12.1 9, 18, 20, 28, 31, 42, 49

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

(9) $\int (3^3 + x^{13})\,dx = \int (27 + x^{13})\,dx$

$$= 27x + \frac{x^{14}}{14} + C$$

(18) $\int (17 + \sqrt{x^3})\,dx$

$$= \int (17 + x^{\frac{3}{2}})\,dx = 17x + \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C$$

$$= 17x + \frac{2x^{\frac{5}{2}}}{5} + C$$

(20) $\int 3\sqrt[3]{x^2}\,dx$

$$= 3\int x^{\frac{2}{3}}\,dx = 3\cdot\frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + C$$

$$= 3\cdot\frac{3x^{5/3}}{5} + C$$

(28) $\int \left(3x^8 + \frac{4}{x^8} - \frac{5}{\sqrt[5]{x}}\right)dx$

$$= \int 3x^8 + 4x^{-8} - 5x^{\frac{-1}{5}}\,dx = \frac{3x^9}{9} + \frac{4x^{-7}}{-7} - 5\frac{x^{\frac{4}{5}}}{\left(\frac{4}{5}\right)} + C$$

$$= \frac{1}{3}x^9 + \frac{-4}{7x^7} - \frac{25}{4}x^{\frac{4}{5}} + C$$

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12.1

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

(2

(31) $\int \frac{x+1}{x^3}\,dx = \int \frac{x}{x^3} + \frac{1}{x^3}\,dx$

$= \int \frac{1}{x^2} + \frac{1}{x^3}$

$= \int x^{-2}dx + \int x^{-3}dx = \frac{x^{-1}}{-1} + \frac{x^{-2}}{-2} + C$

$= \frac{-1}{x} + \frac{1}{2x^2} + C$

(42) $\overline{MR} = -0.05x + 25$

$R?$ $\quad R(x) = \int \overline{MR} = \int -0.05x + 25\,dx$

$R(x) = -\frac{0.05}{2}x^2 + 25x + C$

$R(0) = 0 \Rightarrow \quad 0 = -0.025(0)^2 + 25(0) + C$

$\Rightarrow C = 0$

$40 = 5 + 5 + C$

$C = 30$

$\boxed{R(x) = -0.025x^2 + 25x}$

(49) $\overline{C}'(x) = \frac{1}{4} - \frac{100}{x^2}$, $\overline{C}(20) = 40.$

$\overline{C} = \int \frac{1}{4} - 100x^{-2}\,dx = \frac{1}{4}x - \frac{100x^{-1}}{-1} + C$

$\overline{C}(x) = \frac{1}{4}x + \frac{100}{x} + C \rightarrow \boxed{C = 30}$

12.2 / 8, 16, 26, 32, 34, 44 $\int u^n \cdot u' = \frac{u^{n+1}}{n+1}$ (3

(8) $\int \underbrace{(4x^2-3x)}_{u}^4 \underbrace{(8x-3)\,dx}_{du}$

$u = 4x^2-3x$
$du = 8x-3\ dx$

$= \frac{(4x^2-3x)^5}{5} + C$

(16) $\int (2x^3-x)(x^4-x^2)^6\,dx$

$u = x^4-x^2$
$du = 4x^3-2x$
$= 2(2x^3-x)$

$= \frac{1}{2}\int 2(2x^3-x)(x^4-x^2)^6\,dx$

$= \frac{1}{2}\,\frac{(x^4-x^2)^7}{7} + C$

(26) $\int \sqrt[3]{x^2+2x}\,(x+1)\,dx$

$u = x^2+2x$
$du = 2x+2$
$= 2(x+1)$

$= \int (x^2+2x)^{\frac{1}{3}}(x+1)\,dx$

$= \frac{1}{2}\int (x^2+2x)^{\frac{1}{3}} \cdot 2(x+1)\,dx$

$= \frac{3}{2} \cdot \frac{(x^2+2x)^{\frac{4}{3}}}{4} + C$

(32) $$\int \frac{x^2+1}{\sqrt{x^3+3x+10}}\,dx$$

$$u = x^3+3x+10$$
$$du = 3x^2+3$$
$$= 3(x^2+1)$$

$$= \frac{1}{3}\int 3(x^2+1)\,(x^3+3x+10)^{-\frac{1}{2}}\,dx$$

$$= \frac{1}{3}\,\frac{(x^3+3x+10)^{\frac{1}{2}}}{\frac{1}{2}}$$

$$= \frac{2}{3}(x^3+3x+10)^{\frac{1}{2}} + C$$

---

(34) If $\int g(x)\,dx = (5x^2+2)^6 + C$ Find $g(x)$

$$g(x) = \left((5x^2+2)^6\right)' = 6(5x^2+2)^5 \cdot 10x$$
$$= 60x\,(5x^2+2)^5$$

---

(44) $$\overline{MR} = 60000 - \frac{40000}{(10+x)^2}$$

$$R(x) = \int 60000 - 40000\,(10+x)^{-2}\,dx$$

$$= 60000x - \frac{40000\,(10+x)^{-1}}{-1} + C$$

$$R(x) = 60000x + \frac{40000}{(10+x)} + C \xRightarrow{R(0)=0} C = -4000$$

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- Sec 12.3 : Integrals Involving Exponential and Logarithmic functions.

If u is a function of x, then:

$$\int u^{-1} u' dx = \int \frac{u'}{u} dx = \ln|u| + c.$$

In particular, $\int \frac{1}{x} dx = \ln|x| + c.$

* page : 10, 14, 17, 25, 27, 32, 43.

$\boxed{10}$ Evaluate the integrals:

$$\int x e^{2x^2} dx$$

$$\longrightarrow \frac{1}{4}\int 4x\, e^{2x^2} dx$$

$4x$

$$= \frac{1}{4} e^{2x^2} + c.$$

$\boxed{14}$ $\int \frac{x^3}{e^{4x^4}}\,dx$

$= \frac{1}{-16}\int -16x^3\, e^{-4x^4}\,dx$

$\downarrow$

$-16x^3$

$= \frac{-1}{16}\int -16x^3 e^{-4x^4}\,dx$

$= \frac{-1}{16} e^{-4x^4} + C$

$\boxed{17}$ $\int \frac{3x^2}{x^3+4}\,dx$

لأن قوة المقام 1 نتأكد إذا كانت
مشتقته موجودة ، يكون على ln .

$\longrightarrow \int \frac{3x^2}{x^3+4}\,dx = \ln|x^3+4| + C$

$3x^2$

$\boxed{25}$ $\int \frac{3x^2-2}{x^3-2x}\,dx$

$3x^2-2$

$= \ln|x^3-2x| + C$

27 $\displaystyle\int \frac{Z^2+1}{\underbrace{Z^3+3Z+17}_{3Z^2+3}}\,dZ$

$$= \frac{1}{3}\int 3\,\frac{Z^2+1}{Z^3+3Z+17}\,dZ$$

$$= \frac{1}{3}\ln|Z^3+3Z+17| + C.$$

32 $\displaystyle\int \frac{X^4-2X^2+X}{X^2-2}\,dX$

قوة المقام اقل من البسط
نقسم قسمة طويلة

$$\begin{array}{r|l} & X^2 \\ \hline X^2-2 & X^4-2X^2+X \\ & \underline{-X^4 \pm 2X^2} \\ & X \end{array}$$

$$= \int X^2 + \frac{1}{2}\int \frac{2X}{\underbrace{X^2-2}_{2X}}\,dX$$

$$= \frac{X^3}{3} + \frac{1}{2}\ln|X^2-2| + C.$$

43 Suppose that the marginal revenue from the sale of x units of a product is $MR = 6e^{0.01x}$. What is the revenue from the sale of 100 units?

$\longrightarrow$ $R(x) = \int MR$

$$= \int 6 e^{0.01x}\, dx$$

$$= 6 \int e^{0.01x}\, dx$$

0.01

$$= \frac{6}{0.01} \int 0.01\, e^{0.01x}\, dx$$

$$R(x) = 600\, e^{0.01x} + C$$

we Know $R(0) = 0$:-

$$0 = 600\, e^{0} + C$$

$$0 = 600 + C \longrightarrow C = -600$$

$\therefore$ $R(x) = 600\, e^{0.01x} - 600$

$$R(100) = 600\, e^{0.01(100)} - 600$$

$$= 600e - 600$$

$$= 1030.97 \ \$$$

- Sec 12.4 : Applications of the Indefinite Integral

① $C(x) = \int \overline{MC}$

The value of constant of integration depends on the fixed costs.

constant of integration
لا يساوي التكاليف الثابتة ولكن يعتمد عليها.

We can't determine the total cost from the marginal cost unless additional information is available.

أي يلزمنا معلومات إضافية كي نجد ثابت التكامل.

② $R(x) = \int MR$

To find C or the constant of integration ,, we use

→ $R(0) = 0$

③ Optimal level of production: $\overline{MR} = \overline{MC}$

- page 780: 8, 14, 16, 18, 20, 26

$\boxed{8}$ A certin firm's marginal cost for a product is $\overline{MC} = 6x + 60$, its marginal revenue is $\overline{MR} = 180 - 2x$, and its total cost of production of 10 items is $1000.

a) Find the optimal level of production.

$\longrightarrow \overline{MC} = \overline{MR}$

$$6x + 60 = 180 - 2x$$
$$+2x \qquad\qquad +2x$$

$$8x + 60 = 180 \longrightarrow 8x = 120 \longrightarrow x = 15$$
$$-60 \qquad -60$$

b) Find the profit function.

$P(x) = R(x) - C(x)$

, $R(x) = \int 180 - 2x \, dx = 180x - \frac{2x^2}{2} + C$

we Know $R(0) = 0$

$\boxed{0 = C} \longrightarrow \boxed{R(x) = 180x - x^2}$

$\therefore\ C(x) = \int 6x + 60 \, dx$

$= \frac{6x^2}{2} + 60x + C$

$= 3x^2 + 60x + C$

$C(10) = 1000$ ((given))

$\therefore\ 1000 = 3(10)^2 + 60(10) + C$

$1000 = 900 + C \longrightarrow \boxed{C = 100}$

$-900 \quad -900$

$\therefore\ C(x) = 3x^2 + 60x + 100$

$\rightarrow P(x) = R(x) - C(x)$

$= (180x - x^2) - (3x^2 + 60x + 100)$

$= 120x - 4x^2 - 100$

c) Find the profit or loss at the optimal level of production.

عوض x=15 في profit

$P(15) = 120(15) - 4(15)^2 - 100$

$= 800$