Chapter 13

13.2 10 /20/30/34/38/42/55

(10) $\int_{-1}^{4} 6x-9 \; dx$

$= \frac{6x^2}{2} - 9x = 3x^2-9x \Big]_{-1}^{4}$

$= (48-36) - (3+9)$

$= 12-12 = 0$

(20) $\int_{0}^{4} (3x^2-4)^4 x \, dx$

$u = 3x^2-4$

$u' = 6x$

$= \frac{1}{6}\int_{0}^{4} (3x^2-4)^4 \, 6 \cdot x \, dx$

$= \frac{1}{6} \frac{(3x^2-4)^5}{5} \Big]_{0}^{4} = 5497207.467 + 34.13\ldots$

$= 5497173.337$

(30) $\int_{0}^{1} \frac{3x^3 \, dx}{4x^4+9}$

$\left(\begin{array}{l} u = 4x^4+9 \\ u' = 16x^3 \end{array} \right.$

$\int \frac{u'}{u} = \ln|u|$

$= \frac{3}{16}\int_{0}^{1} \frac{16x^3}{4x^4+9} = \frac{3}{16} \ln|4x^4+9| \Big]_{0}^{1} = \frac{3}{16}(\ln 13 - \ln 9)$

$= 0.069$

(34) $\displaystyle\int_1^4 \frac{4\sqrt{x}+5}{\sqrt{x}}\,dx$

$$= \int_1^4 \frac{4\sqrt{x}}{\sqrt{x}} + \frac{5}{\sqrt{x}}\,dx$$

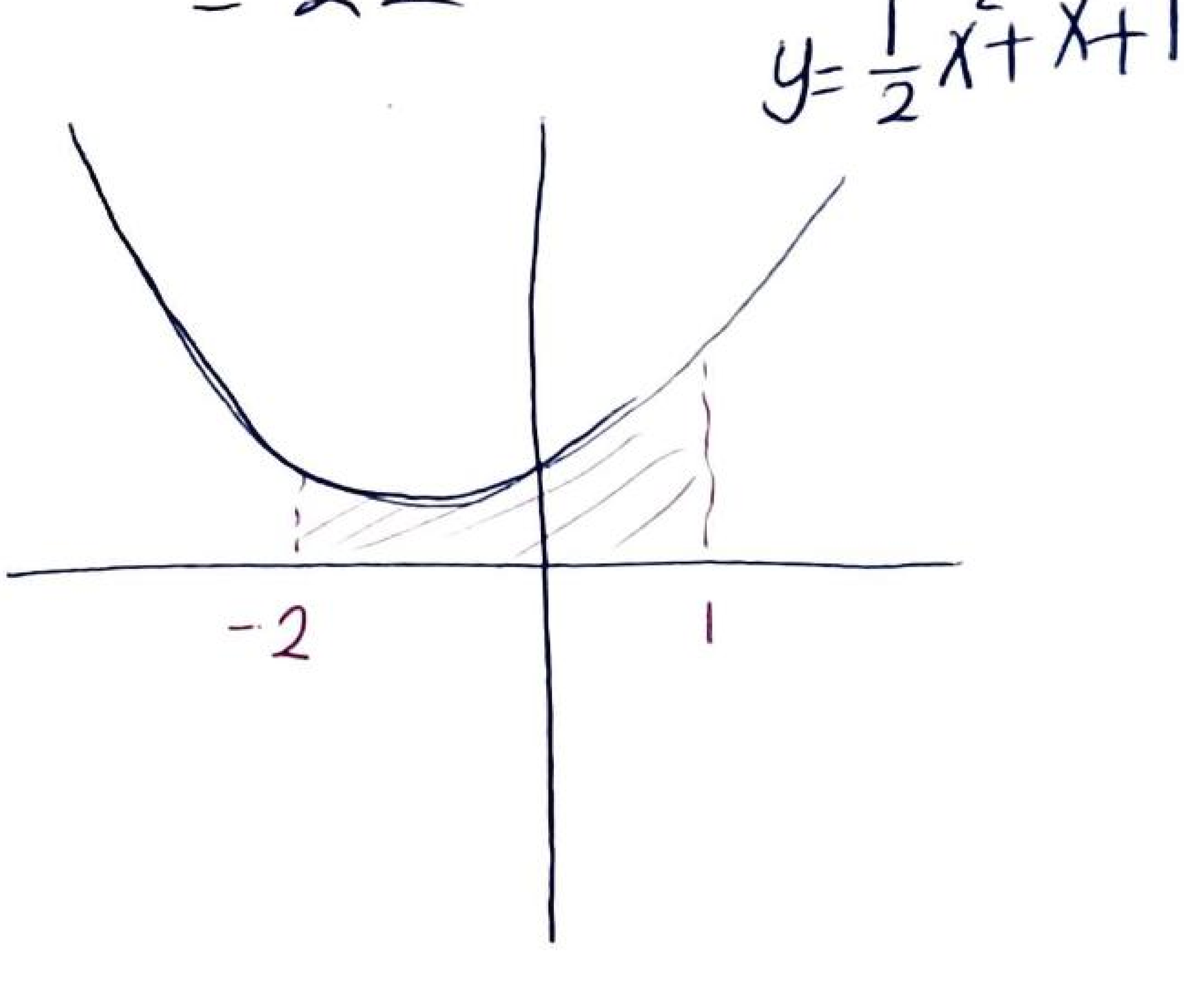
$$= \int_1^4 4\,dx + \int_1^4 5x^{-\frac{1}{2}}\,dx$$

$$= 4x\Big]_1^4 + \frac{5x^{\frac{1}{2}}}{\frac{1}{2}}\Bigg]_1^4 = 12 + 10(2-1)$$

$$= 12 + 10$$

$$= 22$$

(38)



$$A = \int_{-2}^{1} \frac{1}{2}x^2 + x + 1$$

$$= \frac{1}{2}\frac{x^3}{3} + \frac{x^2}{2} + x\Bigg]_{-2}^{1}$$

$$= \left(\frac{1}{6} + \frac{1}{2} + 1\right) - \left(\frac{-8}{6} + \cancel{2} - \cancel{2}\right)$$

$$= \frac{10}{6} + \frac{8}{6} = \frac{18}{6} = 3$$

(42) Find the area between

$y = x^2 + 3x + 2$ and the x-axis

between $x = -1$ to $x = 3$

$$A = \int_{-1}^{3} x^2 + 3x + 2 = \frac{x^3}{3} + \frac{3x^2}{2} + 2x \Big]_{-1}^{3}$$

$$= \left(9 + \frac{27}{2} + 6\right) - \left(\frac{-1}{3} + \frac{3}{2} - 2\right)$$

$$= 28.5 + 0.83$$

$$= 29.333$$

---

(55) sales

$S'(t) = -30t^2 + 360t \quad , \quad 0 \leq t \leq 30$

(a) $t = 0$ to $t = 7$

total sales S $= \int_0^7 -30t^2 + 360t$

$$= -30\frac{t^3}{3} + 360\frac{t^2}{2} \Big]_0^7$$

$$= -10t^3 + 180t^2 \Big]_0^7 = 5390$$

(b) $t = 7$ to $t = 14$

$$S = \int_7^{14} -30t^2 + 360t = -10t^3 + 180t^2 \Big]_7^{14}$$

$$= 7840 - 5390 = 2450$$

Scanned by TapScanner

13.3 6/8/14/17/30/40

(6) $A = \int_{-1}^{1} (x^3+1) - (-x-1) \, dx$

(8) $A = \int_{1}^{4} (x-4) - (x^2-4x) \, dx$

(14) $f(x) = x^2$, $g(x) = \frac{-1}{10}(10+x)$, $x=0$ to $x=3$

$A = \int_0^3 x^2 - \frac{-1}{10}(10+x)$

$= \int_0^3 x^2 + 1 + \frac{x}{10} \, dx$

$= \frac{x^3}{3} + x + \frac{x^2}{20} \Big]_0^3 = 9 + 3 + \frac{9}{20}$

$= 12.45$

(17) $y = \frac{1}{2}x^2$, $y = x^2 - 2x$

$\frac{1}{2}x^2 = x^2 - 2x$

$\Rightarrow \frac{1}{2}x^2 - 2x = 0 \Rightarrow x(\frac{1}{2}x - 2) = 0 \Rightarrow x = 0, 4$

$$A = \int_0^4 \left(\tfrac{1}{2}x^2\right) - \left(x^2 - 2x\right)\,dx$$

$$= \int_0^4 \frac{-1}{2}x^2 + 2x = \frac{-1}{2}\frac{x^3}{3} + \frac{2x^2}{2}\Big]_0^4$$

$$= 5.33$$

---

30 $f(x) = \frac{1}{2}x^3 + 1$ over $[-2, 0]$

Averge value of $f(x) = \frac{1}{b-a}\int_a^b f(x)\,dx$

$$= \frac{1}{0 - -2}\int_{-2}^{0} \frac{1}{2}x^3 + 1\,dx$$

$$= \frac{1}{2}\int_{-2}^{0} \frac{1}{2}x^3 + 1\,dx = \frac{1}{2}\left(\frac{1}{2}\frac{x^4}{4} + x\Big]_{-2}^{0}\right)$$

$$= \frac{1}{2}\left(0 - (2 - 2)\right) = 0$$

---

40 D: $P = 500 + \frac{1000}{q+1}$

Averge price from 49 to 99 units

$$= \frac{1}{99-49}\int_{49}^{99} 500 + 1000$$

$$= \frac{1}{50}\left(500q + 1000\ln|q+1|\Big]_{49}^{99}\right) = \frac{1}{50}\left(54105.17 - 28412.02\right)$$

$$= 513.86$$

Scanned by TapScanner

13.4 19 / 22 / 27 / 36

(19) $D: \; p = 100 - 4x$

Consumer's surplus $\quad EP(\underset{\text{given}}{\overset{x}{\underline{8}}}, 68)$

$$CS = \int_0^8 (100 - 4x)\,dx - (8 \cdot 68)$$

$$= 100x - \frac{\cancel{4}^{2}x^2}{\cancel{2}_{1}}\Big]_0^8 - 544$$

$$= 800 - 128 - 544 = 128$$

---

(22) $D: p = 49 - x^2$

$S: p = 4x + 4$

EP: $S = D$

$4x + 4 = 49 - x^2$

$4x + 4 - 49 + x^2 = 0 \Rightarrow x^2 + 4x - 45 = 0$

$(x+9)(x-5) = 0$

$x = -9$ ✗

$x = 5$ ✓ $\rightarrow p = 4(5) + 4$

$= 24$

$$CS = \int_0^5 49 - x^2 - (5 \cdot 24)$$

$$= \left[49x - \frac{x^3}{3}\Big]_0^5\right] - 120 = 83.33$$

(27) $S: p = 4x^2 + 2x + 2$

equilibrium price $= 422$

$\Rightarrow x = ?$

$422 = 4x^2 + 2x + 2$

$4x^2 + 2x + 2 - 422 = 0$

$4x^2 + 2x - 420 = 0$

$2x^2 + x - 210 = 0$

$x = \dfrac{-1 \pm \sqrt{1 - 4(2)(-210)}}{2(2)}$

$= \dfrac{-1 \pm 41}{4}$

$x_1 = \dfrac{40}{4} = 10$ ✓

$x_2 = \dfrac{-42}{4}$ X

Produce's surplus

EP $(10, 422)$

$PS = (10 \cdot 422) - \int_0^{10} 4x^2 + 2x + 2$

$= 4220 - \left(\dfrac{4x^3}{3} + \dfrac{\cancel{2}x^2}{\cancel{2}} + 2x \Big]_0^{10} \right)$

$= 4220 - (1453.33)$

$= 2766.67$

---

(36) $D: P = 280 - 4x - x^2$

$S: P = 160 + 4x + x^2$

EP $\begin{array}{c} x \quad p \\ (6, 220) \end{array}$

$S = D$

$160 + 4x + x^2 = 280 - 4x - x^2$

$160 + 4x + x^2 - 280 + 4x + x^2 = 0$

$2x^2 + 8x - 120 = 0$

$x^2 + 4x - 60 = 0$

$(x + 10)(x - 6) = 0$

$x = -10$ X

$x = 6$ ✓

? $PS = (6 \cdot 220) - \int_0^6 160 + 4x + x^2 \, dx$

$= 1320 - \left(160x + \dfrac{4x^2}{2} + \dfrac{x^3}{3} \Big]_0^6 \right)$

$= 1320 - 1104$

$= 216$

Scanned by TapScanner