Chapter 2
Sets

Defⁿ:- A set is a collection of well defined objects. Sets are denoted by capital letters $A, B, \dots$ and objects by small letters as $a, b, \dots$ and placed between $\{ \}$ brackets.

Examples:

1. $A = \{1, 2, 5\}$
2. $B = \{2, 4, 6, 8, \dots, 100\}$
3. $C = \{1, 3, 5, 7, \dots\}$
4. $D = \{x / x$ is in math 243, fall 2020$\}$
5. $[-1, 2) = \{x \in \mathbb{R} \mid -1 \leq x < 2\}$
6. $(2, \infty) = \{x \in \mathbb{R} \mid x > 2$
7. $\{x \in \mathbb{R} \mid x^2 = -1\} = \phi = \{ \}$ called phi

Defⁿ:- if A is a set and x is a member of A then we write $x \in A$,
if y is not a member of A then we write $y \notin A$.

Defⁿ:- if A, B are sets then A, B are equal and we write $A = B$ iff they contain same elements.

Defⁿ:- If A, B are sets then

1) A is a subset of B and we write $A \subseteq B$ iff $x \in A \Rightarrow x \in B$.

that is every element in A is in B.

Consequently

$A \not\subseteq B$ iff $\exists x \in A \ni x \notin B$.

2) A is a proper subset of A iff

$A \subseteq B$ and $A \neq B$.

that is every element in A is in B and some element of B is not in A.

Defⁿ:- the number of elements of A is called the cardinality of A and is denoted by $|A|$.

proposition:- Let A be a set, then $\phi \subseteq A$ and $A \subseteq A$.

proof:- follows from the definition

Example:- Find all subsets of $A = \{1,2\}$

Ans. All subsets of A are $\phi, \{1\}, \{2\}, \{1,2\}$.

Defⁿ:- Let A be a set then the set of all subsets of A denoted by $P(A)$.

Ex:- $A = \{1, 2, 3\}$ then

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

Theorem:- if $|A| = n$ then $|P(A)| = 2^n$.

proposition 2.4 (Text) Let A, B, C be sets. If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.

proof:- suppose $x \in A \Rightarrow x \in B$ since $A \subseteq B$

$\Rightarrow x \in C$ since $B \subseteq C$

U: universal set.

Defn:- Let A, B be sets, then

1- $A \cup B$ (A union B) $= \{x \in U \mid x \in A$ or $x \in B\}$

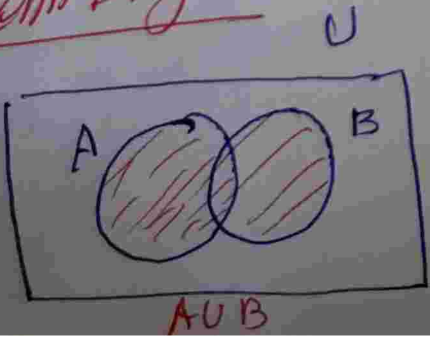
2- $A \cap B$ (A intersection B) $= \{x \in U \mid x \in A$ and $x \in B\}$

3- $A - B$ (A minus B) $= \{x \in A \mid x \notin B\}$

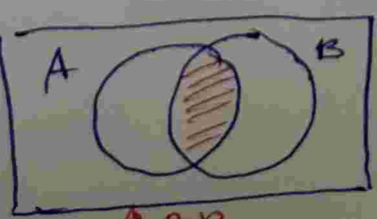
4- $\overline{A}$ (A complement) $= \{x \in U \mid x \notin A\}$.

the above can be illustrated by diagram

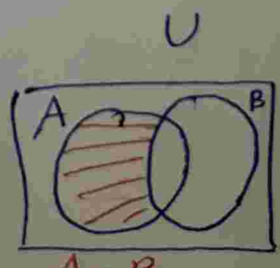
Venn Diagram



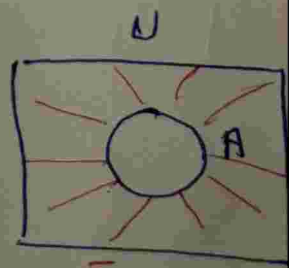
$A \cup B$



$A \cap B$



$A - B$



$\overline{A}$

Let $A = \{1, 2, 5, 7\}$, $B = \{2, 5, 6, 7\}$
$U = \{1, 2, \ldots, 10\}$

then $A \cup B = \{1, 2, 5, 7, 6\}$
$A \cap B = \{2, 5, 7\}$
$A - B = \{1\}$
$\overline{A} = \{3, 4, 6, 8, 9, 10\}$.

Example:- Let $\mathbb{R}$ be real numbers, $A = [1, 3)$, $B = (2, 4]$
then $A \cup B = [1, 4]$, $A \cap B = (2, 3)$
$A - B = [1, 2]$.
$\overline{A} = (-\infty, 1] \cup [3, \infty)$.

Defn:- A and B are disjoint iff $A \cap B = \phi$.

proposition 2.5 (Text) Let A, B, C be sets then
a) $\phi \cap A = \phi$, $\phi \cup A = A$
b) $A \cap B \subseteq A$ c) $A \subseteq A \cup B$.
d) $A \cup B = B \cup A$ and $A \cap B = B \cap A$.
e) $A \cup (B \cup C) = (A \cup B) \cup C$ and $A \cap (B \cap C) = (A \cap B) \cap C$
f) $A \cup A = A = A \cap A$.
g) if $A \subseteq B$ then $A \cup C \subseteq B \cup C$ and $A \cap B \subseteq A \cap C$