Methods of proof:

Defn:
* An integer n is even iff ther is an integer k such that $n = 2k$.

* An integer n is odd iff there is an integer k such that $n = 2k+1$

* An integer n is perfect squar iff there is an integer k such that $n = k^2$.

* ~~apositive integ~~ An integer k divides an integer n and we write $k \mid n$ iff there is an integer t such that $n = k \cdot t$
k is also called adivisor of n or n is amultiple of k.

* Apositive integer p is prime iff $p > 1$ and the only positive divisors of p are 1 and p.
if p is not prime then it is called Composite

Direct methed of proof.
to prove $p \to q$ is true. we assume p and prove q.
See the table.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Example:- Show that if n is even integer then n^2 is even

Proof:- p : n is even

q : n^2 is even

So we need to prove $p \to q$.

So suppose p is true that is suppose n is odd and we prove q that is n^2 is even.

So : suppose n is even $\Rightarrow \exists k \in \mathbb{Z}$; $n = 2k$

$$\Rightarrow n^2 = (2k)^2 = 4k^2 = 2(2k^2)$$
$$= 2k'$$

where $k' = 2k^2 \in \mathbb{Z}$.

Ex:- (th 1.6) Text.

If n is an odd integer then n^2 is odd.

using Direct Method.

Suppose n is odd $\Rightarrow \exists k \in \mathbb{Z}$; $n = 2k+1$

$$\Rightarrow n^2 = (2k+1)^2 = 4k^2 + 4k + 1$$
$$\Rightarrow n^2 = 2(2k^2 + 2k) + 1$$
$$\Rightarrow n^2 = 2k' + 1 \text{ where } k' = 2k^2 + 2k \in \mathbb{Z}.$$
$$\Rightarrow n^2 \text{ is odd.}$$

Contrapositive Method of proof.

Since $p \to q \equiv \sim q \to \sim p$

So to prove $p \to q$ we instead prove $\sim q \to \sim p$ using direct method.

that is we suppose $\sim q$ and prove $\sim p$.

Ex:- Show that if n^2 is even then n is even

p : n^2 is even

q : n is even

Our problem is to prove $p \to q$

Using contrapositive method

So we assume n is not even (i.e n is odd)

i.e $n = 2k+1$ for some $k \in \mathbb{Z}$.

and show n^2 is odd.

Now suppose n is odd

$\Rightarrow \exists k \in \mathbb{Z} ; n = 2k+1$

$$\Rightarrow n^2 = (2k+1)^2 = 4k^2+4k+1$$

$$= 2(2k^2+2k)+1$$

$$= 2k'+1$$

$\Rightarrow n^2$ is odd.

Contradiction Method of prove

To prove $p \to q$ using contradiction method we assume p and $\sim q$ and then reach a Contradiction see table

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Example:- show that

if n is even then n^2 is even.

Proof: by contradiction

Suppose $p \wedge \sim q$ that is suppose

n is even and n^2 is not even (i.e is odd)

$\Rightarrow \exists k \in \mathbb{Z}$ such that $n = 2k$

$\Rightarrow n^2 = (2k)^2 = 4k^2 = 2(2k^2) = 2k'$

$\Rightarrow n^2$ is even which contradict our assumption n^2 is not even.

Ex:- (Th 1.7 page 25)

Suppose $m, b \in \mathbb{R}$, $m \neq 0$ and suppose $f(x) = mx + b$

prove that if $\underset{p}{x \neq y}$ then $\underset{q}{f(x) \neq f(y)}$

Proof:- using Contrapositive

Suppose $\sim q$ that is suppose $f(x) = f(y)$

and we need to prove $x = y$

So $f(x) = f(y) \implies mx + b = my + b$

$\implies mx = my$

$\implies x = y$ (done).

using Contradiction

suppose $x \neq y$ and $f(x) = f(y)$

$\implies x \neq y$ and $mx + b = my + b$

$\implies mx = my$

$\implies x = y$ $\times$. a contradiction