11.1 Derivatives of Logarithmic functions

11 + 15 + 19 + 22 + 32 + 39 + 45

Remember that

$y = \log_a u(x)$

$$\boxed{\bar{y} = \frac{\bar{u}(x)}{u(x) \cdot \ln a}}$$

$y = \ln u(x)$

$$\boxed{\bar{y} = \frac{\bar{u}(x)}{u(x)}}$$

11 Find $\frac{dp}{dq}$ if $p = \ln(q^2 + 1)$

$$\boxed{\frac{dp}{dq} = \frac{2q}{q^2 + 1}}$$

15 (a) $y = \ln \sqrt[3]{x^2 - 1}$

(a) $y = \frac{1}{3} \ln(x^2 - 1)$

$y = \ln (x^2 - 1)^{\frac{1}{3}}$

$y = \frac{1}{3} \ln(x^2 - 1)$

$$\boxed{\bar{y} = \frac{1}{3} \cdot \frac{2x}{x^2 - 1}}$$

19 $P = \ln\left(\frac{q^2-1}{q}\right)$

$P = \ln(q^2-1) - \ln q$

$$\boxed{\frac{dP}{dq} = \frac{2q}{q^2-1} - \frac{1}{q}}$$

22 $y = \ln\left(\frac{3x+2}{x^2-5}\right)^{\frac{1}{4}}$

$y = \frac{1}{4}\ln\left(\frac{3x+2}{x^2-5}\right)$

$= \frac{1}{4}\left[\ln(3x+2) - \ln(x^2-5)\right]$

$$\boxed{y' = \frac{1}{4}\left[\frac{3}{3x+2} - \frac{2x}{x^2-5}\right]}$$

$$\boxed{\begin{array}{l} y = \ln a^n \\ y = n\ln a \end{array}}$$

but

$$\boxed{\begin{array}{l} y = (\ln a)^n \\ \neq n\ln a \end{array}}$$

32 $y = (\ln x)^{-1}$

$y' = -1\cdot(\ln x)^{-2}\cdot\frac{1}{x}$

$$\boxed{y' = \frac{-1}{x(\ln x)^2}}$$

39 $y = x \ln x,\ x > 0$ Find max. minim.

$y' = x \cdot \frac{1}{x} + \ln x \cdot 1$

$y' = 1 + \ln x$

$y' = 0$ or y' is undefined

$1 + \ln x = 0$

$\ln x = -1$

e e

$\boxed{x = e^{-1}}$ critical ✓

$- - - \quad + + +$

0 $\quad \frac{1}{e} = e^{-1}$

Or use the 2nd derivative Test

$y'' = \frac{1}{x}$

$y''\left(\frac{1}{e}\right) = \frac{1}{\frac{1}{e}} = e > 0$

So minimum.

at $x = \frac{1}{e}$, Relative minimum point

$\left(\frac{1}{e}, f\left(\frac{1}{e}\right)\right) = \left(\frac{1}{e}, \frac{1}{e} \cdot \ln e^{-1}\right)$

$= \left(\frac{1}{e}, -\frac{1}{e}\right)$ Rel. Min. point

what is the minimum value $= -\frac{1}{e}$.

45

$$R(x) = \frac{2500x}{\ln(10x+10)}$$

Find the $\bar{MR}$ at $x = 100$

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$$\bar{R}(x) = MR = \frac{\left[\ln(10x+10)\right] * 2500 - 2500x \cdot \left[\frac{10}{10x+10}\right]}{\left(\ln(10x+10)\right)^2}$$

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$$\bar{R}(100) = \frac{\ln(1010) * 2500 - 2500(100) * \frac{10}{1010}}{(\ln 1010)^2}$$

$= \#$ —

11.2 Derivatives of Exponential Functions

8 + 12 + 14 + 20 + 34 + 37 + 43 + 46

$$y = e^{u(x)}$$
$$y' = e^{u(x)} \cdot u'(x)$$

$$y = a^{u(x)}$$
$$y'(x) = a^{u(x)} \cdot u'(x) \cdot \ln a$$

(8) $y = e^{x^2-1}$

$$\boxed{y' = e^{x^2-1} \cdot 2x}$$

(12) $y = e^{\sqrt{x^2-9}}$

$$= e^{(x^2-9)^{\frac{1}{2}}}$$

$$y' = e^{(x^2-9)^{\frac{1}{2}}} \cdot \frac{1}{2}(x^2-9)^{-\frac{1}{2}} \cdot 2x$$

$$= \frac{x}{\sqrt{x^2-9}} \cdot e^{\sqrt{x^2-9}}$$

$$\boxed{y' = \frac{x\, e^{\sqrt{x^2-9}}}{\sqrt{x^2-9}}}$$

$\boxed{14}$

$$y = e^3 + e^{\ln x}$$

$$y = e^3 + x$$

$$y' = 0 + 1 = 1$$

$$\boxed{y' = 1}$$

$\boxed{20}$

$$P = 4q e^{q^3}$$

$$P' = 4q \cdot e^{q^3} \cdot 3q^2 + e^{q^3} \cdot 4$$

$$\boxed{P' = 4e^{q^3}\left[3q^3 + 1\right]}$$

$\boxed{34}$

$$y = 5^{2x-1}$$

$$\boxed{y' = 5^{2x-1} \cdot 2 \cdot \ln 5}$$

$\boxed{37}$

X

$\boxed{46}$ $S = 50\,000\, e^{-0.8t}$

$$\frac{ds}{dt} = 50000\, e^{-0.8t}(-0.8)$$

Example :

$y = x^3$

$\bar{y} = 3x^2$

$y = \pi^3$

$\bar{y} = 0$

$y = \log_8 x$

$\bar{y} = \frac{1}{x \cdot \ln 8}$

$y = \log_5 4$

$\bar{y} = 0$

$y = \ln 4$

$\bar{y} = 0$

$y = e^3$

$\bar{y} = 0$

$y = 4x$

$\bar{y} = 4$

$\cancel{y = 0}$

$y = (\ln 5) \cdot x$

$\bar{y} = \ln 5$

$y = e^2 x$

$\bar{y} = e^2$

$y = (\ln x)^3$

$\bar{y} = 3 \cdot (\ln x)^2 \cdot \frac{1}{x}$

$y = \ln x^3 = 3 \ln x$

$\bar{y} = 3 \cdot \frac{1}{x}$