MATH2351
Outline Solution 9.1-9.8

الدكتورة بتول ردّاد

9.1 Limits

Questions : 14, 22, 28, 32, 38, 52

Pages : 553 – 554

$\boxed{14}$ Complete the table and predict the limit.

$$f(x) = \begin{cases} 4 - x^2, & \text{for } x \le -2 \\ x^2 + 2x, & \text{for } x > -2 \end{cases}$$

$$\lim_{x \to -2} f(x) = ??$$

x	$f(x)$
-2.1	$4-(-2.1)^2 = -0.41$
-2.01	$4-(-2.01)^2 = -0.0401$
-2.001	$4-(-2.001)^2 = -0.004001$
-1.999	$(-1.999)^2+2(-1.999) = -0.001999$
-1.99	$(-1.99)^2+2(-1.99) = -0.0199$

$\longrightarrow$ 0 $\longleftarrow$

$f(x)$	-0.41	-0.0401	-0.004001		-0.001999	-0.0199
x	-2.1	-2.01	-2.001	-2	-1.999	-1.99

نلاحظ التالي
الصور (قيم $f(x)$) تقترب
من الصفر عاليمين وعا
ليسار $x = -2$

so $\lim_{x \to -2} f(x) = 0$

1

$\boxed{22}$ $\lim_{x \to -4} \frac{x^2 - 16}{x + 4} = \frac{0}{0}$

so $\lim_{x \to -4} \frac{x^2 - 16}{x + 4} = \lim_{x \to -4} \frac{(x-4)\cancel{(x+4)}}{\cancel{(x+4)}}$

$= \lim_{x \to -4} (x-4) = -4-4 = -8$

$\boxed{28}$ $\lim_{x \to 10} \frac{x^2 - 8x - 20}{x^2 - 11x + 10} = \frac{0}{0}$

so $\lim_{x \to 10} \frac{x^2 - 8x - 20}{x^2 - 11x + 10} = \lim_{x \to 10} \frac{(x+2)\cancel{(x-10)}}{(x-1)\cancel{(x-10)}}$

$= \lim_{x \to 10} \frac{x+2}{x-1} = \frac{10+2}{10-1} = \frac{12}{9}$

$\boxed{32}$ $\lim_{x \to 2} f(x)$, where $f(x) = \begin{cases} \frac{x^3 - 4}{x - 3} & , x \le 2 \\ \frac{3 - x^2}{x} & , x > 2 \end{cases}$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{3 - x^2}{x} = \frac{3 - (2)^2}{2} = \frac{3-4}{2} = \frac{-1}{2}$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{x^3 - 4}{x - 3} = \frac{(2)^3 - 4}{2 - 3} = \frac{8-4}{-1} = -4$

$\lim_{x \to 2^+} f(x) \neq \lim_{x \to 2^-} f(x)$

so $\lim_{x \to 2} f(x)$ does not exist (DNE)

2

$\boxed{38}$ $\lim_{h \to 0} \frac{2(x+h)^2 - 2x^2}{h}$

$= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) - 2x^2}{h}$

$= \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{2x^2}}{h}$

$= \lim_{h \to 0} \frac{4xh + 2h^2}{h} = \lim_{h \to 0} \frac{\cancel{h}(4x + 2h)}{\cancel{h}}$

$= \lim_{h \to 0} 4x + 2h = 4x + 2(0) = 4x$

$\boxed{52}$ If $\lim_{x \to 5} [f(x) - g(x)] = 8$ and $\lim_{x \to 5} g(x) = 2$, find

a) $\lim_{x \to 5} f(x)$?

$\lim_{x \to 5} f(x) - \lim_{x \to 5} g(x) = 8$

$\lim_{x \to 5} f(x) - \cancel{2} = 8$
$\quad\quad\quad\quad + \cancel{2} \quad +2$

$\lim_{x \to 5} f(x) = 10$

b) $\lim_{x \to 5} \{[g(x)]^2 - f(x)\}$

$= \lim_{x \to 5} (g(x))^2 - \lim_{x \to 5} f(x)$

$= (2)^2 - 10 = 4 - 10 = -6$

3

c) $$\lim_{x \to 5} \left[\frac{2x\, g(x)}{4 - f(x)} \right] = \frac{\lim_{x \to 5} (2x\, g(x))}{\lim_{x \to 5} (4 - f(x))}$$

$$= \frac{2(5)(2)}{4 - 10} = \frac{20}{-6} = -\frac{10}{3}$$

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9.2 Continuous Functions ; Limits at Infinity.

Questions : 8 , 15 , 20 , 30 , 32

Pages : 564 – 565

$\boxed{8}$ Is $f(x) = \begin{cases} x^2+1 & \text{if } x \le 1 \\ 2x^2-1 & \text{if } x > 1 \end{cases}$ Continuous at $x=1$??

we must check if $\lim\limits_{x\to 1^+} f(x) \overset{??}{=} \lim\limits_{x\to 1^-} f(x) \overset{??}{=} f(1)$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 2x^2-1 = 2(1)^2-1 = 1$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x^2+1 = (1)^2+1 = 2$$

$$f(1) = (1)^2+1 = 2$$

$$\lim_{x\to 1^+} f(x) \neq \lim_{x\to 1^-} f(x)$$

So $f(x)$ is discontinuous at $x=1$

$\boxed{15}$ Determine if the following function is continuous.

$$f(x) = \begin{cases} 3 & \text{if } x \le 1 \\ x^2+2 & \text{if } x > 1 \end{cases}$$

– for $x < 1$, $f(x) = 3$ Constant function so $f(x)$ is Continuous for $x < 1$

– for $x > 1$, $f(x) = x^2+2$ polynomial so $f(x)$ is Continuous for $x > 1$

– for $x = 1$, $\lim\limits_{x\to 1^+} f(x) = (1)^2+2 = 3$

$\lim\limits_{x\to 1^-} f(x) = 3$, $f(1) = 3$ so $f(x)$ is continuous at $x=1$

so $f(x)$ is continuous for $x \in \mathbb{R}$

$\boxed{20}$ Determine if the following function is continuous

$$f(x) = \begin{cases} x^2+4 & \text{if } x \neq 4 \\ 8 & \text{if } x = 4 \end{cases}$$

– for $x \neq 4$, $f(x) = x^2+4$ (polynomial) so $f(x)$ is continuous for $x \neq 4$

– at $x = 4$, $f(x) = 4$

$$\lim_{x\to 4} f(x) = \lim_{x\to 4} x^2+4 = (4)^2+4 = 16+4 = 20$$

so $\lim_{x\to 4} f(x) \neq f(4)$

so $f(x)$ is discontinuous at $x=4$

$\boxed{30}$ $$\lim_{x\to\infty} \frac{4x^2+5x}{x^2-4x} = \lim_{x\to\infty} \frac{\frac{4x^2}{x^2} + \frac{5x}{x^2}}{\frac{x^2}{x^2} - \frac{4x}{x^2}}$$

$$= \lim_{x\to\infty} \frac{4 + \frac{5}{x}}{1 - \frac{4}{x}} = \frac{4+0}{1-0} = 4$$

so $y = 4$ is a horizontal asymptote

$\boxed{32}$ $$\lim_{x\to-\infty} \frac{5x^3-8}{4x^2+5x} = \lim_{x\to-\infty} \frac{\frac{5x^3}{x^3} - \frac{8}{x^3}}{\frac{4x^2}{x^3} + \frac{5x}{x^3}}$$

$$= \lim_{x\to-\infty} \frac{5 - \frac{8}{x^3}}{\frac{4}{x} + \frac{5}{x^2}} = \frac{5-0}{0+0} = \frac{5}{0} = \infty$$

so $\frac{5x^3-8}{4x^2+5x} \to \infty$ as $x \to -\infty$

no horizontal asymptotes.

6

9.3 Rates of change and derivatives.

- average rate of change of $f(x)$ from $x=a$ to $x=b$ is $= \dfrac{f(b)-f(a)}{b-a}$

- The instantaneous rate of change of $f(x)$ (commonly called rate of change) $= \lim_{h \to 0} \dfrac{f(x+h)-f(x)}{h} = f'(x)$

- Interpretations of derivative:
  1) Velocity
  2) instantaneous rate of change
  3) marginal revenue
  4) slope of the tangent.

- If a function is differentiable at $x=c$, then f is continuous at $x=c$

Questions: 39, 42

pages: 579

7

39 Suppose total cost from the production of x printers is given by $C(x) = 0.0001x^3 + 0.005x^2 + 28x + 3000$

Find the average rate of change of total cost when production changes:

a) from 100 to 300 printers.

$$= \frac{C(300) - C(100)}{300 - 100} = \cancel{0.0001(300)^3 +}$$

$$C(300) = 0.0001(300)^3 + 0.005(300)^2 + 28(300) + 3000$$
$$= 2700 + 450 + 8400 + 3000$$
$$= 14550$$

$$C(100) = 0.0001(100)^3 + 0.005(100)^2 + 28(100) + 3000$$
$$= 100 + 50 + 2800 + 3000$$
$$= 5950$$

So average rate $= \frac{14550 - 5950}{200} = \frac{8600}{200} = 43$ \$/printer

b) from 300 to 600 printers.

$$\frac{C(600) - C(300)}{600 - 300}$$

$$C(300) = 14550$$

$$C(600) = 0.0001(600)^3 + 0.005(600)^2 + 28(600) + 3000$$
$$= 21600 + 1800 + 16800 + 3000 = 43200$$

average rate $= \frac{43200 - 14550}{300} = \frac{28650}{300} = 95.5$ \$ per printer

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c) interpret the results from parts (a) and (b)

- the average cost per printer when 100 to 300 are produced is \$43 per printer.

- the average cost when 300 to 600 are produced is \$95.5 per printer.

$\boxed{42}$ If total revenue function for a blender is

$$R(x) = 36x - 0.01x^2$$

where x is the number of units sold, what is the average rate of change in revenue R(x) as x increases from 10 to 20 units?

$$\text{average rate of change} = \frac{R(20) - R(10)}{20 - 10}$$

$$R(10) = 36(10) - 0.01(10)^2$$
$$= 360 - 0.01(100)$$
$$= 359$$

$$R(20) = 36(20) - 0.01(20)^2$$
$$= 720 - 4 = 716$$

$$\text{average rate} = \frac{716 - 359}{10} = \frac{357}{10} = 35.7$$

average rate is \$ 35.7 per blender.

9

9.4 Derivative formulas.

1) $f(x) = x^n$, $f'(x) = n\,x^{n-1}$

2) $f(x) = c$, $f'(x) = 0$

3) $f(x) = c \cdot u(x)$, $f'(x) = c \cdot u'(x)$

4) $f(x) = u(x) \pm v(x)$, $f'(x) = u'(x) \pm v'(x)$

Questions: 13, 22, 30, 34, 48

Pages: 588 _ 589

$\boxed{13}$ find the derivative of the function

$$g(x) = 2x^{12} - 5x^6 + 9x^4 + x - 5$$

$$g'(x) = 2 \cdot 12x^{11} - 5 \cdot 6x^5 + 9 \cdot 4x^3 + 1 - 0$$

$$g'(x) = 24x^{11} - 30x^5 + 36x^3 + 1$$

$\boxed{22}$ find the derivative of the function

$$y = 5x^{8/5} - 3x^{5/6} + x^{1/3} + 5$$

$$y' = \cancel{5} \cdot \frac{8}{\cancel{5}} x^{8/5 - 1} - \cancel{3} \cdot \frac{5}{\cancel{6}_{2}} x^{5/6 - 1} + \frac{1}{3} x^{1/3 - 1} + 0$$

$$y' = 8x^{3/5} - \frac{5}{2} x^{-1/6} + \frac{1}{3} x^{-2/3}$$

$$y' = 8x^{3/5} - \frac{5}{2x^{1/6}} + \frac{1}{3x^{2/3}}$$

10

30 write the equation of the tangent line to the curve $f(x) = \frac{x^3}{3} - \frac{3}{x^3}$ at $x = -1$

$\Rightarrow f(x) = \frac{x^3}{3} - 3x^{-3}$

the equation of any line is $y - y_1 = m(x - x_1)$

so we need the slope and a point on the line.

slope of the tangent $= f'(x)\Big|_{x=-1}$

$f'(x) = \frac{1}{\cancel{3}} \cdot \cancel{3}x^2 - 3 \cdot (-3x^{-3-1})$

$= x^2 + 9x^{-4} = x^2 + \frac{9}{x^4}$

$f'(-1) = (-1)^2 + \frac{9}{(-1)^4} = 1 + 9 = 10$

$f(-1) = \frac{(-1)^3}{3} - \frac{3}{(-1)^3} = \frac{-1}{3} + \frac{3 \cdot 3}{1 \cdot 3} = \frac{-1+9}{3} = \frac{8}{3}$

the point is $(-1, \frac{8}{3})$

the equation of the tangent at $x = -1$

$y - \frac{8}{3} = 10(x - {}^{-}1)$

$y - \frac{8}{3} = 10x + 10$

$y = 10x + \frac{10 \cdot 3}{1 \cdot 3} + \frac{8}{3}$

$\boxed{y = 10x + \frac{38}{3}}$

11

34 Find the coordinates of points where the graph of $f(x)$ has horizontal tangents.

$f(x) = 3x^5 - 5x^3 + 2$

- the slope of any horizontal tangent is zero.

so we want to find the point (x_0, y_0) where f' is zero

$f'(x) = 15x^4 - 15x^2$

$f'\big|_{(x_0,y_0)} = 15x_0^4 - 15x_0^2 = 0$

$15x_0^2(x_0^2 - 1) = 0$

$15x_0^2(x_0 - 1)(x_0 + 1) = 0$

So $x_0 = 0$ or $x_0 = 1$ or $x_0 = -1$

$f(0) = 3(0)^5 - 5(0)^3 + 2 = 2$, the point $(0,2)$

$f(1) = 3(1)^5 - 5(1)^3 + 2 = 0$, the point $(1,0)$

$f(-1) = 3(-1)^5 - 5(-1)^3 + 2$

$= -3 + 5 + 2 = 4$, the point $(-1,4)$

we have 3 points where the graph has horizontal tangents.

12

48 The total revenue, in dollars, for a commodity is described by the function

$$R(x) = 300x - 0.02x^2$$

a) What is the marginal revenue when 40 units are sold?

marginal revenue $= R'(x)$

we need to find $R'(40)$

$R'(x) = 300 - 0.02(2x) = 300 - 0.04x$

$R'(400) = 300 - 0.04(400) = 300 - 16 = \$284$

b) interpret your answer to part (a)

the expected change in revenue from the 41^{st} unit is about \$284

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9.5 The Product Rule and Quotient Rule

Product Rule:

If $f(x) = u(x) \cdot v(x)$, then $f'(x) = u(x) \cdot v'(x) + v(x) \cdot u'(x)$

Quotient Rule:

If $f(x) = \dfrac{u(x)}{v(x)}$, with $v(x) \neq 0$, then $f'(x) = \dfrac{v(x) \cdot u'(x) - u(x) \cdot v'(x)}{[v(x)]^2}$

Questions: 3, 7, 16, 22, 41

Pages: 596 - 597

$\boxed{3}$ find the derivative and simplify

$$f(x) = (x^{12} + 3x^4 + 4)(2x^3 - 1)$$

$$f'(x) = (x^{12} + 3x^4 + 4) \cdot (2(3x^2)) + (2x^3 - 1) \cdot (12x^{11} + 3 \cdot 4x^3)$$

$$= (x^{12} + 3x^4 + 4)(6x^2) + (2x^3 - 1)(12x^{11} + 12x^3)$$

$$= 6x^2 \cdot x^{12} + 6x^2 \cdot 3x^4 + 6x^2 \cdot 4 + 2x^3 \cdot 12x^{11} + 2x^3 \cdot 12x^3$$
$$- 1 \cdot 12x^{11} - 1 \cdot 12x^3$$

$$= 6x^{14} + 18x^6 + 24x^2 + 24x^{14} + 24x^6$$
$$- 12x^{11} - 12x^3$$

$$f'(x) = 30x^{14} - 12x^{11} + 42x^6 - 12x^3 + 24x^2.$$

-1-

$\boxed{7}$ find the derivative, but do not simplify.

$$y = (x^2+x+1)(\sqrt[3]{x} - 2\sqrt{x} + 5)$$
$$= (x^2+x+1)(x^{\frac{1}{3}} - 2x^{\frac{1}{2}} + 5)$$
$$y' = (x^2+x+1)\cdot\left(\tfrac{1}{3}x^{\frac{1}{3}-1} - \cancel{2}\cdot\tfrac{1}{\cancel{2}}x^{\frac{1}{2}-1} + 0\right)$$
$$+ (x^{\frac{1}{3}} - 2x^{\frac{1}{2}} + 5)\cdot(2x+1)$$
$$y' = (x^2+x+1)\cdot\left(\tfrac{1}{3}x^{\frac{-2}{3}} - x^{\frac{-1}{2}}\right) + (x^{\frac{1}{3}} - 2x^{\frac{1}{2}} + 5)(2x+1)$$
$$y' = (x^2+x+1)\cdot\left(\frac{1}{3x^{2/3}} - \frac{1}{x^{\frac{1}{2}}}\right) + (x^{\frac{1}{3}} - 2x^{\frac{1}{2}} + 5)\cdot(2x+1)$$

✓ $\boxed{16}$ find $\frac{dy}{dx}$ for $y = 200x - \frac{100x}{3x+1}$ and simplify.

$$y' = \frac{dy}{dx} = 200 - \left[\frac{(3x+1)\cdot 100 - 100x\cdot(3)}{(3x+1)^2}\right]$$
$$\frac{dy}{dx} = 200 - \left[\frac{\cancel{300x} + 100 - \cancel{300x}}{(3x+1)^2}\right]$$
$$\frac{dy}{dx} = 200 - \frac{100}{(3x+1)^2}$$

2

$\boxed{22}$ at the indicated point, find

a) the slope of the tangent Line, and

b) the instantaneous rate of change of the function.

$$y = \frac{x^2+1}{x+3}, \text{ at } (2,1)$$

a) the slope of the tangent at $(2,1) = y'\Big|_{(2,1)}$

$$y' = \frac{(x+3)\cdot(2x) - (x^2+1)\cdot(1)}{(x+3)^2}$$

$$y'\Big|_{(2,1)} = \frac{(2+3)(2)(2) - ((2)^2+1)}{(2+3)^2} = \frac{5\cdot2\cdot2 - 5}{(5)^2} = \frac{20-5}{25}$$

$$= \frac{15}{25} = \frac{3}{5}$$

So the slope of the tangent at $(2,1) = \frac{3}{5}$

b) the instantaneous rate of change at $(2,1) = y'\Big|_{x=2}$

$$= \frac{3}{5}$$

3

$\boxed{22}$ at the indicated point, find

a) the slope of the tangent line,

b) the instantaneous rate of change of the function

$$y = \frac{x^2 - 4x}{x^2 + 2x} \quad \text{at } (2, -\tfrac{1}{2})$$

the slope of the tangent at $(2, -\frac{1}{2}) = y'\Big|_{(2,-\frac{1}{2})}$

$$y' = \frac{(x^2+2x)\cdot(2x-4) - (x^2-4x)\cdot(2x+2)}{(x^2+2x)^2}$$

$$y'\Big|_{(2,-\frac{1}{2})} = \frac{((2)^2 + 2(2))\cdot(2(2)-4) - ((2)^2-4(2))\cdot(2(2)+2)}{((2)^2+2(2))^2}$$

$$= \frac{(4+4)(4-4) - (4-8)(4+2)}{(4+4)^2} = \frac{0-(-4)(6)}{(8)^2}$$

$$= +\frac{24}{64} = \frac{3}{8}$$

the ~~instant~~ slope of the tangent at $(2, -\frac{1}{2}) = \frac{3}{8}$

the instantaneous rate of change $= y'\Big|_{(2,-\frac{1}{2})} = \frac{3}{8}$

u

41 Suppose the revenue (in dollars) from the sale of x units of a product is given by:-

$$R(x) = \frac{60x^2 + 74x}{2x + 2}$$

Find the marginal revenue when 49 units are sold. Interpret your result.

marginal revenue $= R'(x)$

$$R'(x) = \frac{(2x+2)\cdot(60\cdot 2x + 74) - (60x^2 + 74x)\cdot(2)}{(2x+2)^2}$$

$$R'(49) = \frac{(2\cdot 49 + 2)\cdot(120\cdot 49 + 74) - (60\cdot(49)^2 + 74\cdot 49)(2)}{(2\cdot 49 + 2)^2}$$

$$= \frac{100 \cdot (5954) - (147686)(2)}{(100)^2}$$

$$= \frac{595400 - 295372}{10000} = \frac{300028}{10000} \simeq \$30$$

The expected revenue from the sale of the next unit (50^{th}) is about $30.

5

9.7 Using Derivative Formulas.

Questions: 7, 17, 20, 30, 40

Pages: 610

Find the derivatives of the following functions.

Simplify and express the answer using positive exponents only.

$\boxed{7}$ $y = (x^2 - 2)(x+4)$

$$y' = (x^2-2)\cdot(1) + (x+4)\cdot(2x)$$

$$= x^2 - 2 + 2x^2 + 8x = 3x^2 + 8x - 2$$

$\boxed{17}$ $y = \dfrac{(x^2-4)^3}{(x^2+1)}$

$$y' = \frac{(x^2+1) \cdot 3(x^2-4)^2 \cdot (2x) \mp (x^2-4)^3 \cdot (2x)}{(x^2+1)^2}$$

$$y' = \frac{6x(x^2+1)(x^2-4)^2 - 2x(x^2-4)^3}{(x^2+1)^2}$$

6

$\boxed{20}$ $y = [(4-x^2)(x^2+5x)]^4$

$y' = 4[(4-x^2)(x^2+5x)]^3[(4-x^2).(2x+5)+(x^2+5x).(-2x)]$

$= 4[(4-x^2)(x^2+5x)]^3[(4-x^2)(2x+5) - 2x(x^2+5x)]$

$= 4[4x^2+20x-x^4-5x^3]^3[8x+20-2x^3-5x^2-2x^3-10x^2]$

$= 4[-x^4-5x^3+4x^2+20x]^3[-4x^3-15x^2+8x+20]$

$\boxed{30}$ $y = 3x\sqrt[3]{4x^4+3} = 3x(4x^4+3)^{\frac{1}{3}}$

$y' = \cancel{3}x . \frac{1}{\cancel{3}}(4x^4+3)^{\frac{1}{3}-1} . (4.4x^3) + (4x^4+3)^{\frac{1}{3}} . 3$

$y' = x(4x^4+3)^{\frac{-2}{3}} \cdot 16x^3 + 3(4x^4+3)^{\frac{1}{3}}$

$= \frac{16x^4}{(4x^4+3)^{\frac{2}{3}}} + 3(4x^4+3)^{\frac{1}{3}}$

$y' = \frac{16x^4+3(4x^4+3)}{(4x^4+3)^{2/3}}$

7

40 Suppose that the demand function for q units of an appliance priced at \$ p per units is given by:

$$p = \frac{400(q+1)}{(q+2)^2}$$

Find the rate of change of price with respect to the number of appliances.

rate of change of price with respect to $x = \frac{dp}{dq}$

$$\frac{dp}{dq} = \frac{(q+2)^2 \cdot 400 - 400(q+1) \cdot 2(q+2)}{[(q+2)^2]^2}$$

$$\frac{dp}{dq} = \frac{400(q+2)^2 - 800(q+1)(q+2)}{[q+2]^4}$$

$$\frac{dp}{dq} = \frac{\cancel{(q+2)}\,[400(q+2) - 800(q+1)]}{(q+2)^{\cancel{4}\,3}}$$

$$\frac{dp}{dq} = \frac{400q + 8\cancel{00} - 800q - 8\cancel{00}}{(q+2)^3}$$

$$\frac{dp}{dq} = \frac{-400q}{(q+2)^3}.$$

8

9.6 The Chain Rule and the Power Rule

Questions: 10, 16, 23, 32, 41, 42

pages: 603 - 604

Differentiate the following functions:-

$\boxed{10}$ $p(q) = 4(3q^2-1)^4 - 13q$

$$\frac{dp}{dq} = 4 \cdot 4(3q^2-1)^3 \cdot (3 \cdot 2q) - 13$$

$$= 96q(3q^2-1)^3 - 13$$

$\boxed{16}$ $y = \dfrac{1}{(3x^3+4x+1)^{3/2}} = (3x^3+4x+1)^{-\frac{2}{3}}$

$$y' = -\frac{2}{3}(3x^3+4x+1)^{-\frac{2}{3}-1} \cdot (3 \cdot 3x^2+4)$$

$$= \frac{-2}{3}(3x^3+4x+1)^{-\frac{5}{3}}(9x^2+4)$$

$$= \frac{-2(9x^2+4)}{3(3x^3+4x+1)^{5/3}}$$

$$y' = \frac{-18x^2-8}{3(3x^3+4x+1)^{5/3}}$$

9

23 At the indicated point, find:-

a) the slope of the tangent line,

b) the instantaneous rate of change of the function.

$y = (x^3 + 2x)^4$ at $x = 2$

the slope of the tangent at $x=2$ is $y'\big|_{x=2}$

$y' = 4(x^3+2x)^3\,(3x^2+2)$

$y'\big|_{x=2} = 4\left((2)^3+2(2)\right)^3\left(3(2)^2+2\right)$

$= 4(8+4)^3(12+2) = 4(1728)(14) = 96768$

the slope of the tangent at $x=2$ is 96768

the instantaneous rate of change at $x=2$ is 96768

10

$\boxed{32}$ $f(x) = 10 - (x^2 - 2x - 8)^2$

a) Find $f'(x)$.

b) Find x-values for which the slope of the tangent is 0.

c) Find points (x, y) for which the slope of the tangent is 0.

$f'(x) = -2(x^2 - 2x - 8)(2x - 2)$

X-values for which the slope of tangent is 0.

$-2(x^2 - 2x - 8)(2x - 2) = 0$

$-2(x-4)(x+2)(2x-2) = 0$

$x - 4 = 0$ or $x + 2 = 0$ or $2x - 2 = 0$

$x = 4$ $\quad x = -2$ $\quad x = 1$

X-values: $1, -2, 4$

Points $(1, f(1))$, $(-2, f(-2))$, $(4, f(4))$

$f(1) = 10 - (1^2 - 2(1) - 8)^2 = 10 - (1 - 2 - 8)^2 = 10 - 81 = -71$

$f(-2) = 10 - ((-2)^2 - 2(-2) - 8)^2 = 10 - (4 + 4 - 8)^2 = 10 - 0 = 10$

$f(4) = 10 - (4^2 - 2(4) - 8)^2 = 10 - (0) = 10$

Points: $(1, -71)$, $(-2, 10)$, $(4, 10)$

-14-

41 Suppose the weekly sales volume y (in thousands of units sold) depends on the price per unit (in dollars) of the product according to

$$y = 32(3p+1)^{-2/5}, \quad p > 0$$

a) What is the rate of change in sales volume (y) when the price (p) is \$21?

$$\frac{dy}{dp} = 32 \cdot \left(\frac{-2}{5}\right)(3p+1)^{-2/5-1} \cdot (3)$$

$$\frac{dy}{dp} = -\frac{192}{5}(3p+1)^{-\frac{7}{5}}$$

at the price of \$21 :- $\left.\frac{dy}{dp}\right|_{p=21} = -\frac{192}{5}\left(3(21)+1\right)^{-\frac{7}{5}}$

$$= -\frac{192}{5}(64)^{-\frac{7}{5}} = -\frac{192}{5} \cdot \frac{1}{(\sqrt[5]{64})^7}$$

$$= -\cancel{\frac{192}{5}} \cdot \cancel{\frac{1}{(2.3)}} = -\cancel{\frac{192}{5}} \cdot \cancel{\frac{1}{2.639}} = -0.1137$$

b) Interpret your answer to part (a)

If the price changes by \$1 (to \$22), the weekly sales volume will changes by approximately −0.114 thousand unit.

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42 A chain of auto service stations has found that its monthly sales volume y (in thousands of dollars) is related to the price p (in dollars) of an oil change according to

$$y = \frac{90}{\sqrt{p+5}} \quad , \quad p > 10$$

a) What is the rate of change of sales volume (y) when the price (p) is \$ 20 ?

$$\frac{dy}{dp} = 90 \cdot -\frac{1}{2} (p+5)^{-\frac{1}{2}-1} \cdot 1$$

$$= -45 (p+5)^{-3/2} = \frac{-45}{(p+5)^{3/2}}$$

$$\left.\frac{dy}{dp}\right|_{p=20} = \frac{-45}{(20+5)^{3/2}} = \frac{-45}{(\sqrt{25})^3} = \frac{-45}{(5)^3} = \frac{-45}{125}$$

$$= -0.36$$

b) interpret your answer to part (a)

If the price changes by \$1 (to \$21) , the monthly sales volum will changes by approximately -0.36 thousands of dollars.

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9.8 Higher - Order Derivatives.

Questions: 6, 18, 27, 28, 36

Pages: 615 - 616

$\boxed{6}$ find the second derivative.

$$y = 3x^2 - \sqrt[3]{x^2} = 2x^2 - x^{2/3}$$

$$y' = 2.2x - \frac{2}{3} x^{-\frac{1}{3}}$$

$$y'' = 4 \quad - \frac{2}{3} \cdot (-\frac{1}{3}) \, x^{-\frac{1}{3}-1}$$

$$= 4 \quad + \frac{2}{9} x^{-\frac{4}{3}} \quad = 4 + \frac{2}{9\sqrt[3]{x^4}}$$

$\boxed{18}$ find $y^{(4)}$ if $y = x^6 - 15x^3$

$$y' = 6x^5 - 15 \cdot 3x^2 = 6x^5 - 45x^2$$

$$y'' = 6.5x^4 - 45.2x \quad = 30x^4 - 90x$$

$$y''' = 30.4x^3 - 90 = 120x^3 - 90$$

$$y^{(4)} = 120.3x^2 \quad = 360 \, x^2$$

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$\boxed{27}$ find $f''(3)$ for $f(x) = x^3 - \frac{27}{x} = x^3 - 27x^{-1}$

$f'(x) = 3x^2 - 27\cdot(-1)x^{-2} = 3x^2 + 27x^{-2}$

$f''(x) = 3.2x + 27(-2)x^{-3} = 6x - 54x^{-2}$

$= 6x - \frac{54}{x^2}$

$f''(3) = 6(3) - \frac{54}{(3)^2} = 18 - \frac{54}{9} = 18 - 6 = \boxed{12}$

$\boxed{28}$ find $f''(-1)$ for $f(x) = \frac{x^2}{4} - \frac{4}{x^2} = \frac{x^2}{4} - 4x^{-2}$

$f'(x) = \frac{1}{4}\cdot 2x - 4(-2)x^{-3} = \frac{1}{2}x + 8x^{-3}$

$f''(x) = \frac{1}{2} + 8(-3)x^{-4} = \frac{1}{2} - 24x^{-4}$

$= \frac{1}{2} - \frac{24}{x^4}$

$f''(-1) = \frac{1}{2} - \frac{24}{(-1)^4} = \frac{1}{2} - 24 = \frac{1}{2} - \frac{48}{2} = \frac{-47}{2}$

$= \boxed{-23.5}$

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36 Suppose that the revenue (in dollars) from the sale of a product is given by

$$R = 70x - 0.5x^2 - 0.001x^3$$

where x is the number of units sold. How fast is the marginal revenue $\overline{MR}$ changing when $x = 100$?

$$R = 70x - 0.5x^2 - 0.001x^3$$

$$\overline{MR} = R' = 70 - 0.5(2x) - 0.001(3x^2)$$

fast of the marginal revenue when $x = 100$

is $(\overline{MR})'\Big|_{x=100} = R''\Big|_{x=100}$

$$R'' = -0.5(2) - 0.001(3.2x)$$

$$R''\Big|_{x=100} = -0.5(2) - 0.001(6(100))$$

$$= -1 - 0.001(600)$$

$$= -1 - 0.6 = \boxed{-1.6}$$

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