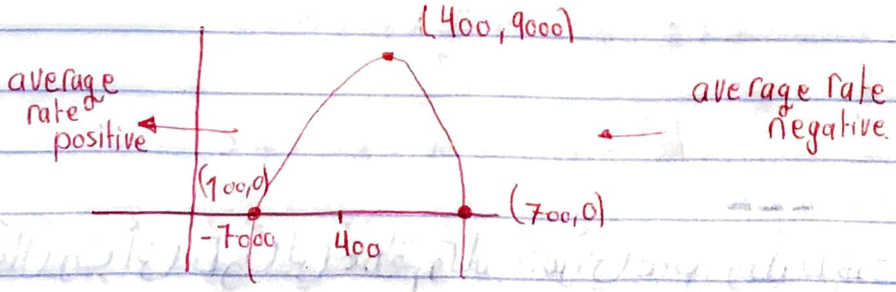
$$\left(\frac{-b}{2a}, p\left(\frac{-b}{2a}\right) \right)$$

$$\frac{-b}{2a} = \frac{-80}{2(-0.1)} = \frac{+80}{0.2} = 400$$

$$p(400) = 80(400) - 0.1(400)^2 - 7000$$

$$= 32000 - 16000 - 7000$$

$$= \boxed{9000}.$$



(b) - Find the vertex of the graph. is it a maximum point or a minimum point:

solution:

maximum point

(400, 900).

E: 17.

suppose a company has fixed cost of \$28,000 and variable cost per unit of $\frac{2}{5}X + 222$ dollars, where X is the total number of unit produced. suppose futher that the selling price of its product is $1250 - \frac{3}{5}X$ dollars per unit.

ⓐ. Find the break-even points

solution:

*المعطيات:

* fixed cost → 28000

* variable cost → $\frac{2}{5}X + 222$

* revenue → $P = 1250 - \frac{3}{5}X$

break-even = revenue = cost

$$B(X) = C(X)$$

$$\therefore B(X) = p \cdot X$$

$$\left(1250 - \frac{3}{5}X\right) \cdot X$$

$$= \boxed{1250X - \frac{3}{2}X^2} \Longrightarrow B(X) = -\frac{3}{5}X^2 + 1250X.$$

$$\therefore C(X) = aX + b$$

$$\left(\frac{2}{5}X + 222\right) \cdot X + 28000$$

$$= \boxed{\frac{2}{5}X^2 + 222X + 28000}$$

$$\therefore \text{break even} = B(X) = C(X)$$

$$= -\frac{3}{5}X^2 + 1250X = \frac{2}{5}X^2 + 222X + 28000$$

$$= \frac{-1X^2}{-1} + \frac{1028X}{-1} \frac{\pm 28000}{-1} = \frac{0}{-1}$$

$$X^2 - 1028X + 28000 = 0$$

"قَالَ رَبِّ أَنَّىٰ يَكُونُ لِي غُلَامٌ وَكَانَتِ امْرَأَتِي عَاقِرًا وَقَدْ بَلَغْتُ مِنَ الْكِبَرِ عِتِيًّا" 6

"قَالَ كَذَٰلِكَ قَالَ رَبُّكَ هُوَ عَلَيَّ هَيِّنٌ وَقَدْ خَلَقْتُكَ مِن قَبْلُ وَلَمْ تَكُ شَيْئًا".

... ...

E: 32

The supply and demand for a product are given by $2p - q = 50$ and $pq = 100 + 20q$, respectively. find the market equilibrium point:

solution:

$S: 2p - q = 50 \rightarrow 2p = 50 + q \Rightarrow \boxed{p = 25 + \frac{q}{2}}$.

$D: pq = 100 + 20q \Rightarrow \boxed{p = \frac{100 + 20q}{q}}$.

$$\frac{\left(25 + \frac{q}{2}\right)}{1} = \left(\frac{100 + 20q}{q}\right)$$

$$\left(25 + \frac{q}{2}\right) q = 100 + 20q$$

$$25 + \frac{q^2}{2} = 100 + 20q$$

$$25q + \frac{q^2}{2} - 100 - 20q = 0$$

$$2 \times \left(\frac{q^2}{2} + 5q - 100 = 0\right)$$

$$q^2 + 10q - 200 = 0$$

$$(q + 20)(q - 10) = 0$$

$$q + 20 = 0 \text{ or } q - 10 = 0$$

$$\boxed{q = -20} \text{ or } \boxed{q = 10}.$$

$$p = 25 + \frac{q}{2}$$

$$= 25 + \frac{10}{2} \Rightarrow 25 + 5 = 30$$

$\therefore (10, 30)$.

E: 34. For the product in problem 32, if a $ 12.50 tax is placed on production and passed through by the supplier, find the new equilibrium point:

solution:

$$S_{new} = p = 25 + \frac{q}{2} + 12.5 = 37.5 + \frac{q}{2}$$

$$\therefore D: p = \frac{100 + 20q}{q}$$

$$\frac{37 + \frac{q}{2}}{1} \times \frac{100 + 20q}{q}$$

$$\therefore \left(37 + \frac{q}{2}\right) q = 100 + 20q$$

$$37q + \frac{q^2}{2} - 100 - 20q = 0$$

$$\left(17q + \frac{q^2}{2} - 100 = 0\right) 2$$

$$35q + q^2 - 200 = 0$$

$$\boxed{q^2 + 35q - 200 = 0}$$

$$(q + 40)\ (q - 5) = 0$$

$$\boxed{q = -40} \text{ or } \boxed{q = 5}$$

X ✓

$$p = 37.5 + \frac{q}{2}$$

$$= 37.5 + \frac{5}{2} \Rightarrow 37.5 + 2.5 = \boxed{40}$$

$$\therefore (5, 40)$$

ⓑ- Find the maximum revenue:

solution:

$B(X) = \frac{-3}{5}X^2 + 1250X$

$\boxed{a = \frac{-3}{5}}$, $\boxed{b = 1250}$

∴ vertex $= X = \frac{-b}{2a}$, $f(X) = \frac{-b}{2a}$

$= \frac{-(1250)}{\frac{2}{1}(\frac{-3}{5})} \Rightarrow \frac{-1250}{\frac{-6}{5}} \rightarrow -1250 \div \frac{-6}{5}$

$= -1250 \cdot \frac{5}{-6}$

$\boxed{X = 1041{,}6}$

ⓒ- Form the profit function from the cost and revenue functions and find maximum profit:

solution:

profit function:

$\Rightarrow$ revenue - cost

$\boxed{P(X) = B(X) - C(X)}$

$\boxed{p.X}$ variable cost + fixedcost

$\boxed{aX+b}$

$p(X) = -1X^2 + 1028X - 28000 = 0$

$\boxed{a = -1}$, $\boxed{b = 1028}$.

∴ vertex $= X = \frac{-b}{2a}$, $f(X) = \frac{-b}{2a}$

$= \frac{-(1028)}{-2} = \frac{+1028}{+2} = (514)$.

$-1(514)^2 + 1028(514) - 28000$

$= -1028 + [528]3 - 28000$

$= -1028 + 500{,}392$

$= \$\boxed{499{,}3}$

ⓓ- what price will maximize the profit: → $\$\boxed{499{,}3}$

solution:

$\$\boxed{499{,}3}$.

∴ Ziad alulu.

جامعة بيرزيت

BIRZEIT UNIVERSITY

Mathematics Department

MATH 2351

Handout # 2: Prepared by Mohammad Madiah
Sections 1.6 and 2.3 additional problems

....

1. The cost of manufacturing 100 units of a product is \$3000. When 600 units are produced, the cost is \$5000. Find the **cost equation** (assuming linear cost model).
2. Suppose that consumers will demand 100 units of a product when the price is \$10 per unit, and 120 units when the price is \$8 per unit. Assuming that price p and quantity q are linearly related, find the **price** at which 90 units are demanded.
3. Suppose that a manufacturer sell a product for \$12 per unit. If the fixed cost is \$1600 and the variable cost is \$8 per unit find the **profit (or loss)** of selling 500 units.
4. Suppose a manufacturer will not market any unit of a product if the unit price is \$120 or lower, but is willing to market 50 at \$180 per unit. Find the linear **supply** equation.
5. If the revenue function is $R(x) = 80x - 0.2x^2$, find the **quantity demanded** when the price is \$40.
6. The cost and revenue functions for producing x number of battery packages are given by $C(x) = 5x + 60{,}000$ and $R(x) = 25x$; respectively. Find the **break-even point**
7. How **many items** does the company have to manufacture and sell to **not lose** money if the revenue function is $R(x) = 200x - 0.25x^2$ and the cost function is $C(x) = 40x + 9975$? What is the **maximum profit**?
8. A company has a profit function given by $p(x) = -100x^2 + 1000x - 2400$. Find the sales levels (i.e. x-values) where the company is **not losing money**.
9. Suppose you are given the supply and demand curves, respectively, $p - 4x = 5$ and $2p + 4x = 162$ (x = # of units, p = price in dollars).
   a. At p = \$53, is there a shortage or surplus?
   b. Is the price likely to increase from \$53 or decrease from it?
   c. What is the equilibrium point?
10. Suppose consumers purchase q units of a manufacturer's product when price per unit (in dollars) is $60 - 0.5q$. If no more than 75 units can be sold, find the number of units that must be sold in order that the revenue be \$1000
11. A manufacture sells his product at \$23 per unit. His fixed cost is 418000 and his variable cost per unit is \$18.5. The level of production at which the manufacture break-even is
12. If the supply and demand functions for a product are given by $6p - q = 60$ and $(p + 2)q = 4830$, respectively, find the price that will result in market equilibrium.
13. If the supply function is $p = x + 5$, and the demand function is $p = 25 - x^2$, find the **equilibrium point**.

Section 5.1
(Exponential Functions)

5.1/ exponential functions *الاقترانات الأسية.

* $\boxed{Form} \Rightarrow f(x) = a^x$

* use a calculator to evaluate each expressions:

1. (a) - $10^{0.5}$ (b) - $5^{-2.7}$
2. (a) - $10^{3.6}$ (b) - $8^{-2.6}$
3. (a) - $3^{\frac{1}{3}}$ = ثلاث قوة افتح قوس $\frac{1}{3}$ سكر القوس. (b) - e^2
4. (a) - $2^{\frac{11}{6}}$ (b) - e^{-3}

: solution:

1. (a) - $= \boxed{3.16}$ (b) - $= \boxed{0.012}$
2. (a) - $= \boxed{3.98}$ (b) - $= \boxed{4.48}$
3. (a) - $= \boxed{1.4}$ (b) - $= \boxed{7.3}$ ← $\boxed{shift \to e}$ ← على الآلة حاسبة.
4. (a) - $= \boxed{3.56}$. (b) $= \boxed{0.04}$.

5.2 / Logarithmic and properties. *اقترانات اللوغاريتمك.

* Form $\xrightarrow{\text{الصيغة}}$ Logarithmic function:

: $\boxed{f(x) = \log_a x}$, $a > 0$, $a \neq 1$

* examples:

* write $\boxed{64 = 4^3}$, Logarithmic function form. *أكتب الـ $64 = 4^3$ على شكل اقتران لوغاريتمي.

solution:

: Logarithmic function $\Rightarrow f(x) = \log_a x$

$64 = 4^3$, $\boxed{a = 4}$, $\boxed{x = 64}$

: $f(x) = \log_4 64$

* solve: $\log_4 64 = ??$

solution:

$\log_4 64 = \boxed{3}$ → انا الـ a الـ 4 كم رفعناها قوة لحتى صارت الـ x الـ 64 ؟ $\boxed{\text{ثلاث}}$

وهكذا...

* solve: $\log_9 81 = ??$

solution:

$\log_9 81 = \boxed{2}$.

* write $\log_4\left(\frac{1}{64}\right) = -3$, in exponential function. أكتب الـ $\log_4\left(\frac{1}{64}\right)=-3$ على شكل الاقتران الأسي *

solution:

exponential function $\Rightarrow$ $\boxed{f(x) = a^x}$

$\log_4\left(\frac{1}{64}\right) = -3$

$\therefore \; 4^{-3} = \frac{1}{64}$.

* Rule:

$\log_a x = b \quad , \quad \boxed{x = a^b}$.

* example:

Find x if $\log_2 x = 4$.

solution:

$\log_2 x = 4 \Rightarrow \boxed{a=2}, \boxed{x=?}, \boxed{b=4}$.

$x = a^b \Rightarrow 2^4 = \boxed{16} \rightarrow x$.

* example:

Evaluate $\log_2 8$.

solution:

$\log_2 8 = \boxed{3}$.

انه الـ 2 كم رفعناها قوة لحتى اعطاني أنه الـ X = 8؟ = (3)

* example:

Evaluate $\log_5\left(\frac{1}{25}\right)$

solution:

$\log_5\left(\frac{1}{25}\right) = ?\ \boxed{-2}$

لما تكون الـ X كسر فالـ b بتكون سالب "علل سالب"

وايضا لما يكون الـ X نسبة .

* Common and natural Logarithmis:

* Common Logarithmis: $\log x \xrightarrow{\text{mean}} \boxed{\log_a x} \longrightarrow \log_{10} x$

* natural Logarithmis: $\ln x \xrightarrow{\text{mean}} \boxed{\log_e x}$

* Rules:

$\log_a a^{x} = x$ $a > 0,\ a \neq 1$

* example:

Use property I to simplify each of the following:

1. $\cancel{\log_4 3^4}$ $\log_4 4^3$

solution $\log_4 4^3 \Rightarrow \log_a a^x = x \Rightarrow \boxed{3}$

2. $\ln e^x$

solution: $\ln e^x \Rightarrow \log_e e^x$

$\ln e^x = \boxed{x}$.

* Rule:

1. because $\underbrace{a^1 = a}$, we have $\underbrace{\log_a a = 1}$

2. because $\underbrace{a^0 = 1}$, we have $\underbrace{\log_a 1 = 0}$

* Rule:

$a^{\log_a x} = x$, $a > 0,\ a \neq 1$

* Rules:

$$\log_a\left(\frac{X}{Y}\right) = \log_a X - \log_a Y$$

* example:

Evaluate $\log_3\left(\frac{9}{27}\right)$

solution:

$$\log_3\left(\frac{9}{27}\right) = \log_3 9 - \log_3 27$$

$$= 2 - 3$$

$$= \boxed{-1}.$$

* example:

find $\log_{10}\left(\frac{16}{5}\right)$, if $\log_{10} 16 = 1.2041$ and $\log_{10} 5 = 0.6990$ (to 4 decimal places).

solution: $\log_{10}\left(\frac{16}{5}\right) = \log_{10} 16 - \log_{10} 5$

$$= 1.2041 - 0.6990$$

$$= \boxed{0.5051}.$$

* Rule:

$$\log_a\left(X^a\right) = a \cdot \log_a X$$

* example:

* simplify $\log_3(9^2)$.

solution:

$$\log_3(9^2) = 2 \cdot \log_3 9$$

$$= 2 \cdot 2$$

$$= \boxed{4}.$$

* examples:

simplify : $\ln 8^{-4}$, if $\ln 8 = 2.0794$ (to 4 decimal places):

solution:

$$\ln 8^{-4} = -4 \cdot \ln 8$$
$$= -4 \cdot (2.0794)$$
$$= \boxed{-8,3176} .$$

: Ziad alulu .

"فَأَمَّا مَنْ أَعْطَىٰ وَاتَّقَىٰ * وَصَدَّقَ بِالْحُسْنَىٰ * فَسَنُيَسِّرُهُ لِلْيُسْرَىٰ" ،

"وَأَمَّا مَنْ بَخِلَ وَاسْتَغْنَىٰ * وَكَذَّبَ بِالْحُسْنَىٰ * فَسَنُيَسِّرُهُ لِلْعُسْرَىٰ".

...

* exercises:

حل اسئلة السيكشن 5.2 / "اسئلة مختارة"

*Use the definition of a logarithmic function to rewrite each equation in exponential form:

① $4 = \log_2 16$

solution: exponential form $\Rightarrow a^x$

$\boxed{2^4 = 16}$

② $4 = \log_3 81$

solution:

$3^4 = 81$

③ $\frac{1}{2} = \log_4 2$

solution:

$4^{\frac{1}{2}} = 2$

④ $-2 = \log_3 \left(\frac{1}{9}\right)$

solution:

$3^{-2} = \left(\frac{1}{9}\right)$

* solve for x by writing the equation in exponential form:

① $\log_3 X = 4$

solution:

$3^4 = X \Rightarrow 81$

② $\log_{16} X = -\frac{1}{2}$

solution:

$16^{-\frac{1}{2}} = X \Rightarrow 0.25$

③. $\log_{25} X = \frac{1}{2}$

solution: $25^{\frac{1}{2}} = X \Rightarrow 5$

* write the equation in Logarithmic form:

①. $2^5 = 32$

solution:

Logarithmic form $\Rightarrow \log_a X = b$

$\log_2 32 = 5$

②. $4^{-1} = \frac{1}{4}$

solution: $\log_4 \frac{1}{4} = -1$

* evaluate each logarithmic by using properties of Logarithms and the following facts:

$\log_a X = 3.1$, $\log_a Y = 1.8$, $\log_a Z = 2.7$

①. (a) $\log_a (XY)$

solution:

$\log_a (XY) = \log_a X \cdot \log_a Y$

$= (3.1) + (1.8)$

$= \cancel{\boxed{5.58}}$. $\boxed{4.9}$

(b) $\log_a \left(\frac{X}{2}\right)$

solution:

$\log_a \left(\frac{X}{2}\right)$

$= \log_a X - \log_a Z$

$= 3.1 - 2.7$

$= \boxed{0.4}$.

②. (c) $\log_a (X^4)$

solution:

$\log_a (X^4) \Rightarrow \log_a X^4$

$4 . \log_a X$

$= 4 . (3.1)$

$= \boxed{12.4}$.

(D) $\log_a \sqrt{Y}$

solution:

$\log_a \sqrt{Y} \Rightarrow \log_a Y^{\frac{1}{2}}$

$= \frac{1}{2} \cdot \log_a Y$

$= \frac{1}{2} \cdot (1.8)$

$= \boxed{0.9}$.

~~$\log_a Y^{\frac{1}{2}}$~~

~~$\frac{1}{2} \cdot \log_a$~~

~~$= \frac{1}{2} \cdot (3.1)$~~

~~$= (1.55)$~~

: Ziad aLoLu.

Section 6.1

(Simple Interest)

6.1/ simple interest: sequence تسلسل : فائدة بسيطة *

* simple interest ⟹ $\boxed{I = p.R.T}$.

① - I يرمز → interest. "in dollar" للفائدة
② - p يرمز → principal. "in dollar" للأساس
("written as decimal" تكتب ك عشر) ← ③ - R يرمز → ~~Rate~~ annual interest rate. معدل فائدة سنة
④ - T يرمز → time "in years". للوقت

* examples: (1).
If \$ 8000 is invested for 2 years at an annual interest rate of 9 %, how much interest will be received at the end of the 2-year period.

solution:

المعطيات
* $\boxed{p = 8000}$ "p → principal "invested"
* $\boxed{T = 2}$ years
* R = 9% → $\boxed{0.09}$

المطلوب ال interest

∴ simple interest → $I = p.R.T$
$= 8000.(0.09).2$
$I = \boxed{1440}$ in dollar
ب الوحدة.

* example: (2).
If \$4000 is borrowed for 39 weeks at an annual interest rate of 15%, how much interest is due at the end of the 39 weeks?

solution:

المعطيات
* p = 4000
* R = 15% → 0.15
* T → time: $\boxed{39 \text{ weeks}}$ → $\frac{39}{52} = \boxed{0,75}$

المطلوب ال interest

∴ simple interest ⟹ $I = p.R.T$
$= 4000.(0,15).(0,75)$
$= \$\boxed{450}$. dollar

لاحظ:
* التحويل من أشهر إلى السنة ← بنقسم عدد الأشهر على (12).
* التحويل من أسابيع إلى السنة ← بنقسم عدد الأسابيع على (52).

* Future value(FV): القيمة المستقبلية *

* Rule → $\boxed{S = p + I}$. $I = p \cdot R \cdot T$

* حل الأسئلة الأوت لاين: "6, 8, 21"

$\boxed{E:6}$. $800 is invested for 5 years at an annual simple interest rate of 14%.

ⓐ- How much interest will be earned?

solution:

المطلوب الـ interest:

$I = p \cdot R \cdot T$

$= 800 \cdot (0,14) \cdot 5$

$= \$\boxed{560}$.

* المعطيات:

p: $800

T: $\boxed{5}$

R: 14% ⇒ 0.14

ⓑ- What is the future value of the investment at the end of the 5 years?

solution:

المطلوب الـ future value:

$FV = P + I$

$= 800 + 560$

$= \$\boxed{1,360}$.

P = $800

I اللي طلعناها → $560

$\boxed{E:8}$. $1800 is invested for 9 months at an annuale sample interest rate of 15%?

* المعطيات:

$p = \$1800$

$R = 15\% \rightarrow \boxed{0.15}$

$T = \frac{9}{12} = 0.75$

ⓐ- How much interest will be earned?

solution:

المطلوب الـ interest

$I = p \cdot R \cdot T$

$= 1800 \cdot (0,15) \cdot (0,75)$

$= \$\boxed{202,5}$

: Ziad aLuLu.

(b) - What is the future value of the investment after 9 months ؟

solution:

المطلوب futurevalue.

p = \$1800
I = \$202,5

$FV = p + I$

$= 1800 + 202,5$

$= \$\boxed{2,002,5}$

$\boxed{E:21}$

If \$5000 is invested at 8% annual simple interest, how Long does it take to be worth \$9000 ؟

بدي ال (T) ؟

solution:

~~FV = p + I~~

~~I = p . R . T~~

~~9000 = 5000 . (0.08) . T~~

~~9000 = 400T~~

~~400 400~~

~~22,5 = T~~

* المعطيات :

p = \$5000

R = 8% ⇒ (0.08)

S/FV = 9000

T = ؟

* عشان نوصل (T) منحل على هاد القانون

solution

: $\boxed{FV = p + I}$

$= 5000 + p . R . T$

$= 5000 + [5000 . 0.08 . T]$

$= 5000 + 400T$

$9000 = \cancel{5000} + 400T$

$- 5000 \quad - 5000$

$\frac{4000}{400} = \frac{400T}{400}$

$10 = T$

T = 10 years

: ziad alulu.

قال تعالى : "وأن ليس للإنسان إلا ما سعى".

Section 6.2
(Compound Interest)

* 6.2 / compound interest *فائدة المركبة.

* Future value (annual compound):

القيمة المستقبلية (المركب سنوي):

: annual compounding $\Rightarrow$ $\boxed{S = p(1+R)^n}$, $n \to T$: year.

* example:

If $3000 is invested for 4 years at 9% compounded annually, how much interest is earned?

solution:

* compounded annually " interest " *المعطيات:

$$S = p(1+R)^n$$

* $p = 3000$
* $R = 9\% \Rightarrow 0.09$
* $n = 4$ Years.

$$= 3000(1+0.09)^4$$

$$= 3000 \cdot (1{,}09)^4$$

$$= 3000 \cdot (1{,}41)$$

$$= \$\boxed{4{,}23}.$$

* Future value (periodic compounding)

الدورية

القيمة المستقبلية (مضاعفة الدورية):

: periodic compounding $\Rightarrow$ $\boxed{S = p\left(1+\frac{R}{m}\right)^{m.T}}$.

ال $\boxed{m=2}$ إذا كان المطلوب semi annually

و ال $\boxed{m=4}$ إذا كان المطلوب semi quarterly

وال $\boxed{m=12}$ إذا كان المطلوب semi monthly

* the total number of compounding periods:

: $\boxed{n = mt}$

1- compounded annually $\boxed{m=2}$.
2- compounded monthly $\boxed{m=12}$
3- compounded quarterly $\boxed{m=4}$

* the interest rate per compounding periods:

: $\boxed{i = \frac{r}{m}}$

* example:

For each of the following investment, find the interest rate per period, i, and the number of compounding periods, n:

a- 12% compounded monthly for 7 years:

solution:

1- the interest rate... $\Rightarrow I = \frac{R}{m}$

: $R = 12\% \Rightarrow \boxed{0.12}$

: compounded monthly $\Rightarrow \boxed{m = 12}$

$\frac{0.12}{12} = \boxed{0.01} \rightarrow I$

2- the number... $\Rightarrow n = mT$

$\boxed{84} = 12.7$

b- 7.2% compounded quarterly for 11 quarterly:

solution:

1- the interest rate... $\Rightarrow I = \frac{R}{m}$

: $R = 7.2\% = \boxed{0.072}$

: compounded quarterly $\Rightarrow m = 4$

: $\frac{0.072}{4} = \boxed{0.018}$

2- the number... $\Rightarrow n = mT \rightarrow$

$\boxed{11} = 4.(2.75)$

مهم
لما يكون الحاكي
11 quarterly
نقسم 11 ÷ 4
$= \boxed{2.75}$

$\frac{11}{4} = \frac{4.T}{4}$

$\boxed{2.75} = T$

* Future value (continuous compounding):

* القيمة المستقبلية (المضاعفة المستمرة):

∴ continuous componding ⇒ $\boxed{S = p \cdot e^{R.T}}$.

* example: ①

Find the future value if $1000 is invested for 20 years at 8 %, compounded continuously:

solution:

*المعطيات:

$p = 1000$

$R = \frac{8}{100} \Rightarrow \boxed{0.08}$

$T = 20$ years

compounded continuously

$\Rightarrow \boxed{S = p \cdot e^{R.T}}$

$= 1000 \cdot e^{(0.08)20}$

$= 1000 \cdot e^{1.6}$

$1000 \cdot (4,95303)$

$= \$\boxed{4,953,03}$.

* example: ②

What amount must be invested at 6,5% , compounded continuously, so that it will be worth $25,000 after 8 years:

p = ??

solution:

compounded continuously

$\Rightarrow S = p \cdot e^{R.T}$

$= p \cdot e$

$25,000 = p \cdot e^{(0.065)8}$

$25,000 = p \cdot e^{0.52}$

$\frac{\$25,000}{1,68202} = \frac{p \cdot (1,68202)}{1,68202}$

$\boxed{\$14,863.01 = p}$ ✓

$p = ??$

$R = 6,5\% \Rightarrow \boxed{0.065}$

$T = 8$ years.

$S = 25,000$

* example : ③.

How much more will you earn if you invest \$1000 for 5 years at 8% compounded continuously ① instead of at 8% compounded quarterly ②?

solution :

①. Compounded continuously , $P = 1000$, $R = 0.08$, $T = 5$

$$\therefore S = P \cdot e^{RT}$$

$$= 1000 \cdot e^{(0.08)5}$$

$$= 1000 \cdot e^{0.4}$$

$$= 1000 \cdot (1,491824)$$

$$= \$\boxed{1,491,824}.$$

②. compounded quarterly : $P = 1000$, $R = 0,08$, $T = 5$

$$\therefore S = P\left(1 + \frac{R}{m}\right)^{mT} \qquad \therefore \boxed{m = 4}$$

$$= 1000\left(1 + \left(\frac{0.08}{4}\right)\right)^{20}$$

$$= 1000\left(1 + (0.02)\right)^{20}$$

$$= 1000\,(1.02)^{20}$$

$$= 1000\,(1,48594)$$

$$= \$\boxed{1,485,94}.$$

∴ Thus the extra interest earned by compounding continuously is

$$1,491,824 - 1,485,94$$

$$= \boxed{\$5.87}.$$

قال تعالى :" وَمَا أَسْأَلُكُمْ عَلَيْهِ مِنْ أَجْرٍ إِنْ أَجْرِيَ إِلَّا عَلَىٰ رَبِّ الْعَالَمِينَ ".

مهم

$\underline{20{,}000} = (2 \cdot 10{,}000) = S$ يعني الـ

مضاعفة

*example:

How Long does it take an investment of \$10,000 to double if it is invested at:

Time : n بلانا فترة الـ

(a) 8%, compounded annually:

solution:

*المعطيات:

$p = 10{,}000$

$n = ??$

$R = 8\% \Rightarrow \boxed{0.08}$

compounded annually

$$\Rightarrow S = p\,(1 + R)^n$$

$$= 10{,}000\,(1 + 0.08)^n$$

$$\frac{20{,}000}{10{,}000} = \frac{10{,}000\,(1.08)^n}{10{,}000}$$

بندخل على قانون اللوغاريتم لـ ضرب الطرفين بـ Ln

$$2 = 1.08^{n}$$

$$\frac{\ln 2}{1.08 \ln} = \frac{n \cdot 1.08 \ln}{1.08 \ln}$$

$$\frac{\ln 2}{\ln 1.08} = n$$

$$\frac{0.69}{0.076} = \boxed{9.078 \Rightarrow n} \Rightarrow \text{"9.0 years"}.$$

(b). 8%, compounded continuously?

solution:

*المعطيات:

$p = 10{,}000$

$T/n = ??$

$R = 8\% \Rightarrow 0.08$

$S = 20{,}000$

compounded continuously

$$\Rightarrow \cancel{S = p(1}$$

$$S = p \cdot e^{R \cdot T}$$

$$\frac{20{,}000}{10{,}000} = \frac{10{,}000 \cdot e^{(0.08)T}}{10{,}000}$$

ضرب الطرفين بـ Ln

$$\ln 2 = \cancel{e}^{0.08T}\ \cancel{\ln}$$

$$\ln 2 = 0.08T$$

$$\frac{0.69}{0.08} = \frac{0.08\,T}{0.08}$$

$$\boxed{8.625 = T} \Rightarrow T \approx 8.7 \text{ "years"}.$$

: ziadaLULU.

* حل أسئلة الأوت لاين: "10, 16, 18, 28, 35"

$\boxed{E: 10}$. What is the future value if \$8600 is invested for 8 years at 10% compounded semiannually?

solution:

∴ compounded semi annually $\Rightarrow \boxed{m=2}$

$S = p\left(1+\frac{R}{m}\right)^{Tm}$

$= 8600\left(1+\left(\frac{0.1}{2}\right)\right)^{2\cdot 8}$

$= 8600\left(1+\left(\frac{0.1}{2}\right)\right)^{16}$

$= 8600(1+(0.05))^{16}$

$= 8600(1.05)^{16}$

$= 8600(2.1828)$

$= \boxed{18,772.7}$ \$

* المعطيات:

p = \$8600

R = 10% ⇒ 0.1

T = 8

$\boxed{E: 16}$. What present value amounts to \$300,000 if it is invested at 7%, compounded semiannually, for 15 years?

solution:

∴ compounded semiannually: $\Rightarrow \boxed{m=2}$

$S = p\left(1+\frac{R}{m}\right)^{mT}$

$= 300{,}000\left(1+\left(\frac{0.07}{2}\right)\right)^{2\cdot(15)}$

$= 300{,}000(1+0.035)^{30}$

$= 300{,}000(2.80679)$

$= \boxed{842{,}038.1}$ \$

* المعطيات:

p = 300,000

R = 7% ⇒ 0.07

T = 15

E: 18.

Find the interest that will result if $8000 is invested at 7%, compounded continuously, for 8 years.

solution:

compounded continuously

$$S = P \cdot e^{R \cdot T}$$
$$= 8000 \cdot e^{(0.07)8}$$
$$= 8000 \cdot e^{0.56}$$
$$= \$\boxed{14,005,38}.$$

*المعطيات:

$P = \$8000$

$R = 7\% \Rightarrow 0.07$

$T = 8$ years.

E: 28.

6% compounded continuously, 6% compounded semi-annually, 6% compounded monthly:

* rank each interest rate and compounding scheme in order from highest yield to lowest yield.

المطلوب: ترتيب كل سعر فائدة والمركب بالترتيب من أعلى عائد الى أدنى عائد.

solution: compounded continuously. أعلى فائدة

compounded monthly.

compounded semi annually أقل فائدة

E: 35.

How Long (in years) would $700 have to be invested at 11.9%, compounded continuously, to earn $300 interest:

solution:

How long ($t = ??$)

$$S = p e^{rt}$$
$$1000 = 700 \cdot e^{0.119t}$$

$S = P + I$

$S = 700 + 300$

$S = 1000$

$$\frac{1000}{700} = e^{0.119t} \Rightarrow \ln\left(\frac{1000}{700}\right) = \ln\left(e^{0.119t}\right)$$
$$\Rightarrow \ln\left(\frac{1000}{700}\right) = 0.119t$$
$$\Rightarrow T = \frac{\ln\left(\frac{1000}{700}\right)}{0.119}$$
$$\boxed{t \approx 3 \text{ years}}.$$

$\ln e = 1$

"إِنْ أَجْرِيَ إِلَّا عَلَى اللَّهِ"

: Ziad alulu.

جامعة بيرزيت

BIRZEIT UNIVERSITY

Mathematics Department

MATH 2351

Handout # 4 Prepared by Mohammad Madiah
Sections 5.1, 5.2, 6.1 and 6.2 Additional Problems

....

1. **Evaluate the following:**
   - a. $(32)^{\frac{-3}{5}}(125)^{\frac{5}{3}}$
   - b. $\log 1000 - 10\log\sqrt{100}$
   - c. $\log_{\sqrt{10}} 10^4$
   - d. $\log_2 \frac{1}{256}$
   - e. $\dfrac{\log_a 1024}{\log_a 625}$
2. **Solve for x.**
   - a. $(x+3)^{\frac{3}{4}} = 64$
   - b. $(x+2)^{\frac{2}{5}} = 15$
   - c. $e^{2x} = 10$
   - d. $e^{(\ln 0.7)x} = 0.1$
   - e. $\ln x + \ln(2x-1) = 0$
   - f. $\log_2 x - \log_2(x-8) = 3$
   - g. $\log_2 \sqrt{x} = 3$
   - h. $\log_2 4 + \log_2(x-1) = 1$
3. If $3600 is invested for 42 months at a simple interest rate of 5.5%
   - a. How much interest will be earned?
   - b. What is the future value of the investment after 42 months?
   - c. How long does it take the investment to be worth $7200
4. Find the future amount for $P invested at 2.5% simple interest for 72 months.
5. If $15000 is invested at an annual rate of interest of 4.8%, What is the amount after 10 years if the compounding take place compounding
   - a. Annually
   - b. Semiannually
   - c. Quarterly
   - d. Monthly
   - e. Continuously
6. You have $28500 for investment.
   - a. What is your future value if you invest this money for 6 years at an annual rate of 10.5% compounded quarterly?
   - b. How long will it take your money to grow to $38000 in account paying 7.5% compounded continuously?

7. How long would it take an investment to double if it is invested at
   a. 4.8% simple interest?
   b. 4.8% compounded annually.
   c. 4.8% compounded quarterly.
   d. 4.8% compounded continuously.
8. What is the present value for $6500 payable in 4 years at 12% interest compounded semiannually?
9. How long will it take for $5500 to grow to $40300 at an interest rate of 4.8% compounded continuously
10. What annual rate of interest you seek if you want to double your investment in 6 years, if the amount is:
    a. Compounded continuously
    b. Compounded monthly.