MATH2351
Outline Solution 6.1&6.2

الدكتورة بتول ردّاد

6.1 Simple interest

$$I = Prt$$

$$S = P + I = P + Prt = P(1+rt)$$

Questions: 6, 8, 21, Page 376 - 377

6 $\$800$ (P) is invested for 5 (t) years at an annual interest rate of 14% ($r = 14\% = \frac{14}{100} = 0.14$). $S=P+Prt$ Simple

a) How much interest will be earned? I??

$$I = Prt = 800(0.14)(5) = \$560$$

b) What is the future value (S??) of the investment at the end of the 5 years?

$$S = P + I = 800 + 560 = \$1360$$

8 $\$1800$ (P) is invested for 9 months ($t = \frac{9}{12}$ years) at an annual interest rate of 15% ($r = 15\% = \frac{15}{100} = 0.15$). $S=P+Prt$ simple

a) How much interest will be earned? I??

$$I = Prt = 1800(0.15)\left(\frac{9}{12}\right) = \$202.5$$

b) What is the future value (S??) after 9 months?

$$S = P + I = 1800 + 202.5 = \$2002.5$$

-1-

21 If $\$5000$ (P) is invested at 8% ($r = \frac{8}{100} = 0.08$) annual simple interest, how long (t??) does it take to be worth $\$9000$ (S)?

$S = P + Prt$

$9000 = 5000 + 5000(0.08)t$

$-5000 \quad -5000$

$4000 = 5000(0.08)t$

$\frac{4000}{400} = \frac{400}{400}t$

$10 \text{ years} = t$

(2)

6.2 Compound interest.

- compound annually : $S = P(1+r)^t$
- Compound m times per year : $S = P(1+\frac{r}{m})^{mt}$
- compound continuously : $S = P e^{rt}$

Questions : 10, 16, 18, 28, 35

Pages : 389 - 390

Q 10 What is future value (S??) if $8600 (P) is invested for 8 (t) years at 10% ($r = \frac{10}{100} = 0.10$) compounded semiannually? ($m = 2$)

Compounded Semiannally (2 times per year)

$$S = P(1+\frac{r}{m})^{mt}$$

$$S = 8600\left(1+\frac{0.10}{2}\right)^{2(8)}$$

$$= 8600\,(1+0.05)^{16} = 8600\,(1.05)^{16}$$

$$= 8600\,(2.18287)$$

$$= 18772.72146 \simeq 18772.72\ \$$$

16 What present value (P??) amounts to $S = \$300\,000$ if it is invested at 7% ($r = \frac{7}{100} = 0.07$) compounded semiannually ($m = 2$) for 15 (t) years?

$$S = P(1+\frac{r}{m})^{mt}$$

$$300\,000 = P\left(1+\frac{0.07}{2}\right)^{2(15)}$$

$$300\,000 = P(1.035)^{30}$$

$$\frac{300\,000}{2.806793} = \frac{P(2.806793)}{2.806793} \rightarrow \$\,106883.55 = P$$

18 Find the interest (I ??) that will result if $8000 (P) is invested at 7% ($r = \frac{7}{100} = 0.07$) compounded continuously ($S = Pe^{rt}$), for 8 (t) years.

$$S = P e^{rt}$$
$$= 8000\ e^{(0.07)(8)}$$
$$= 8000\ e^{0.56} = 8000(1.7506725) = \$14005.38$$

$$I = S - P$$
$$= 14005.38 - 8000 = \$6005.38$$

28 6% compounded continuously (أسرع نوع من الفوائد دائماً), 6% compounded semiannually ($m = 2$), 6% compounded monthly ($m = 12$)

Rank each interest rate and compounding scheme in orded from highest yield to lowest.

Solution: 6% compounded continuously, 6% compounded monthly, 6% compounded semiannually

(4)

35 How long (in years) would $700 have to be invested at 11.9 %, compounded continuously, to earn $300 interest?

t ?? P $S = P e^{rt}$ $r = \frac{11.9}{100} = 0.119$ $I = 300$

$S = P + I = 700 + 300 = \$1000$

$S = P e^{rt}$

$\frac{1000}{700} = \frac{700}{700} e^{0.119t}$ (divide both sides by 700)

$1.42857 = e^{0.119t}$ (take ln for both sides)

$\ln 1.42857 = \ln e^{0.119t}$

$\frac{0.3566739}{0.119} = \frac{0.119t}{0.119}$ (divide both sides by 0.119)

$2.997259 = t$

$t \approx 3$ years.

(5)