

49] The DeWitt company has found that the total rate of change of its average cost for a product is $\bar{C}' = \frac{1}{4} - \frac{100}{x^2}$.

where x is the number of units and cost is in dollars. The average cost of producing 20 units is \$40.

a) Find the average cost function.

$$\begin{aligned}\bar{C} &= \int \left(\frac{1}{4} - \frac{100}{x^2} \right) dx \\ &= \int \left(\frac{1}{4} - 100x^{-2} \right) dx \\ &= \frac{1}{4}x - \frac{100x^{-1}}{-1} + C \\ &= \frac{1}{4}x + \frac{100}{x} + C\end{aligned}$$

To find C , we have the average cost of producing 20 units is 40\$. ($\bar{C}(20) = 40$).

$$40 = \frac{1}{4}(20) + \frac{100}{20} + C$$

$$\rightarrow 40 = 5 + 5 + C$$

$$40 = \cancel{10} + C$$

$$-10$$

$$\rightarrow \boxed{C = 30}$$

$$\text{So, } \boxed{\bar{C} = \frac{1}{4}x + \frac{100}{x} + 30}$$