# 1.3 Quantifiers

Let $P(x)$ : $x+2=6$

then $P$ is not a proposition but if we replace $x$ by 1 then

$P(1)$ : $1+2=6$ is a proposition which is false

$P(4)$ : $4+2=6$ is a proposition which is true -

so $P(x)$ is called propositional function

A propositional function can be a proposition if we add a quantifier that is

for every value of $x \in A$ we have $P(x)$ written $(\forall x \in A)(P(x))$

and this will be true iff it is true for any value of $x \in A$. this quantifier is called Universal

A is called set of meenings

and will be false if it is false for at least one of the values of $x$ in A.

EX 1) $(\forall x \in \mathbb{N})(x+2=6)$ is false since $P(1)$ is false.

2) $(\forall x \in \mathbb{N})(x^2 \geq 0)$ is true since it is true for any $x \in \mathbb{N}$

Second quantifier Called Existential

$(\exists x \in A)(P(x))$ : there is $x \in A$ such that $P(x)$ or
for some $x \in A$ we have $P(x)$

This quantifier is true if $P(x)$ is true for at least one of the values in A.
and it is false if it is false for any value of $x \in A$

<u>Example:-</u> 1) $(\exists x \in \mathbb{N})(x+2=6)$ is true since
for $x=4$, $P(4)$: $4+2=6$ is true.
2) $\exists(x \in \mathbb{N})(2x=7)$ is false since
no value or all values of $x \in \mathbb{N}$ $2x=7$ is false.

<u>Ex:</u> which of the following is true and which is false

a) $(\forall x \in \mathbb{R})(x^2>0)$

b) $(\exists x \in \mathbb{R})(2x=5)$

c) all students in math 243, first semester 2020 are male

d) Some student in math 243 born is november.

e) $(\exists x \in \mathbb{Q})(x^2=2)$

Negation of quantifiers:

$$\sim\left((\forall x)(P(x))\right) \equiv (\exists x)(\sim P(x)).$$

$$\sim\left((\exists x)(P(x))\right) \equiv (\forall x)(\sim P(x)).$$

Ex:- write the negation of the statements in the previous example.

Ans. a) $\sim\left((\forall x\in\mathbb{R})(x^2>0)\right) \equiv (\exists x\in\mathbb{R})(x^2\leq 0).$

b) $\sim\left((\exists x\in\mathbb{R})(2x=5)\right) \equiv (\forall x\in\mathbb{R})(2x\neq 5).$

c) the negation is
Some students in math 243, first semester 2020 are not male.

the ngation is

d) all students in math 243 not born in november

e) the negation is

$(\forall x\in\mathbb{Q})(x^2\neq 2).$

We may have more than quantifier in the proposition as follows.

a) $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x+y=5)$ which is true.

b) $(\exists x \in \mathbb{R})(\forall y \in \mathbb{R})(x+y=5)$ which is false

c) $(\exists x \in \mathbb{R})(\exists y \in \mathbb{R})(x+y=10)$ which is true

d) $(\forall y \in \mathbb{R})(\forall x \in \mathbb{R})(xy>0)$ which is false

the negation of the above statements

a) negation: $(\exists x \in \mathbb{R})(\forall y \in \mathbb{R})(x+y \neq 5)$ which is false

b) negation $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x+y \neq 5)$ which is true.

c) negation: $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(x+y \neq 10)$ which is ~~true~~ false

d) negation: $(\exists y \in \mathbb{R})(\exists x \in \mathbb{R})(xy \leq 0)$ which is true.