[34] If $f(x,y) = x^3 + \ln(xy-1)$, find:-

a) $\frac{\partial^2 f}{\partial x^2}$ → "f_{xx}"

$$\frac{\partial f}{\partial x} = 3x^2 + \frac{y}{xy-1}$$

$$\frac{\partial^2 f}{\partial x^2} = 6x + \frac{-y^2}{(xy-1)^2}$$

b) $\frac{\partial^2 f}{\partial y^2}$ "f_{yy}"

$$\rightarrow \frac{\partial f}{\partial y} = \frac{x}{xy-1}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{-x^2}{xy-1}$$

c) $\frac{\partial^2 f}{\partial x \partial y}$

$$\rightarrow \frac{\partial f}{\partial x} = 3x^2 + \frac{y}{xy-1}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{(xy-1)(1) - y(x)}{(xy-1)^2}$$

$$= \frac{xy-1-yx}{(xy-1)^2} = \frac{-1}{(xy-1)^2}$$

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